Study of Planck law with a small USB grating spectrometer

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Abstract

In this paper an experiment studying Planck radiation law is presented. The spectra of a heated furnace and of a halogen lamp under various conditions were measured with a small USB grating spectrometer and fitted with a Planck law. The temperature determined from the fit was then compared with the results of comparative temperature measuring techniques (e.g. with the measurement of filament temperature from the temperature dependence of filament resistance).

We have shown, that even spectrometers with limited wavelength range may be successfully used to study Planck distribution. The temperature obtained from the fit of Planck law to the measured spectra was within the uncertainty close to the results of comparative techniques.

1 Introduction

Black-body radiation is an important topic in secondary-school quantum physics. Out of the physical effects discussed when failure of classical physics is demonstrated and a requirement of a new quantum theory is raised (blackbody radiation, atomic spectra, photoelectric effect and Compton scattering), blackbody radiation is the most complicated [1]. Since the derivation of Planck law from the theory is beyond the possibilities of secondary school education, an experiment, in which Planck law can be studied, is particularly needed. However, the spectra attainable with available didactic apparatuses, utilizing a thermopile detector, are claimed to be only qualitative [2].

On the other hand, small USB grating spectrometers have become relatively standard equipment in student physical laboratories, at universities and even at secondary schools. Although observation of coloured spectra with the naked eye provides beautiful impressions, which cannot be surpassed by measurement with any digital instrument, the USB spectrometer is appropriate when quantification of the amount of light at different wavelengths is needed. Furthermore, it can visualize the spectral lines, which are invisible to the naked eye – in ultra-violet (UV) and infra-red (IR) regions. Finally, in comparison with an optical bench setup, the light sources can be weaker, total classroom black-out is not needed and, with a help of laptop, the experiments can also be performed outdoors.

The spectral range of the simplest spectrometer types available to schools is usually only a little larger than that which is visible – usually from 350 to 900 nm. However, the spectral range 180 – 1100 nm is achievable with a silicon CCD detector, especially when the chip sensitivity is enhanced in UV with a special coating. The combination of the wide spectral coverage, a small size of a chip and of a grating plus small focal length mirrors limits the wavelength resolution to a few nanometres. Such a resolution inhibits observation of the true profiles of the atomic spectral lines. E.g., the sodium doublet spectral line (with wavelengths 588.995 and 589.5924 nm) will not be resolved.

The low resolution does not play an important role when the studied light has a continuous spectrum. Small spectrometers are therefore very useful in studies of absorption and emission of light in condensed systems – liquids and solids. Absorption of light in dyes and luminescence in light-emitting diodes belong to these typical applications.

A spectrometer measuring light in the IR region can be used to diagnose the emission of radiation from heated bodies. Since at normal temperatures the emission of radiation is only in IR, high temperature sources (at least 1000 K) are needed in such experiments. A heated furnace, flame or filament of a halogen lamp seem to be the most accessible sources.

The aim of this article is to show, how a small spectrometer with a spectral range 200 – 1100 nm may be used to investigate the spectrum of heated bodies. With the help of the spectrometer, we can demonstrate not only Planck radiation law, but we can also determine the blackbody temperature from the optical spectra. The latter provides a direct insight into remote temperature measurement, used e.g. in environmental or astrophysical applications.

2 Planck law

The emission of radiation from heated bodies (incandescence) is manifested with a continuous optical spectrum. Besides real heated bodies, a model of a so-called black body, absorbing all the light at all wavelengths, plays a fundamental role, since its spectral dependence of intensity is directly described by Planck distribution. This distribution expressed particularly either as energy density ρ (energy of radiation present in unit volume), radiance L (radiation power emitted from unit projected perpendicular surface into unit solid angle) or irradiance M (radiation power emitted from unit surface). Since for isotropic radiation and ideal diffuse surface

$$L = \frac{1}{4\pi}\rho c, \qquad M = \pi L,$$

the expressions of Planck law have the forms

$$\begin{aligned}
\rho_\mathrm{b}(\lambda) &= \frac{8\pi hc}{\lambda^5}\frac{1}{\mathrm{e}^{\frac{hc}{k_\mathrm{b}T\lambda}}-1} \qquad (\mathrm{Jm^{-3}\,m^{-1}}),\\
L_\mathrm{b}(\lambda) &= \frac{2hc^2}{\lambda^5}\frac{1}{\mathrm{e}^{\frac{hc}{k_\mathrm{b}T\lambda}}-1} \qquad (\mathrm{Wm^{-2}\,sr^{-1}\,m^{-1}}),\\
M_\mathrm{b}(\lambda) &= \frac{2\pi hc^2}{\lambda^5}\frac{1}{\mathrm{e}^{\frac{hc}{k_\mathrm{b}T\lambda}}-1} \qquad (\mathrm{Wm^{-2}\,m^{-1}}).
\end{aligned}$$

where h is Planck constant (6.625 $\times$ 10^{-34} J s), c vacuum speed of light (2.998 $\times$ 10^8 m s^{-1}) and k_b Boltzmann constant (1.38 $\times$ 10^{-23} J K^{-1}). Here, the energy is expressed per unit wavelength interval (last m^{-1} or nm^{-1} in units) in a form convenient for the measurements in the visible spectral region. From the formulas usually derived for energy per frequency interval (Hz^{-1}) they are obtained as

$$L(\lambda) = L(\nu)\left|\frac{\mathrm{d}\nu}{\mathrm{d}\lambda}\right|.$$

The differentiation of Planck law with respect to the wavelength and the integration of Planck law over all wavelengths give two important laws:

