Lissajous figures drawn on phosphorescent foil by Blackburn pendulum

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Lissajous figures are easily demonstrated with two function generators and an oscilloscope with a XY mode. Applying sinusoidal signals from signal generators on the X and Y input of an analogue oscilloscope with a cathode-ray tube, the signals are first amplified and then applied on two mutually perpendicular deflection systems. The electron beam in the tube is subjected to two perpendicular deflections before landing on a fluorescent screen and, when the frequencies of both sinusoidal signals are in a ratio of small natural numbers, it draws Lissajous figures. In digital storage oscilloscopes, both the X and Y signals are measured with a fast analogue-to-digital converter and stored in a digital form in the memory. Then, the Lissajous figures are a product of a plotting program running in the oscilloscope. This is very similar to a pure simulation of the perpendicular oscillations performed in mathematical software or in applets. Since the analogue oscilloscopes have been gradually replaced with digital ones, other experiments showing perpendicular oscillations may be preferred.

The Y-suspended double pendulum, also known as a Blackburn pendulum, belongs to the classical and best known examples of perpendicular mechanical oscillations [1]. A funnel-shaped bob of the pendulum, hung on a double suspension, is filled with sand, flour or other loose material. Whilst the pendulum is oscillating, the sand runs out from the bob down onto black paper, tracing the trajectory of the motion. Due to the different lengths of suspension in mutually perpendicular directions, the pendulum swings with two independent frequencies simultaneously.

In our experiment, we used a luminescent technique similar to that of an analogue oscilloscope to visualise the trajectory of the Blackburn pendulum. Since the oscillation period of the pendulum is in the order of a second, a phosphorescent material (i.e. with long-lasting photoluminescence) must be used. Strong phosphorescence is a feature of pigments based e.g. on strontium aluminate doped with rare earth element. Such pigments are commonly used in photoluminescent safety signs. They are also added into foils with larger dimensions, such as $40\,\mathrm{cm} \times 60\,\mathrm{cm}$, which is the format just suitable for our experiment. The foils require the excitation with a light of a wavelength shorter than that of a green foil luminescence – our foil is especially sensitive to violet and ultraviolet light. A widely available class 3R violet laser pointer with a wavelength of 405 nm and a light power below 5 mW produces an intensive and long-lasting phosphorescence of the foil.

The schema of our Blackburn pendulum is shown in figure 1. The laser pointer is fixed into the aluminium cylinder which is hung on the double suspension as in the case of the standard Blackburn pendulum. The ratio of the frequencies of the perpendicular oscillations may be varied by changing the ratio of the lengths on the double suspension with a movable button. The phosphorescent foil is placed underneath.

Typical photographs of Lissajous figures taken in the dark are shown in figure 2. The figures strongly resemble the oscillogram produced with an analogue cathode-ray oscilloscope. E.g., the higher intensity of the luminescence in the extreme positions shows that the oscillator spends more time at the full amplitude of the oscillations. A sinusoidal temporal dependence in each spatial coordinate may be also demonstrated when the pendulum is swung in one direction only and the foil is moved in a perpendicular direction at a constant speed (see figure 3).

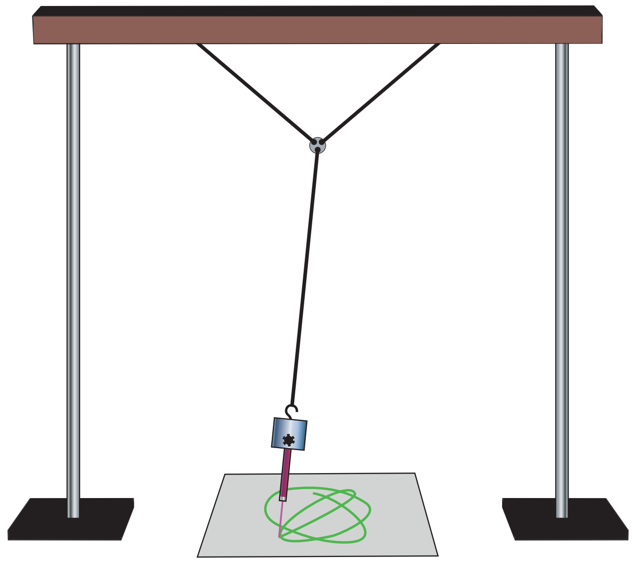

Figure 1: Schema of the Blackburn pendulum with a violet laser. A phosphorescent foil is placed underneath.

