Omni-directional transformation-optics cloak made from lenses and glenses

TOMÁŠ TYC¹, STEPHEN OXBURGH², EUAN N. COWIE², GREGORY CHAPLAIN², GAVIN MACAULEY², CHRIS D. WHITE², AND JOHANNES COURTIAL²,*

¹Institute of Theoretical Physics and Astrophysics, Masaryk University, Kotlarska 2, 61137 Brno, Czech Republic
²School of Physics & Astronomy, College of Science & Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom
*Corresponding author: johannes.courtial@glasgow.ac.uk

We present a design for an omni-directional transformation-optics (TO) cloak comprising thin lenses and glenses (generalised thin lenses) [Chaplain et al., ...]. It should be possible to realise such devices in pixellated form. Our design is a piecewise non-affine generalisation of piecewise affine pixellated-transformation-optics devices [Oxburgh et al., Proc. SPIE 9193, 91931E (2014) and J. Opt. (in press, 2016)]. It is intended to be a step in the direction of TO devices made entirely from lenses, which should be readily realisable on large length scales and for a broad range of wavelengths.

1. INTRODUCTION

Transformation optics [1, 2] is the science of using a material structure to distort light-ray trajectories within the structure, thereby changing the apparent shape and/or size of any object inside it. The actual structure is said to be in physical space, the apparent structure as seen from the outside is called electromagnetic (EM) space. In the famous invisibility cloak [2], a physical-space void inside the structure and anything inside the void is made to appear infinitely small when viewed from outside the cloak (it is infinitely small in EM space), while any object behind the cloak is seen undistorted. This idea quickly took off, leading for example to different experimental realisations that use artificial metamaterial structures [3–7], natural crystals [8–10], and lenses and mirrors [11, 12]. The ideas of transformation optics have even been applied to other branches of physics, resulting for example in transformation thermodynamics [13, 14], acoustic cloaking [15], elastic cloaking [16], and seismic cloaking [17].

The original suggestion was to realise TO devices using metamaterials, engineered structures with subwavelength-size features that allow their optical properties to be controlled, but it was quickly realised that such structures that work for all visible light and at macroscopic length scales would be difficult to realise. The reasons include the immense practical difficulties of manufacturing a macroscopic, three-dimensional, spatially-varying, bespoke nano structure. There are also fundamental difficulties: ideal cloak structures have been built, but only on the scale of a few wavelengths (e.g. [7]), and the requirements and control of loss and bandwidth limitations that would allow significant size increases are daunting [18–20].

These difficulties led researchers to investigate alternative realisations that are much easier to fabricate and which work for all visible light and at macroscopic length scales, but at the cost of compromising performance. Approximations of the material properties have been shown to introduce visible imperfections [21–26]; a number of the simplified devices work only for light incident from a limited range of directions; and in all cases the cloaking is “ray-optical”, which means that these cloaks alter the phase of transmitted light.

Our own interest in transformation optics stems from our research into light-ray-direction-changing micro-structured sheets called telescope windows that can be combined into approximations to TO devices. Telescope windows [27] such as pairs of confocal microlens arrays [28, 29] in which pairs of microlenses — one from each array — form telescopes that act as the “pixels” of the sheets. It can be shown that the light-ray-direction changes that can be achieved in this way — pixellated generalised refraction — could lead to wave-optically forbidden light-ray fields if the sheets were not pixellated [27, 30]. The generalised laws of refraction that can be achieved in this way, albeit only in pixellated form, allow very general stigmatic imaging.

We recently defined a glens to be a planar interface that changes light-ray direction like an idealised thin lens, but generalised to have two independent focal lengths on the two sides of the lens [31]. If a glens is realised, approximately, in the form of a telescope window, then due to the pixellation the imaging is not stigmatic, but integral [32], and the approximate glens has
other imperfections such as the appearance of additional, and usually unwanted, images [33, 34]. On the plus side, telescope windows can be manufactured inexpensively on metre scales\(^1\), and they work for all visible light.

We recently investigated the imaging properties of glenses in the homogeneous limit [36] and showed that these are so general that structures of homogeneous glenses can form omni-directional transformation-optics devices [37, 38]. Realisations of such devices in terms of telescope windows have all the advantages and disadvantages of the telescope windows themselves, including integral imaging instead of stigmatic imaging, a limited field of view, and additional images. Additionally, they do not preserve the phase of transmitted light, and are therefore merely ray-optical. What sets them apart from other ray-optical transformation-optics devices is that they can be be built cheaply, on large length scales.