1. Wien displacement law of intensity maximum

$$\lambda_\mathrm{max}T = \mathrm{konst} = 2.88\times10^{-3}\,\mathrm{m\,K},$$

2. Stefan–Boltzmann law of total radiation intensity

$$M_\mathrm{b} = \sigma T^4, \qquad \sigma = 5.67\times10^{-8}\,\mathrm{Wm^{-2}K^{-4}}.$$

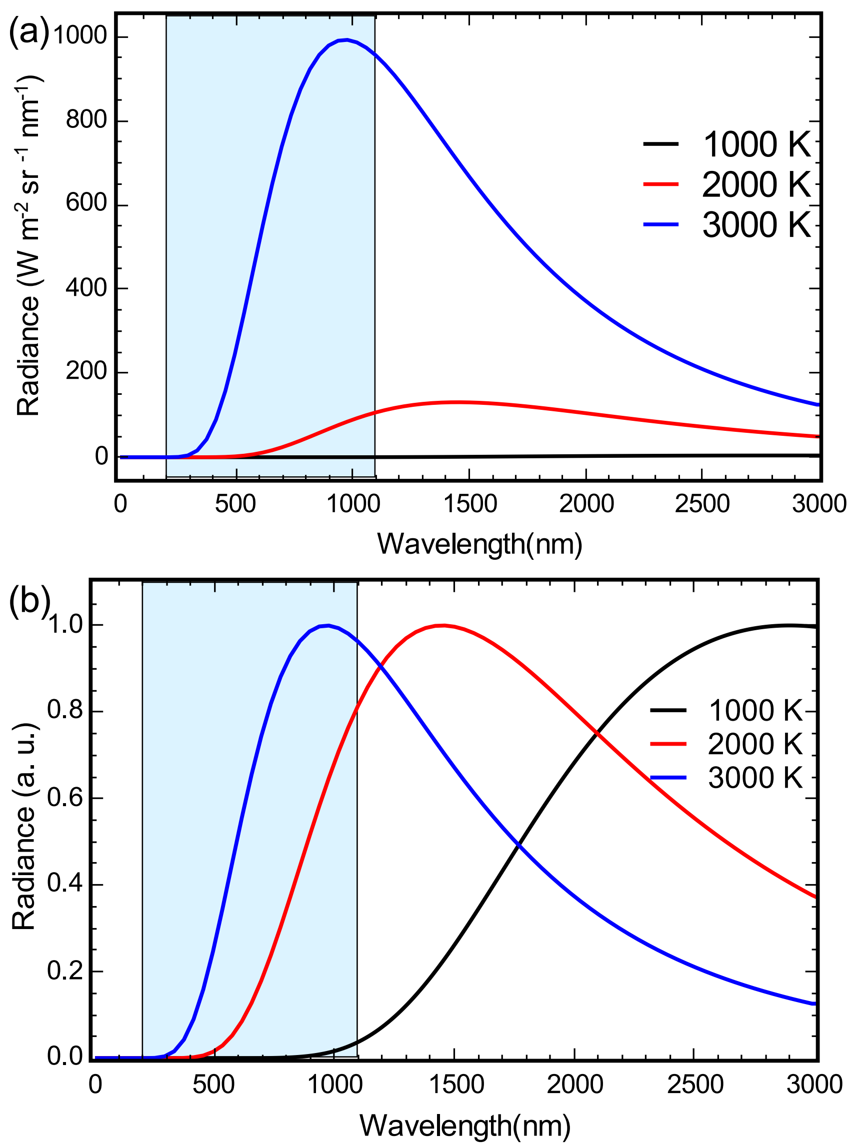

Figure 1: Planck law plotted (a) as radiance $L(\lambda)$ in absolute values and (b) normalized to unit intensity maximum for three temperatures accessible with common incandescent sources. Region of visible wavelengths and region accessible with spectrometer are demarked in the plot.

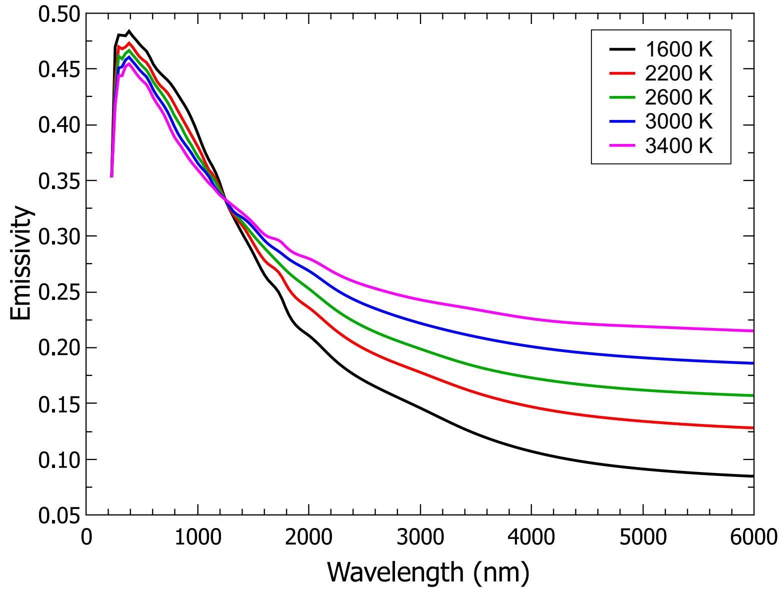

Figure 2: Emissivity of tungsten as a function of wavelength and temperature. The data were taken from [4].