Concluding remark

Our violet laser pointer is intensive enough to produce bright figures which can be observed in a classroom without a room blackout. However, lasers in class 3R (power output below 5 mW for wavelength in 400 –700 nm) might be potentially hazardous for the eyes [2, 3]. The laser in this class (3R) should therefore be treated with care, looking directly into the laser beam must be avoided. The experiment will be safer when the distance of the laser pointer from the phosphorescent foil is small and any surfaces around the setup with a specular reflection are avoided. Since laser pointers with higher power outputs (class 3B) are freely available on the market and the specified power can generally differ from the true value, it is recommended for a class experiment to buy a laser pointer with the output power certificate or to check the output power first with a light power meter.

For a higher safety level, a laser with an output power up to 1 mW (class 2) can be used, but in this case as a minimum a partial blackout will probably be needed. Some ambient light present in the room is still helpful, since it masks previous figures and enables repeated use of a foil. In total darkness, the persistent phosphorescence of the foil can still be observed for many hours after the exposure (see figure 4).

References

[1] Whitaker R J 1991 A Note on the Blackburn Pendulum *American Journal of Physics* **59** 330-3.

[2] Träger F (Ed.) 2007 *Springer Handbook of Lasers and Optics* Springer Science+Business Media, New York p 1265.

[3] IEC Standard 60825-1 Ed. 2 2007 *Safety of laser products – Part 1: Equipment classification and requirements*.