The structure of this paper is as follows. First, in section 2, we review the properties of glenses. In section 3 we introduce a physical-space structure of glenses, and the EM-space structure this represents. It contains a central square whose size differs in physical and EM space; in the theoretical limit when the size of the EM-space square is zero, the structure is a ray-optical cloak.

We then show that the structure indeed maps between physical and EM space as intended. Specifically, we show that any possible combination of surfaces that a ray traversing the cloak can encounter images every point back to itself. Assuming that the structure images as intended we construct, in section 4, the cardinal points of these glenses and show that the glenses link- corresponding corners of the outer and inner squares are actually lenses. Using the cardinal points we then show, in section 5, that the structure indeed images as intended, and that our earlier assumption is therefore true and our argument consistent. In section 6, we confirm these results using ray-tracing simulations of the cloak. Section 7 is a concluding discussion of our findings.

2. REVIEW OF GLENSES

Glenses have recently been defined to be planar interfaces that change the direction of transmitted light rays like ideal thin lenses that possess different focal lengths on the two sides of the interface [31]. Like thin lenses, glenses do not offset the position of transmitted light rays. The generalisation from a thin lens to a glens at first appears rather small, but it is this small generalisation that makes the resulting interfaces the most general imaging elements of their kind: it can be shown that glenses are the most general planar light-ray-direction-changing interfaces that image all of object space into all of image space and vice versa (one-to-one and onto) [39]. In fact, as curved light-ray-direction-changing interfaces can map object space one-to-one-and-onto into image space only trivially [40], glenses are the most general light-ray-direction-changing interfaces (of any shape) that image between all of object space and image space.

Like a thin lens, a glens has an optical axis that is perpendicular to the plane of the glens and on which all the principal points lie. Unlike a thin lens, the two sides of a glens are different, which is why it is important to identify the sides. In Ref. [31] this was done by placing on the optical axis an axis of a Cartesian coordinate system, with its origin in the glens plane, and labelling the two sides of the glens by the sign of this coordinate there. The corresponding axial coordinate was called \( a \). Light rays travelling on the side of the glens where \( a \) is positive were said to be travelling in positive space, those travelling on the other side in negative space.

Fig. 1 shows a diagrammatic representation of a glens, its optical axis, its cardinal points, and different principal rays. Following one of the conventions introduced in Ref. [31], the positive side of the glens is identified by a \('+'\) on that side of the line indicating the glens plane, the negative side is identified by a \('−'\). Three types of principal ray are shown: type 1 is parallel to the optical axis in negative space, and passes through the positive focal point, \( F^+ \), in positive space; type 2 passes through the negative focal point, \( F^- \), in negative space, and is parallel to the optical axis in positive space; and type 3, which travels in the direction of the glens’s nodal point, \( N \), and passes straight through the glens. The two focal points and the nodal point, together with the principal point, \( P \), which lies at the intersection between the glens plane and the optical axis, are the cardinal points of the glens.

A glens is fully characterised by its \( a \) axis and the positive and negative focal lengths, \( f^+ \) and \( f^- \), which are defined as the \( a \) coordinates of the corresponding focal points. A thin lens with focal length \( f \) is then a glens with \( f^- = -f \) and \( f^+ = f \). The \( a \) coordinate of the nodal point, \( N \), is defined as the nodal distance, \( n \), which is related to the focal lengths by the equation

\[ n = f^- + f^+. \]  

(1)

The light-ray-direction change in glenses is of a type that, unless accompanied by a ray offset, can lead to wave-optically forbidden light-ray fields [30]. The experimental realisation of an approximation of a glens [41], an example of a Gabor superlens [42], achieved the light-ray-direction change through transmission through micro-telescopes, which also add a (small) offset to the rays. The micro-telescopes can be seen as the pixels of the Gabor superlens, which is why we refer to it as a pixellated realisation of a glens.