Note, that Stefan–Boltzmann law is obtained by integration over the wavelengths and also over the relevant solid angle and therefore σ of this value is the only constant in expression for irradiance.

The spectral radiance $L_{\mathrm{b}}(\lambda)$ for three temperatures accessible with common incandescent sources is shown in figure 1 (a), the normalized distributions with unit intensity maxima are shown in figure 1 (b). Obviously, only at temperatures of $\approx$ 3000 K and higher can the distribution maximum be registered with a spectrometer range 200 – 1100 nm. Using Wien displacement law, the intensity maximum lies at 2898 nm for 1000 K, at 1440 nm for 2000 K, at 966 nm for 3000 K and at 781 nm for 3687 K (melting point of pure tungsten [3]).

Without an absolute intensity measurement, the registration of intensity maximum simplifies principally the correct determination of temperature from the measured spectra. Otherwise, the fitting procedure must rely only on the different shape of the distributions in monotonic $0 - \lambda_{\max}$ region at different temperatures. Anyway, the relative spectrometer sensitivity must be known.

The deviation of emission of radiation of real bodies from Planck law is described by an emissivity ε

$$L(\lambda; T) = \varepsilon(\lambda; T) L_{\mathrm{b}}(\lambda; T),$$

where $L(\lambda, T)$ is the radiance of a real-body surface with thermodynamic temperature T at wavelength λ and $L_{\mathrm{b}}(\lambda, T)$ is the radiance of black body with the same temperature and at the same wavelength. Emissivity of black body is always a unit, otherwise $\varepsilon < 1$.

With a constant emissivity with respect to the wavelength, the radiator is called a grey body. Otherwise it is called a selective radiator. E.g., in the case of tungsten the emissivity in visible light weakly depends on the wavelength, decreasing towards the longer wavelengths (see figure 2). Therefore, the tungsten filament lamp is a slightly more efficient light source than the black body at the same temperature, although emitting only fraction of the total blackbody radiation intensity.

Without absolute intensity measurement, the constant emissivity < 1 of tungsten cannot be determined. The only important influence on the measured spectra may therefore arise from the spectral dependence of the emissivity. In order to resolve, whether the tungsten may be handled as grey body (no emissivity correction is needed) or selective radiator (the measurements must be compared with $L_{\mathrm{W}}(\lambda) = \varepsilon_{\mathrm{W}}(\lambda) L_{\mathrm{b}}(\lambda)$), Planck distribution with and without emissivity correction was calculated for 3000 K and plotted normalized in figure 3 (a). The spectral emissivity of tungsten for temperature 3000 K was taken in these calculations (see again figure 2). Moreover, a relative difference between the curves

$$\frac{L_{\mathrm{W}}^{\mathrm{n}}(\lambda) - L_{\mathrm{b}}^{\mathrm{n}}(\lambda)}{L_{\mathrm{W}}^{\mathrm{n}}(\lambda)}$$

is shown in figure 3b), with superscript n denoting normalizated distribution with unit intensity maximum. It can be seen, that the relative differences are generally lower than $18\,\%$. This is comparable with the typical intensity calibration error, which may be about $10\,\%$. Thus, the tungsten filament can be handled satisfactorily as the black body and the spectral dependence of emissivity can be neglected.

3 Experimental set-up

An electric furnace and tungsten filament halogen lamp were used as investigated incandescent sources. The furnace, operated at temperature of up to 1470 K, was made of corundum with Kanthal heating element reeled on the external side of the muffle. External parts of the furnace were made of stainless steel. A programmable regulator allowed the temperature to be set and measured with a built-in N-type thermocouple with 5% uncertainty. A small hole in the front could be used as a blackbody simulator (see figure 4 (a)). Another hole in the rear wall enabled control temperature measurement with an additional K-type thermocouple (to check the temperature homogeneity etc.). The temperatures measured with both thermocouples were the same within the measurement uncertainty.

Tungsten-filament halogen lamps belong to the the most available and applicable incandescent light sources operated at high temperatures. The halogen lamp is generally preferred to standard tungsten lamp, since the halogen cycle ensures the transport of tungsten atoms from the neighbourhood of the glass walls back to the filament and thus enables the operation of the lamp at a higher filament temperature. It is probably better to avoid types such as ECO or with a UV-stop filter. The infrared coating on glass bulb of the halogen lamp reflects IR radiation back to the filament, reducing the necessary power input. On the other end of visible spectrum, the UV-stop filter doped quartz glass absorbs mostly UV-C and UV-B radiation, but also substantially reduces UV-A radiation [5]. Although absorption from both coatings in the visible spectrum is probably negligible, it is better to use a bulb without such modifications. Osram HLX 64642 24 V/150 W halogen lamp for slide and overhead projectors with no reflector nor luminaire was fixed in a standard holder (figure 4b) and sourced from a stabilized DC power supply.

There are also several types of lamps with the same voltage and wattage, differing in output luminous fluxes and lifetimes. In this case, the lamp with the highest luminous flux and the lowest lifetime is expected to have the highest filament temperature at nominal voltage.