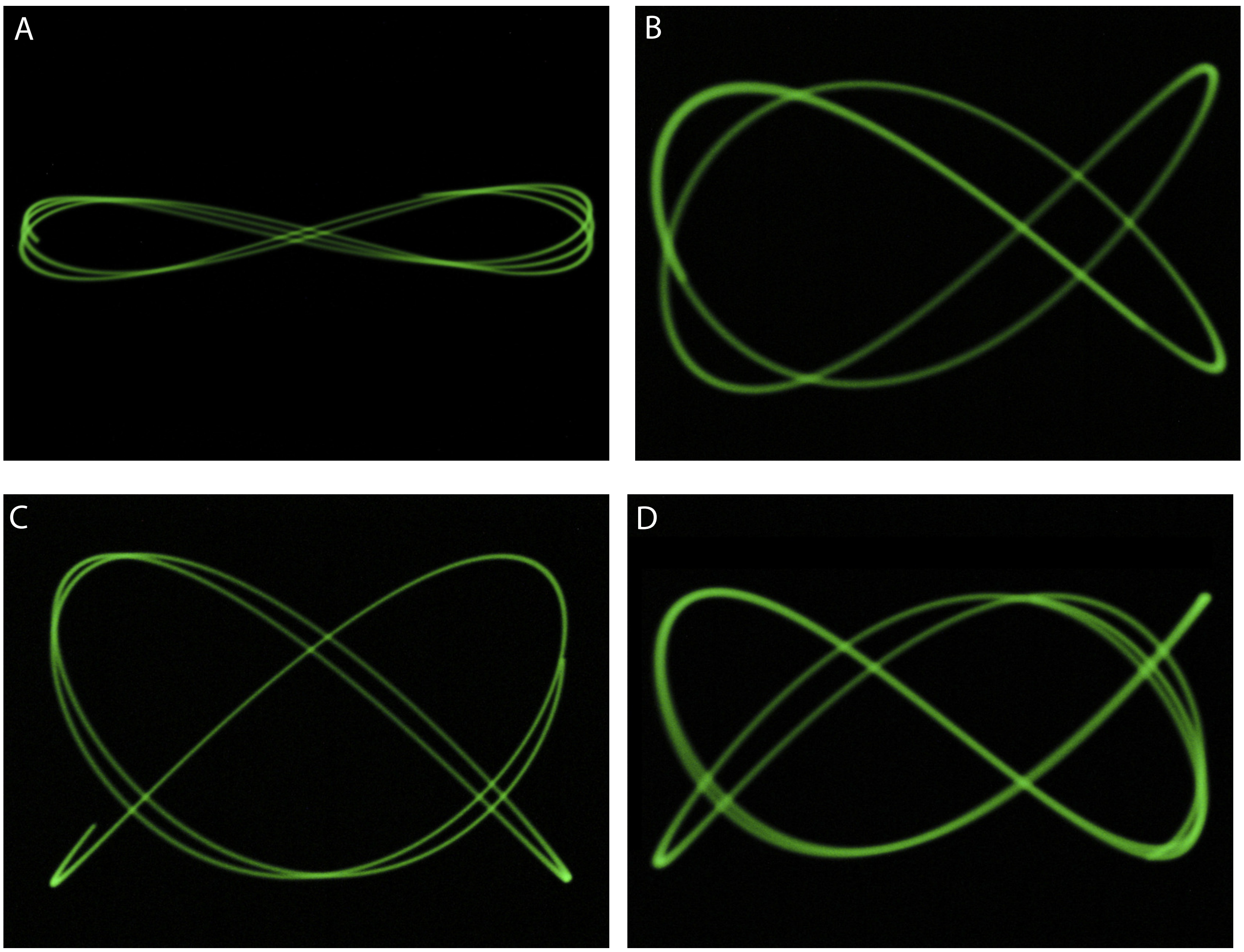

Figure 2: Lissajous figures obtained with our setup in a dark. The ratios of the frequencies were A) 1:2, B) 2:3, C) 3:4 and D) 3:5. The ratios of the lengths in the mutually perpendicular directions were 1:4, 4:9, 9:16 and 9:25, respectively.

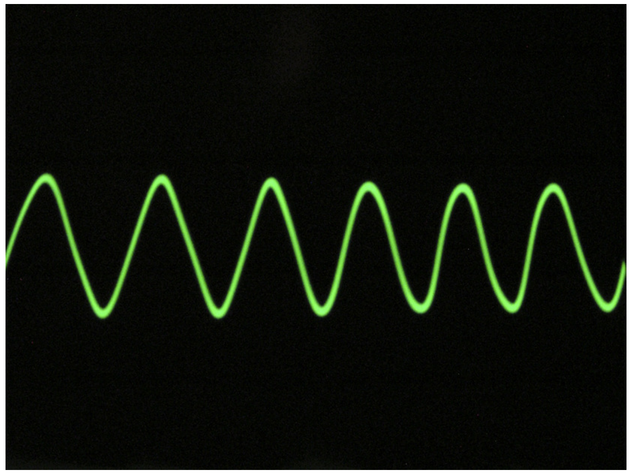

Figure 3: Demonstration of a sinusoidal temporal dependence of a 1D oscillation. The pendulum is swung in one direction only and the foil underneath is moved in a perpendicular direction at a constant speed.

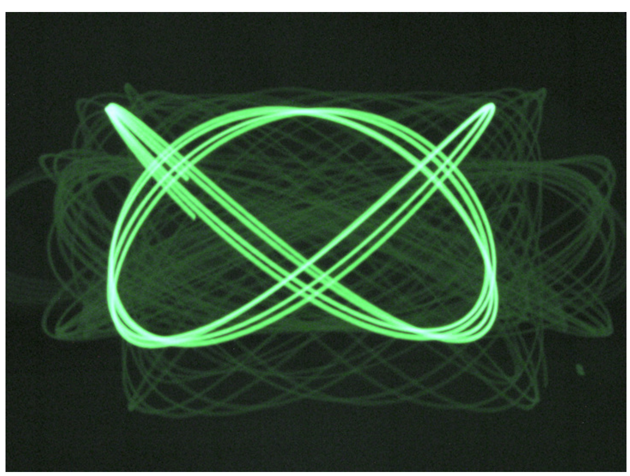

Figure 4: Phosporescent foil with several succesive exposures.