\(^1\)Rowlux Illusion Film [35] has a closely related structure, is inexpensive and available on metre scales.
Fig. 2. Structure of a two-dimensional square cloak in physical space (solid cyan lines) and electromagnetic (EM) space (dotted red lines). Physical space is divided into 6 polygon-shaped regions, R₀ to R₅. Region R₀ is the outside of the cloak, in which physical space and electromagnetic space are identical; region R₅ is the inside of the cloak. Each straight line dividing two regions represents a glens; the glens separating regions Rᵢ and Rⱼ is called Gᵢⱼ. A few of the vertices of the regions are also marked. Three or more regions meet there; the vertex where regions Rᵢ, Rⱼ, Rₖ, ... meet is labelled Vᵢⱼₖ...

3. CLOAK STRUCTURE

Fig. 2 sketches a two-dimensional structure of glenses. (We will generalise this structure to three dimensions in due course.) It consists of four glenses on the sides of an outer square of side length L, four glenses on the sides of an inner square of side length a'L, and which shares its centre and orientation with the outer square, and four glenses linking corresponding corners of the outer and inner square.

The glenses divide physical space into polygonal regions, called R₀ (the outside of the device), R₁ to R₄, and R₅ (the inside of the inner square). Each glens separates two of these regions; the glens separating regions Rᵢ and Rⱼ is labelled Gᵢⱼ. The vertices of the polygonal regions, where 3 or more regions (Rᵢ, Rⱼ, Rₖ, ...) meet, are labelled Vᵢⱼₖ...

Fig. 2 also sketches the structure of the corresponding EM space. The EM-space equivalent of the outer square coincides with its physical-space counterpart; the EM-space equivalent of the inner square is a smaller square of side length a'L whose centre and orientation coincide with those of the other squares. The vertices of the EM-space polygons are labelled such that vertex Vᵢⱼₖ̃ is the point where the EM-space counterparts of regions Rᵢ, Rⱼ, Rₖ, ..., meet.

4. CONSTRUCTION OF THE CARDINAL POINTS

We now construct the cardinal points of all glenses in the cloak structure described in the previous section.

First we analyse the glenses of the outermost square. For symmetry reasons, they are all the same, symmetrically arranged, so it is enough to analyse one of them, namely the left glens, G₀₁. It images the point V₁₄₅ to V₁₄₅' and V₁₂₅ to V₁₂₅'. Its nodal point must therefore lie at the intersection of the straight lines V₁₄₅V₁₄₅' and V₁₂₅V₁₂₅', which is the centre of the cloak, marked N in Fig. 3(a). The same argument holds for all other outer surfaces.

We can calculate the other cardinal points as follows. Fig. 3(a)
shows two rays, marked ‘1’ and ‘2’, which are incident from the left in the direction of point $V'_{145}$. The left glens, $G_{01}$, redirects them such that they travel in the direction of $V_{145}$. Ray 1 is chosen such that it is initially parallel to the optical axis (which is perpendicular to the glens plane and passes through N), which means it (or its straight-line continuation) passes through the positive focal point, $F_{01}^+$, after redirection. $F_{01}^+$ also lies on the optical axis, which fully determines its position. Ray 2 is chosen such that it is parallel to the optical axis after redirection, which means that, before redirection, it (or its straight-line continuation) must have passed through the negative focal point, $F_{01}^-$. Like $F_{01}^+$, $F_{01}^-$ lies on the optical axis, which again fully determines its position.

Next, we analyse glens $G_{15}$, which lies on the left side of the inner square. We consider a light ray incident on the cloak along a straight line through N. Light ray 3 in Fig. 3(b) is such a ray. Because N is the nodal point of the left outermost surface, the ray passes straight through, intersecting the inner square at I$_1$. On the other side, after transmission through the rightmost surface $G_{03}$, it must continue along the same straight-line trajectory. But this passes through the nodal point of that surface also, so the ray must have passed straight through it, which means it must have intersected the right surface $G_{35}$ of the inner square at I$_2$. Within the inner square, the ray must have travelled from I$_1$ to I$_2$, which means that it must have travelled along its original straight-line trajectory there. Thus the inner surfaces have not deflected that ray, so it must pass through the nodal point of the inner surfaces. Repeating this argument for other rays through N leads to the result that N is the nodal point of the surfaces of the inner square also. This means that the optical axes of all the four glenses on the sides of the inner square pass through N.