The large advantage of the tungsten lamp is that the temperature of the filament may be deduced independently from the temperature dependence of tungsten resistance. Thus, the voltage on the filament and the current flowing through were measured with multimetres. The measured resistance

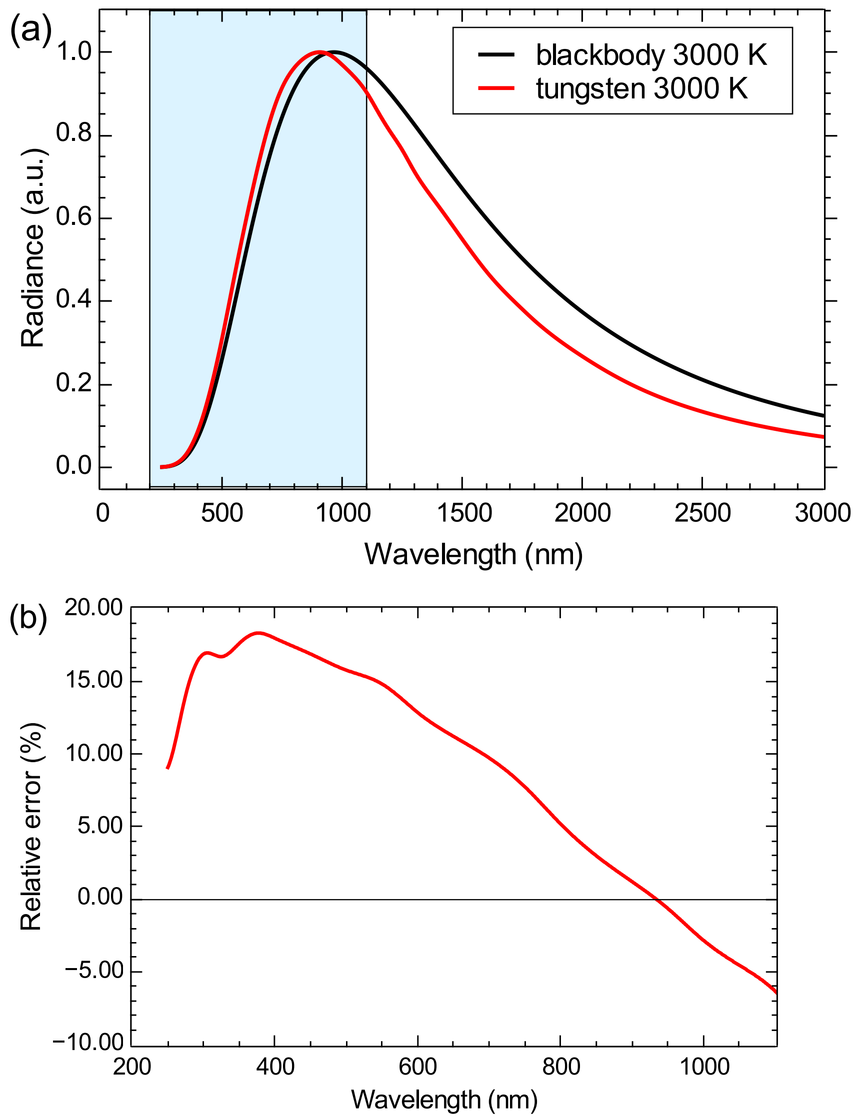

Figure 3: Comparison of black body ($L_\mathrm{b}(\lambda)$) and tungsten ($\varepsilon(\lambda)L_\mathrm{b}(\lambda)$) for temperature 3000 K. The curves were normalized according to their maxima (a). A relative difference between the normalized distributions (b).

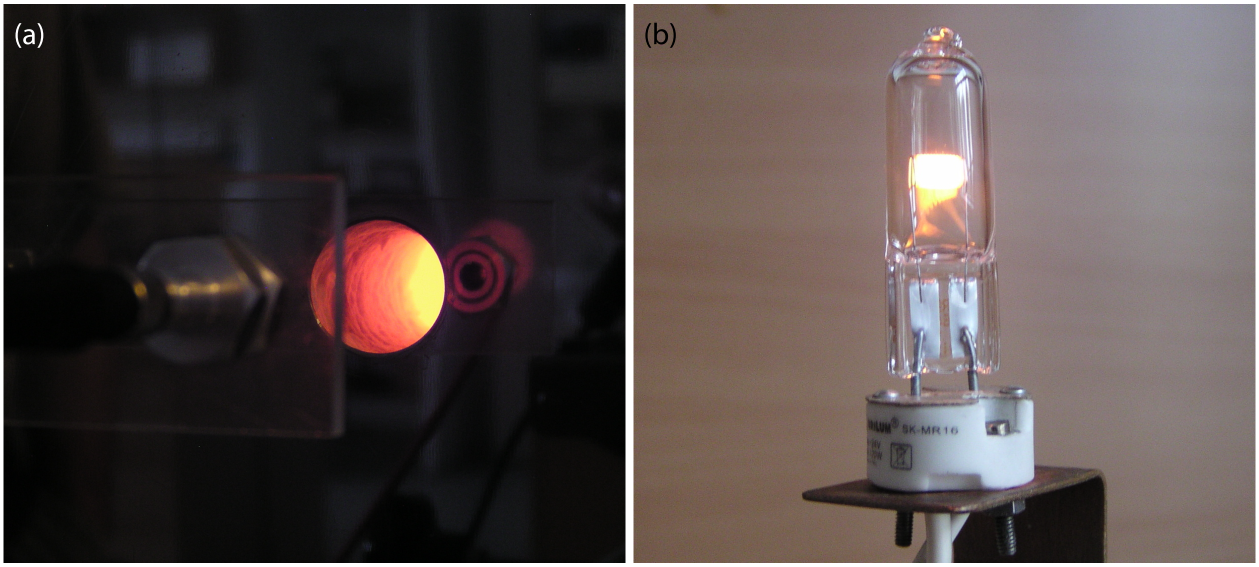

Figure 4: Heated electric furnace with a hole in the front side simulating the black body (a). A plastic-glass holder holds an optical fibre collecting the light for the spectrometer. Halogen tungsten filament lamp (b).

of the filament at room temperature $R_{293} \approx 0.25\,\Omega$ (293 K) was also checked with a ohmmeter. The resistance of the external wiring (the same circuit with the bulb shorted out) was found to be about 10 % of R_{293} and it was therefore subtracted from the measured resistance.

