Comparison of two variants of integration of the problem from the example 1.115 from the webpage
https://is.muni.cz/do/rect/el/estud/prif/js17/pocetni_praktikum1/web/ch01_s04.html
(explanatory physical description - see Example 1.114) :

A charge element of the one-dimensional bar can be written as

$$\mathrm{d}Q = \tau\,\mathrm{d}x, \tag{1}$$

where τ is the homogeneous one-dimensional (longitudinal) charge density and $\mathrm{d}x$ is the longitudinal length-element of the bar, oriented along the x axis. The electrostatic potential at the point P at a perpendicular distance D from the end of the bar, generated by this charge element, can be written as

$$\mathrm{d}\phi = \frac{1}{4\pi\epsilon_0}\frac{\mathrm{d}Q}{r} \equiv \frac{1}{4\pi\epsilon_0}\frac{\tau\,\mathrm{d}x}{\sqrt{D^2+x^2}}, \tag{2}$$

the overall potential at this point will be

$$\phi = \frac{\tau}{4\pi\epsilon_0}\int\limits_0^L \frac{\mathrm{d}x}{\sqrt{D^2+x^2}}. \tag{3}$$

By the simple substitution $x = yD$, $\mathrm{d}x = \mathrm{d}y\,D$, we get

$$\phi = \frac{\tau}{4\pi\epsilon_0}\int\limits_0^{L/D} \frac{\mathrm{d}y}{\sqrt{1+y^2}}, \tag{4}$$

and by the subsequent substitution $y = \tan z$, $\mathrm{d}y = \mathrm{d}z/\cos^2 z$, we get

$$\phi = \frac{\tau}{4\pi\epsilon_0}\int\limits_0^{\mathrm{atan}\,(L/D)} \frac{\mathrm{d}z}{\cos z}. \tag{5}$$

For the following procedure, we can select one of the following two options:

1. by extending the fraction in the integrand by the expression $\cos z$, we get

$$\phi = \frac{\tau}{4\pi\epsilon_0}\int\limits_0^{\mathrm{atan}\,(L/D)} \frac{\cos z\,\mathrm{d}z}{\cos^2 z} = \frac{\tau}{4\pi\epsilon_0}\int\limits_0^{\mathrm{atan}\,(L/D)} \frac{\cos z\,\mathrm{d}z}{1-\sin^2 z}, \tag{6}$$

using substitution $\sin z = t$, $\cos z\,\mathrm{d}z = \mathrm{d}t$, we obtain

$$\begin{aligned}\phi &= \frac{\tau}{4\pi\epsilon_0}\int\limits_0^{\sin[\mathrm{atan}\,(L/D)]} \frac{\mathrm{d}t}{(1-t)(1+t)} = \frac{\tau}{8\pi\epsilon_0}\int\limits_0^{\sin[\mathrm{atan}\,(L/D)]} \left(\frac{1}{1-t}+\frac{1}{1+t}\right)\mathrm{d}t \\ &= \frac{\tau}{8\pi\epsilon_0}\ln\left|\frac{1+t}{1-t}\right|\Bigg|_0^{\sin[\mathrm{atan}\,(L/D)]},\end{aligned} \tag{7}$$

where, using a simple geometric consideration, we can rewrite the upper integration limit as

$$\sin\,[\mathrm{atan}\,(L/D)] = \frac{L}{\sqrt{L^2+D^2}}. \tag{8}$$

By setting the limits expressed in this way, we get

$$
\begin{aligned}
\phi &= \frac{\tau}{8\pi\epsilon_0}\ln\left|\frac{1+\dfrac{L}{\sqrt{L^2+D^2}}}{1-\dfrac{L}{\sqrt{L^2+D^2}}}\right| = \frac{\tau}{8\pi\epsilon_0}\ln\left|\frac{\sqrt{L^2+D^2}+L}{\sqrt{L^2+D^2}-L}\right| = \frac{\tau}{8\pi\epsilon_0}\ln\left(\frac{\sqrt{L^2+D^2}+L}{D}\right)^2 \\
&= \frac{\tau}{4\pi\epsilon_0}\ln\left(\frac{\sqrt{L^2+D^2}+L}{D}\right),
\end{aligned} \tag{9}
$$

where we get the third expression by extending the fraction in the logarithm argument in the second expression by its numerator.