Ray 4 in the same figure 3(b) allows construction of the object-sided focal point $F'_{23}$ of the glens $G_{15}$. The ray is constructed such that it passes first through the left sides of the outer and inner squares, then through the right sides of the same squares. From the above arguments it follows that all of those glenses share a common optic axis. Outside the cloak, it travels parallel to the common optic axis. By symmetry, inside the inner square it also travels parallel to the optic axis. When the ray hits the glens on the left side of the outer square, $G_{01}$, arriving from the negative side and travelling parallel to the optical axis, it gets redirected such that it subsequently passes through the positive focal point $F_{01}^+$. It then hits the glens on the left side of the inner square, $G_{15}$, from the negative side, which redirects it such that it is afterwards parallel to the optic axis. This means that it must have come from the direction of the positive focal point $F_{15}^+$ of $G_{15}$ coincides with that of the positive focal point $F_{01}^+$ of $G_{01}$. The location of the positive focal point $F_{15}^+$ can be constructed using the locations of $F_{15}^+$, N, and the relationship between the focal distances and the nodal distance, Eqn (1). The cardinal points of the glenses on the other surfaces of the inner square can be found by symmetry.

Before we proceed further, we notice a useful property of a system of glenses that will be used extensively below. In particular, to find the imaging properties of a particular glens, we may use even rays that actually do not pass through it but that would do so if the glens were extended beyond its actual size. This follows from the fact that the image of a given point created by the glens it determined uniquely by any portion of that glens, and does not change if the glens is extended.

We can now apply this principle to analyse glens $G_{14}$, which is representative of the diagonal glenses. To do that, we extend glens $G_{14}$ along the dashed diagonal line shown in Fig. 3(c), and also extend glens $G_{04}$ along the dashed horizontal line shown in the same figure. This way, ray 5 now intersects all three glenses $G_{01}$, $G_{14}$ and $G_{04}$. As this ray passes through N, which is the nodal point of glenses $G_{01}$ and $G_{04}$, it passes through them undeviated. But for the ray to continue along its original straight-line trajectory after transmission through all three glenses, it has to be undeviated by $G_{14}$ also, so the nodal point of $G_{14}$ must lie somewhere along the ray. The same argument applies if we rotate ray 5 slightly around N, which implies that N is the nodal point of $G_{14}$. This way we see that N is the nodal point of all glenses of the cloak. Further, N lies on (the continuation of) $G_{14}$, and so $G_{14}$ as well as the other diagonal glenses $G_{12}$, $G_{23}$ and $G_{34}$ are actually lenses. We also employ ray 6 that is normally incident on $G_{14}$ from the direction of $F_{01}^-$. That means that it must have been normally incident on $G_{01}$. After transmission through the cloak, it must continue along its original straight-line trajectory. For symmetry reasons, the ray must therefore be horizontal after transmission through $G_{14}$. But as the ray was normally incident on $G_{14}$, it must pass through the image-sided focal point $F'_{14}$ after transmission through it. The point $F'_{14}$ can therefore be constructed as the intersection between this ray and the optical axis of the lens $G_{14}$ shown as dashed-dotted line in 3(c).

5. PROOF OF IMAGING OF ALL POINTS BACK TO THEMSELVES

Having found the properties of all the glenses of the cloak, we also have to show that any spatial point will be imaged to itself by the cloak, no matter which possible combination of surfaces it may encounter upon traversing the cloak. We do this in two steps. First, we show that all combinations of glenses that a ray may encounter images every point. Second, we show that the image of every point coincides with the point itself.

The first step is easy: a glens images any point in object space into a corresponding point in image space [39]. Transmission through any other glenses simply re-images the image from the previous glens(es). As all surfaces in the cloak are glenses, any combination of these automatically images any point.

The second step is more complicated. We use the result from the first step, namely that any point is being imaged. This implies that all light rays that intersect at a point $Q$ (the object) before transmission through the cloak again intersect at a point $Q'$ (the image) after transmission. To find where this image position is, we only need to find the intersection of any two of these rays; all others then automatically intersect there also. We pick each of the rays such that it is a member of a family of light rays that is sufficiently general so that the object position becomes completely arbitrary, and we do this separately for any combination of glenses that may be encountered.