The temperature dependence of tungsten resistivity is approximated in range 100 – 3600 K with formula [6]

$$\begin{aligned} \varrho(T_*) &= -0.968 + 19.274\,T_* + 7.826\,{T_*}^2 - 1.8517\,{T_*}^3 + 0.2079\,{T_*}^4 \\ [T_*] &= 1000\,\mathrm{K}, \quad [\varrho] = 10^{-8}\Omega\,\mathrm{m}. \end{aligned}$$

The estimated uncertainty of the data used for polynomial expression was about $\pm 3\%$ and the dependence was corrected for thermal expansion. Since in our case the measured quantity is the resistance, in which thermal expansion is included, the uncorrected resistivity dependence should be used instead. However, in the case of tungsten the difference is about 2 % (see figure 5). Dividing the resistivity dependence with the room temperature value

$$\varrho_{293} = 5.28 \times 10^{-8}\ \Omega\,\mathrm{m}\ (293\,\mathrm{K}),$$

an inverse function to $\varrho(T)/\varrho_{293}$ may be fitted with a simple parabolic formula

$$T = 154.6 + 186.4(\varrho/\varrho_{293}) - 1.314(\varrho/\varrho_{293})^2\ \mathrm{K},$$

from which the temperature is evaluated when the ratio of measured resistances is substituted instead of ϱ/ϱ_{293}. Note, that due to the large temperature range the formula is only approximate and does not provide precise values especially at low temperatures.

The optical spectra in 200 – 1100 nm wavelength range were measured with small USB grating spectrometer Avaspec 3648 with 300 lines/mm grating and linear CCD array with 3648 pixels. The

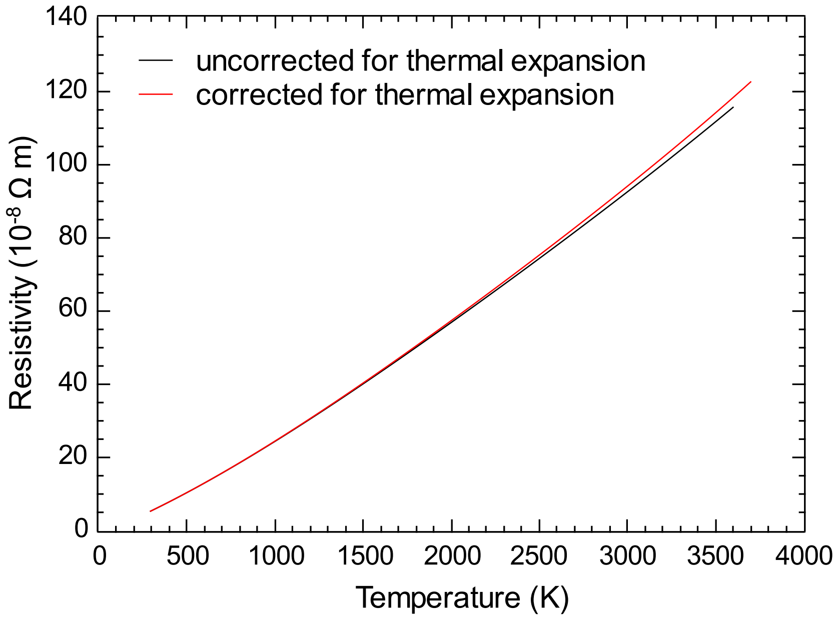

Figure 5: Temperature dependence of tungsten resistivity. The uncorrected curve, showing in principle the dependence of resistance on the temperature, differs from the true resistivity dependence due to thermal expansion.

fixed slit width was $10\,\mu$m. The light from the studied sources was collected with an optical fibre. The spectrometer was also equipped with filters providing the removal of higher spectral orders.

The instrument was intensity-calibrated with a deuterium and tungsten halogen lamp with tabulated true spectrum.

The spectra corrected for instrument sensitivity was fitted in QtiPlot [7] with Marquardt-Levenberg algorithm in order to determine the blackbody temperature. The fitted function was parametrized with temperature T and scalling coefficient A:

$$f(\lambda) = \frac{A}{\lambda^5} \frac{1}{\mathrm{e}^{\frac{hc}{k_\mathrm{b} T \lambda}} - 1}.$$

4 Typical results and discussion

4.1 Furnace

At first, the furnace with the highest allowable temperature 1470 K was studied. After reaching the maximal temperature, the furnace was switched off and allowed to cool down. The spectra of the heated furnace and the thermocouple temperature were then measured periodically. The corrected spectra (for different integration time and instrument spectral sensitivity) are displayed in figure 6 together with their fit with Planck law. As expected, the intensity decayed during the cooling in absolute scale. The curves were monotonically increasing towards the higher wavelengths, having their intensity maxima in the IR region inaccessible to our spectrometer.