2. By the so-called universal substitution $\tan(z/2) = t$, from which, using simple geometric considerations with trigonometric expressions for double arguments, we derive

$$
\sin z = \frac{2t}{1+t^2}, \quad \cos z = \frac{1-t^2}{1+t^2}, \quad \mathrm{d}z = \frac{2\,\mathrm{d}t}{1+t^2}. \tag{10}
$$

After substituting, Equation (5) becomes

$$
\phi = \frac{\tau}{2\pi\epsilon_0}\int\limits_0^{\tan\left[\frac{\operatorname{atan}(L/D)}{2}\right]} \frac{\mathrm{d}t}{(1-t)(1+t)} = \frac{\tau}{4\pi\epsilon_0}\ln\left|\frac{1+t}{1-t}\right|\Bigg|_0^{\tan\left[\frac{\operatorname{atan}(L/D)}{2}\right]}, \tag{11}
$$

where, however, the half-argument of the tangent in the upper limit represents a greater complication than the sine of a similar argument in the previous solution. We solve this by rewriting the tangent in the upper limit as sine/cosine, extending the cosine of the given expression and using trigonometric relations for the double argument of sine and the square of cosine,

$$
\tan\left[\frac{\operatorname{atan}(L/D)}{2}\right] = \frac{\sin\left[\dfrac{\operatorname{atan}(L/D)}{2}\right]\cos\left[\dfrac{\operatorname{atan}(L/D)}{2}\right]}{\cos\left[\dfrac{\operatorname{atan}(L/D)}{2}\right]\cos\left[\dfrac{\operatorname{atan}(L/D)}{2}\right]} = \frac{\sin\left[\operatorname{atan}(L/D)\right]}{1+\cos\left[\operatorname{atan}(L/D)\right]}. \tag{12}
$$

Substituting into Equation (11), we get the expression

$$
\phi = \frac{\tau}{4\pi\epsilon_0}\ln\left|\frac{1+\cos\left[\operatorname{atan}(L/D)\right]+\sin\left[\operatorname{atan}(L/D)\right]}{1+\cos\left[\operatorname{atan}(L/D)\right]-\sin\left[\operatorname{atan}(L/D)\right]}\right|, \tag{13}
$$

whose argument we extend using $1-\cos\left[\operatorname{atan}(L/D)\right]+\sin\left[\operatorname{atan}(L/D)\right]$. Using simple rearrangements, we then get

$$
\phi = \frac{\tau}{4\pi\epsilon_0}\ln\left|\frac{1+\sin\left[\operatorname{atan}(L/D)\right]}{\cos\left[\operatorname{atan}(L/D)\right]}\right|. \tag{14}
$$

Using Equation (8) and the analogous relation

$$
\cos\left[\operatorname{atan}(L/D)\right] = \frac{D}{\sqrt{L^2+D^2}}. \tag{15}
$$

we can rewrite Equation (14) as (we need not use the absolute value anymore, since the following argument of logarithm is always positive)

$$
\phi = \frac{\tau}{4\pi\epsilon_0}\ln\left(\frac{1+\dfrac{L}{\sqrt{L^2+D^2}}}{\dfrac{D}{\sqrt{L^2+D^2}}}\right) = \frac{\tau}{4\pi\epsilon_0}\ln\left(\frac{\sqrt{L^2+D^2}+L}{D}\right), \tag{16}
$$

we thus get the (expected) identical solution to Equation (9). However, the solution with use of the universal substitution is in this case obviously more laborious and time-consuming.