Fig. 4(a) investigates transmission through the glenses on the left and right sides of both the inner and outer squares, i.e.,
for the glens combination \((G_{01}, G_{14}, G_{35}, G_{03})\). We choose the first ray, marked ‘1’ in Fig. 4(a), to pass through \(N\), which is the nodal point of all glenses in the cloak and which therefore passes through the cloak undeviated. The four glenses under consideration have a common optical axis, and it is advantageous to choose the second ray (ray 2 in Fig. 4(a)) as the one that is initially parallel to this optical axis. This ray is redirected by \(G_{01}\) such that it passes through \(F_{01}^+\), whose position is identical to that of \(F_{15}^-\) (see section 4), and so it becomes parallel again to optical axis of \(G_{15}\) after passing through \(G_{15}\). The same happens in reverse when the ray continues through the glenses \(G_{35}\) and \(G_{03}\). This way, beyond the cloak both rays 1 and 2 continue along their original straight-line trajectories, which means that the image \(Q'\) of their intersection point \(Q\) coincides with \(Q\) itself. Moreover, ray 1 can be rotated around \(N\) while ray 2 can be shifted vertically, to move their intersection \(Q\) arbitrarily. We thus see that the combination of these four surfaces images as required.

Fig. 4(b) deals with the glens combination \((G_{01}, G_{14}, G_{34}, G_{03})\). As before, ray 1 passes through \(N\), so it is undeviated. We then choose ray 2 to be initially horizontal, and by exactly the same argument as was used in the previous section for ray 6 in Fig. 3(c) we find that beyond the cloak this ray continues undeviated. The rest of the argument goes the same way as in the previous case.

Fig. 4(c) treats the glens combination \((G_{01}, G_{14}, G_{04})\). Ray 1 passes through \(N\) again and is hence undeviated. We choose ray 2 to be initially parallel with the dashed-dotted diagonal line. To find its direction beyond \(G_{01}\), we use the fact that two initially parallel rays incident upon a glens continue beyond it such that their prolongations pass through common point \(I\) in the image focal plane. This image plane is parallel to \(G_{01}\) and contains the focal point \(F_{01}^+\), and is shown as a vertical dashed line in Fig. 4(c). The point \(I\) can be found as the intersection of the image plane with the ray parallel to ray 2 and passing through \(N\). Now consider the triangle with vertices \(I\) and \(F_{01}^-\) and angles \(45^\circ\) at these vertices (the lower shaded triangle in Fig. 4(c)). Its third vertex with angle \(90^\circ\) then coincides with the focal point \(F_{14}^-\) of glens \(G_{14}\), which follows from the argument in the previous section. Next we note that this triangle is identical to the triangle with the same angles and with vertices \(F_{14}^+\) and \(N\) (the second shaded triangle in the figure), so the distance of the point \(I\) from the nodal point \(N\) of the lens \(G_{14}\) is twice the focal length of \(G_{14}\). Consequently, the point \(I\) will be imaged to a point \(I'\) on the dashed-dotted diagonal at a distance of two focal lengths behind \(N\) (the 4f imaging). This ensures that the ray trajectory is mirror symmetric with respect to the plane of \(G_{14}\), and as the ray was incident along a normal to \(G_{14}\), it leaves along the same normal.

Finally, we use a very similar argument to demonstrate the equivalence of transmission through the lens \(G_{14}\) and through the combination of glenses \(G_{15}\) and \(G_{25}\). We will investigate two particular types of rays that can be made to intersect anywhere and show that the diagonal lens \(G_{14}\) changes both rays in the same way as the combination of \(G_{15}\) and \(G_{25}\). It will then follow that they image any point to the same position, which will in turn show that they redirect any ray in the same way. Fig. 5 shows the geometry. As usual, we pick ray 1 to pass through the common nodal point \(N\), so it passes through undeviated. As ray 2 we pick a ray that approaches from the point \(I\) defined above that is located two focal lengths in front of the lens \(G_{14}\). Lens \(G_{14}\) will simply redirect it such that it passes through the point \(I'\), as

---

**Fig. 4.** Imaging properties of the glens combinations encountered along different types of ray trajectories through the cloak.
To test and demonstrate our findings, we have programmed the positions of the vertices of all surfaces that form the cloak, and also their electromagnetic-space counterparts. We then derived imaging requirements from the vertex positions in the two spaces; for example, glens \( G_{15} \) must image vertex(145,202),(883,814) and the pair of glenses \( G_{15} \) and \( G_{14} \) is then redirected by glens \( G_{45} \) towards \( I' \). This completes the proof of equivalence of lens \( G_{14} \) and the pair of glenses \( G_{15} \) and \( G_{45} \).