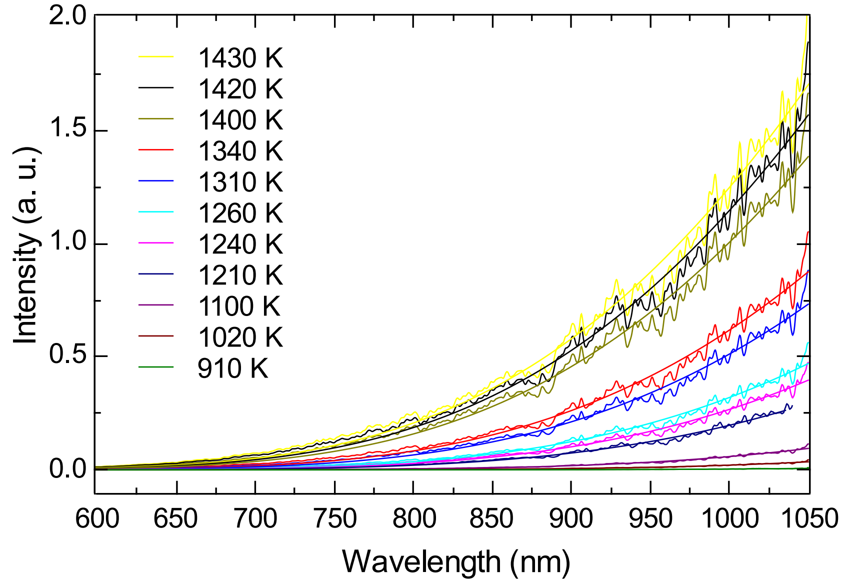

Figure 6: Optical spectra of the heated furnace corrected for instrument sensitivity. The temperature measured with thermocouple ranged 910 – 1410 K.

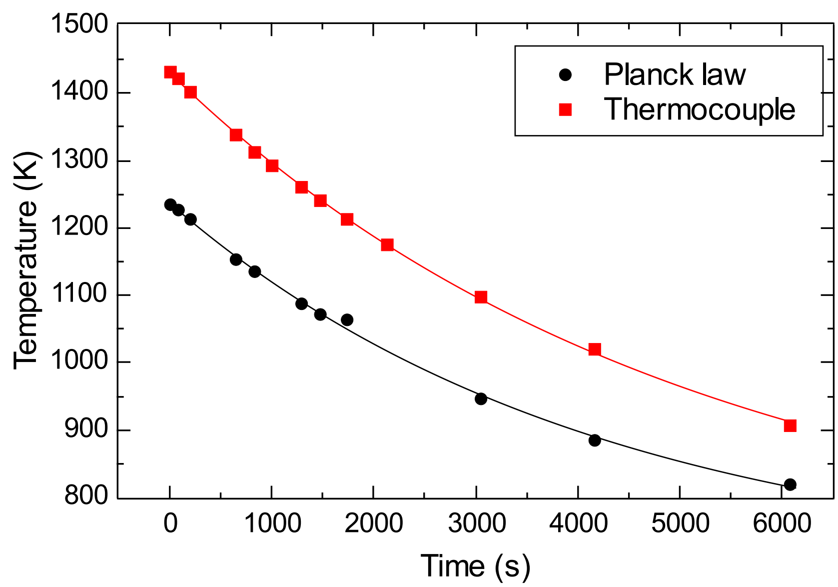

Figure 7: Cooling of the switched-off heated furnace. Comparison of the temperature determined from Planck law with the temperature measured with thermocouple.

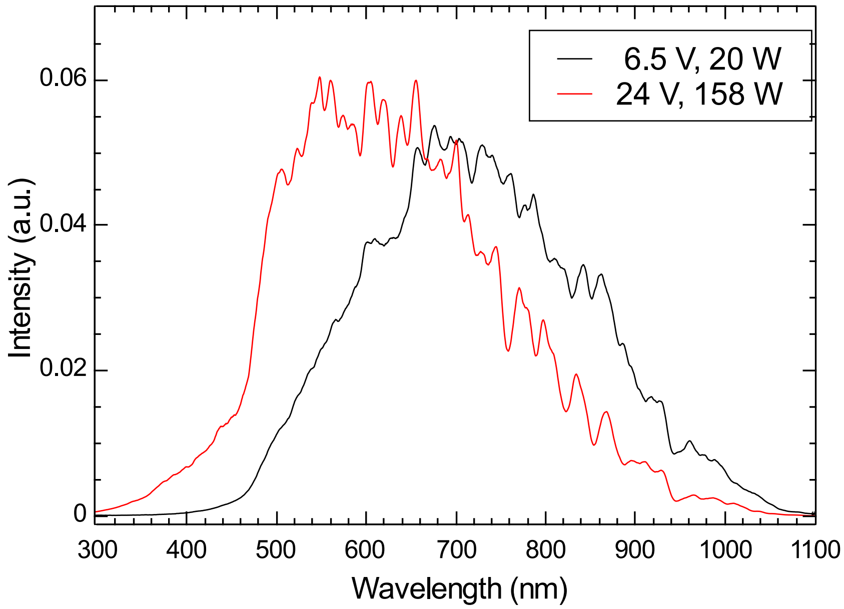

Figure 8: Uncorrected measured spectra of light emitted by a halogen lamp under voltages 6.5 and 24 V. The decrease of the intensity towards higher wavelengths is due to the decreasing instrument sensitivity in this region.

The temperature determined from the fit of the corrected spectra with Planck law is displayed with the corresponding thermocouple temperature as a function of time in figure 7. The correspondence of both curves is more than evident. The temporal dependence of both temperatures may be fitted with an exponential formula, manifesting Newtonian law of cooling. Despite the high temperature inside, the isolated furnace is probably cooled mainly by heat conduction. The relatively large difference between the values (100 –200 K) is probably due to the small part of Planck law, registered with the spectrometer (see curve for 1000 K in figure 1 (b)). A disadvantage of our furnace was the large time needed for heating of the furnace to the final temperature, caused by the relatively large dimensions. On the other hand, the furnace was a suitable representative of a cavity used in the derivation of Planck law.