We see that any spatial point is imaged to itself by any possible combination of glenses that a ray can penetrate when passing through the cloak. This shows that the device indeed works as a cloaking device. Although it cannot make an object disappear completely, it can make it look much smaller than it actually is.

6. RAY-TRACING SIMULATIONS

To test and demonstrate our findings, we have programmed the cloak outlined above into our custom raytracer Dr TIM [43, 44]. The capability to simulate light-ray transmission through glenses, and the capability to map between positive and negative space, is already part of Dr TIM [31]. This enabled us to visualise the view through the cloak.

Our simulations represent a number of physical effects incorrectly. First of all, the calculation of shadows is greatly simplified: surfaces are either shadow-throwing or not, and if there is a shadow-throwing surface in the straight line between a point on another surface and one of the point light sources then that shadow-throwing surface casts a shadow on that point on the other surface. This simple treatment does not correctly represent the effect of surfaces that change the direction of transmitted light rays. Transmission through the glenses neglects absorption [34] and diffraction effects associated with the realisation in the form of a Gabor superlens.

The cloak was programmed by defining the (physical-space) positions of the vertices of all surfaces that form the cloak, and also their electromagnetic-space counterparts. We then derived imaging requirements from the vertex positions in the two spaces; for example, glens \( G_{01} \) must image vertex position \( V_{145} \) in negative space to the position \( V_{145} \) in positive space (see Fig. 3(a)). Using the procedure described in A, Dr TIM then determine the glens parameters from these imaging requirements.

Fig. 6 shows that, within the limitations of our simulation, the cloak design works: the inner cube, and the sphere placed inside it, appear at reduced size, while any object behind the cloak is seen in the same direction as it would be without the cloak (but slightly dimmer, as all glens surfaces were made to be slightly absorbing in order to become visible in the simulations). Fig. 7 shows the cloak working from a different virtual camera position, consistent with the cloak’s omni-directionality. Fig. 7(b) shows an example of a cloak in which the inner cube and the sphere inside it appear reduced to a different apparent size, demonstrating that the reduction factor can, in principle, be chosen arbitrarily.

7. DISCUSSION AND CONCLUSIONS

Glenses are defined as idealised interfaces that change light-ray direction precisely as required, without offsetting light rays or introducing loss. This paper is about an omnidirectional cloak made from glenses (of which a few are lenses), and if the glenses work as defined then the cloak is perfect, as demonstrated by the simulations in the previous section.

However, Gabor superlenses — the only experimental realisation of pixellated glenses to date — suffer from imperfections, which would affect the functioning of any cloak built from them. One imperfection is the Gabor superlenses’ limited field of view, which translates into a limited field of view of the cloak, which is therefore not omnidirectional. Another imperfection is that not all transmitted light changes direction as required. Such light either leads to additional images (if it is allowed to pass through the cloak) or a reduced transmission coefficient (if it is absorbed). Note that the fraction of light that passes through such a cloak as desired gets smaller as the factor by which the central square (cube) appears shrunk increases, just like in the cloak made from homogeneous glenses [38]. A third imperfection is the limited quality of the image formed by a Gabor superlens, which is due to a combination of fundamental effects (diffraction, pixel visibility) and practical effects (aberrations of the simple lens design; dispersion; ...). This optical quality of the image formed by individual Gabor superlenses will need to improve significantly before imaging through combinations of such devices becomes experimentally palatable.