4.2 Halogen lamp

An example of the measured spectra of light emitted by a halogen lamp at 6.5 and 24 V is shown in figure 8. Owing to different filament temperatures, the spectra differ considerably in their relative shape. However, the observed intensity maxima cannot be assigned to the maxima of Planck distribution $\lambda_{\rm max}$, since the maxima are produced by a combination of increase of radiation towards higher wavelengths and decrease of the instrument sensitivity in 500 – 1100 nm region. The calibration of the instrument was therefore necessary.

The voltage applied on the lamp was increased gradually and the current and optical spectra were measured. The whole volt-ampere (VA) characteristic is shown in figure 9. The VA dependence is clearly non-linear due to the change of filament resistance with the temperature. At 0 V (0 W, 20°C), the resistance of the filament with supply wires was $\approx 0.25\,\Omega$. At 24 V (158 W), the resistance

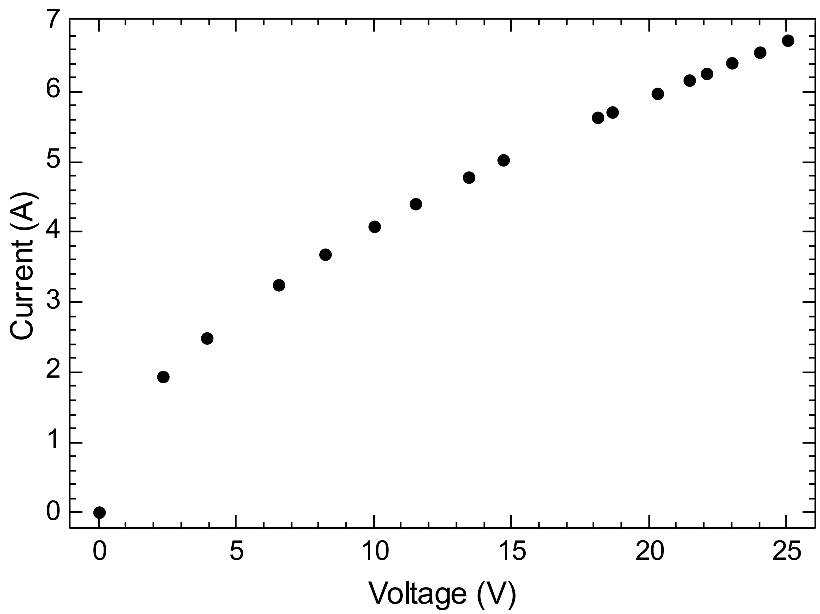

Figure 9: Volt-ampere characteristic of tungsten halogen lamp. The dependence is clearly non-linear due to the change of filament resistance with the temperature.

reached 3.7 Ω, about 15$\times$ larger value than at room temperature.

The corrected spectra of the tungsten halogen lamp fitted with Planck distributions are shown in figure 10. Since the intensity changes in range of 10^5, both linear (a) and logarithmic (b) plot of intensities are displayed. It may be seen in the linear plot, that from 18 V an intensity maximum appears in the infrared region. However, the direct observation of the $\lambda_{\max}$ is complicated owing to the noise in this region. Since with increasing voltage the intensity curve shifts towards the visible region – thus increasing more where the spectrometer is the most sensitive, the integration time had to be lowered accordingly. This resulted in lower measured intensity in IR (compare measured spectra in figure 8) and the larger noise in this region at higher voltages.

The temperatures determined from the fit of corrected spectra with Planck distribution and from the thermal dependence of resistance are plotted against the incoming electric power in figure 11. The curves are very similar, exhibiting a temperature saturation caused by increasing heat losses at higher temperatures. The temperature determined from the optical spectra is a little larger than expected. This may be due to the assumption of the tungsten filament being a grey body. An incorporation of the tungsten spectral emissivity into the calculations would decrease the determined Planck temperature.

The systematic 8-15% difference between the values is acceptable, taking into account the uncertainty of spectrometer intensity calibration (10 %) and the possible errors in evaluation of temperature from the filament resistance. Since at room temperature the resistance of the external circuit (wiring, socket etc) subtracted from the measured value is about 10 %, the uncertainty of determined temperature reaches a similar value.

The nominal luminous flux 5000 lm at power 150 W of used halogen lamp specified by its producer corresponds to luminous efficacy 33 lm/W, which can be obtained with tungsten filament heated at $\approx$ 3100 K [4]. A lamp with shorter life (50 h instead of 300 h), having nominal luminous flux 6000 lm at the same power, would have luminous efficacy 40 lm/W and the corresponding