In section 2 it was pointed out that glenses perform light-ray direction changes that, unless accompanied by an offset, can result in wave-optically forbidden light-ray fields, which is why practical realisations of glenses need to offset the rays. This can be seen as a violation of Liouville’s theorem: any bundle of parallel rays incident on the cloak’s inner cube in electromagnetic space will be altered by the cloak such that, inside the inner cube (in physical space), the rays have the same direction but their distance has been stretched. In direction space, each ray is unchanged, but in position space the volume of the beam has been magnified, resulting in a change in phase-space volume. Upon transmission through the remainder of the cloak, the phase-space volume gets restored to its original size. Such a light beam would, in the simplest case, enter the inner cube by passing first through the glens at a face of the outer cube and then the glens at the corresponding face of the inner cube, but the combination these two glenses, which share a nodal point, is precisely the glens telescope discussed in Ref. [31], where it was already pointed out that such a device violates Liouville’s theorem.
Fig. 6. Simulation of the cubic glens cloak. (a) A sphere inside the cloak is seen at a fraction of its actual size. The head behind the cloak is partially seen through the cloak, but appears in its actual position and at its actual size. The glenses have been made slightly absorbing so that the cloak can just be seen. (b) For comparison, the sphere and head are shown with the cloak removed. (c) A cylinder frame indicates the structure of the cloak. Wherever two or more glenses meet, a red cylinder is placed. The figure was calculated for $L = 2$ (in units of the floor-tile side length), $a = 0.8$, and $a' = 0.4$, which means the central cube appears to be half ($a'/a$) of its actual size. The simulation was performed with an extended version of Dr TIM [43, 44].

Fig. 7. Cubic glens cloak. (a) The same as the cloak shown in Fig. 6(a), but seen from a different direction. (b) Like (a), but with the parameters chosen such that the central cube appears to be a tenth of its actual size ($a' = 0.08$; like before, $a = 0.8$). The simulation was performed with an extended version of Dr TIM [43, 44].
One of the macroscopic cloaks listed in the introduction [12] comprises a simple series of lenses. These lenses image any object seen through all four lenses back to its original position, so such an object is seen undistorted. The cloak has been labelled “paraxial” as it only works for rays that travel close to the optical axis of the lenses. An interesting exercise would be to add lenses around these lenses such that the cloak becomes omnidirectional. It is not clear whether or not this is possible.

More desirable still would be to design TO devices made purely from lenses. Such a device would avoid the difficulties in manufacturing lenses, or even metamaterials; it would also avoid the limitations of lenses (such as diffraction and loss) and metamaterials (limited wavelength range; loss; …). It would also be intellectually satisfying by realising the exotic concept of TO — developed in the context of metamaterials — with components as familiar as lenses.

ACKNOWLEDGEMENTS

This work was supported by the UK’s Engineering and Physical Sciences Research Council [grant numbers EP/K503058/1 and EP/M010724/1]. T. Tyc acknowledges support of the grant P201/12/G028 of the Czech Science Foundation.

A. CALCULATING GLENS PARAMETERS FROM TWO PAIRS OF CONJUGATE POINTS

We used our custom raytracer Dr TIM [44] to simulate the view through the cloak described in this paper. The parameters of the lenses that form the cloak have not been programmed into Dr TIM, but instead a procedure by which these parameters are being calculated from the cloak’s imaging properties. This requires finding the parameters of a glens, given the glens plane and two conjugate pairs of points, Q− and Q+, and R− and R+ (see Fig. 8). The glens plane is given in terms of a position vector that corresponds to the glens plane.

![Fig. 8. Construction of the cardinal points of a glens in a given plane from two pairs of conjugate points. The glens images Q− and Q+ into each other, and R− and R+. PQ− and PQ+ are the orthographic projections of the positions Q− and Q+ into the glens plane.](image)

The required glens does not exist. In Dr TIM’s implementation, a Java exception is being thrown in this case.

Once the nodal point has been found, the principal point P can be calculated: it is simply the orthographic projection of N into the glens plane.

Next, we can calculate the positive focal length, f+, which is the a coordinate of the positive focal point, F+. F+ can be constructed as the point where the optical axis intersects the straight line between Q− and PQ−, the orthographic projection into the glens plane of Q−. As before, if no such intersection exists, a Java exception is thrown. The corresponding focal length is then

\[ f^+ = (F+ - P) \cdot \hat{a}, \]

where F+ and P are the position vectors that correspond to F+ and P. The negative focal length, f−, can be calculated similarly by calculating F− as the intersection of the straight line through PQ+ and Q− with the optical axis, and then calculating the a coordinate of F−.

Finally, Dr TIM checks that the glens with the calculated parameters indeed images both object-image pairs as required. This procedure is implemented in the setParametersUsingTwoConjugatePairs method of the GlensHologram class in the optics.raytrace.surfaces package.

REFERENCES


42. D. Gabor, “Improvements in or relating to optical systems composed of lenticules,” UK Patent 541,753 (1940).