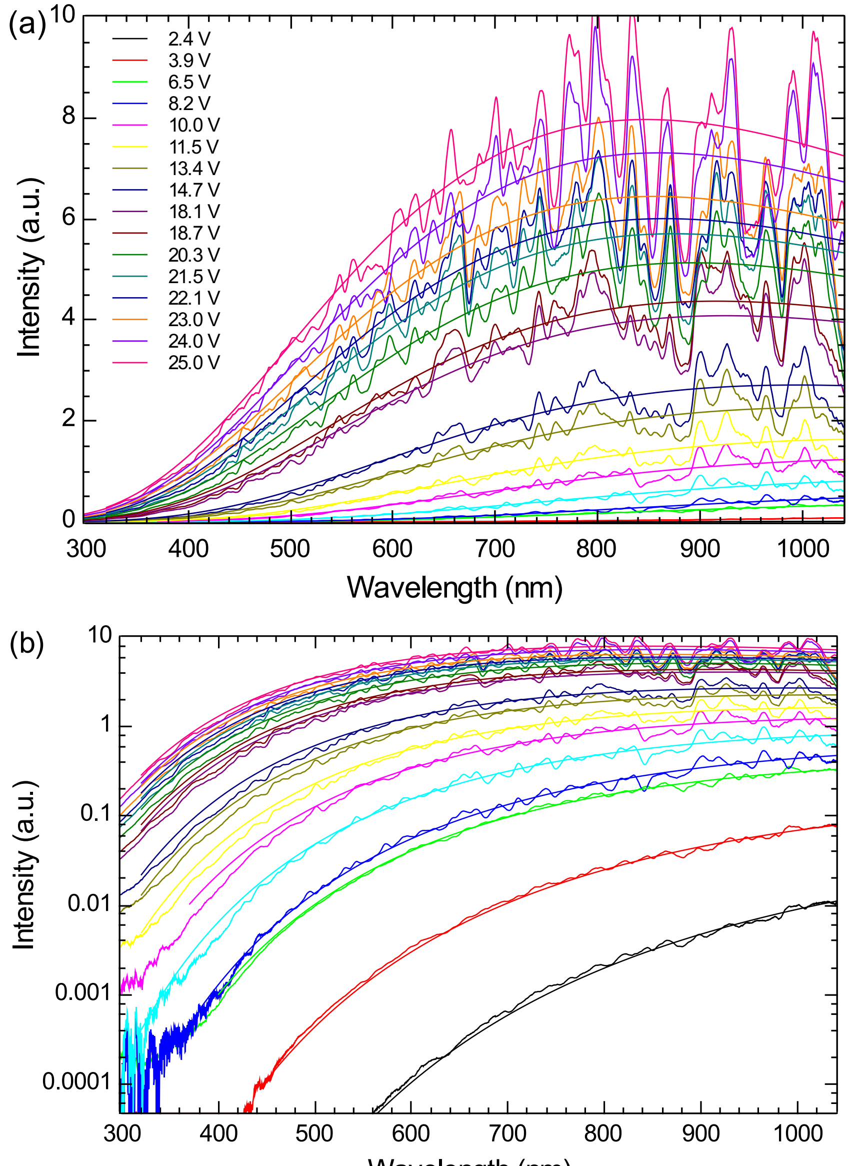

Figure 10: Corrected spectra of tungsten halogen lamp for various voltages in linear plot (a) and log plot (b). The colours of curves in the logarithmic plot correspond to the same voltages as in the linear plot. The spectra were fitted with Planck distribution.

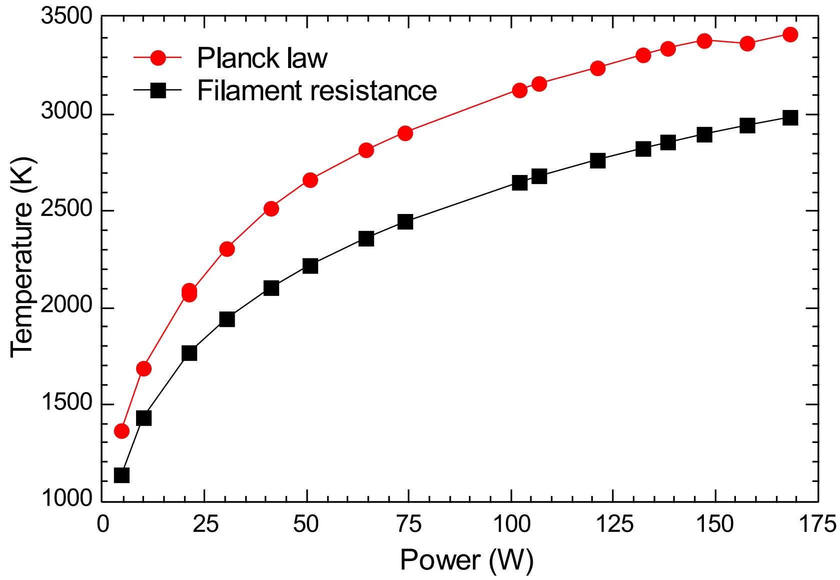

Figure 11: Comparison of filament temperatures determined from Planck law and from the thermal dependence of resistance.

filament temperature should reach approx. 3200 K. Thus, the determined values of the filament temperature are in a good agreement with the expected ones within the estimated uncertainties.

As mentioned in section 1, the spectral range of some school spectrometers is narrower, e.g. up to 850 and 950 nm. Assuming similar spectral sensitivity at common wavelengths, the possibility to use such instruments in the present experiment may be evaluated by fitting the spectrum only in regions accessible with these spectrometers. The comparison of the obtained results is shown in figure 12. Using the reduced wavelength range, the determined values of temperature are 100 K lower, agreeing even more with the resistance measurement. This may be due to the low sensitivity of silicon detectors when approaching 1100 nm and also due to slightly incorrect calibration of our instrument in wavelengths above 780 nm, caused by extrapolation of the calibration curve above this wavelength. In whole, small spectrometers available at secondary schools are therefore also suitable, especially when the halogen lamp is operated at a nominal voltage.

5 Conclusion

We have shown, that small optical spectrometers may be used to determine the temperature from the fit of Planck law to the measured spectra. The only necessity is to calibrate the spectrometer for a varying spectral sensitivity. In our case, a standard calibration lamp with a tabulated true spectrum was used. However, reversing the described technique a self-made calibration may be performed with no additional requirements. Calculating Planck distribution for a temperature determined from the resistance ratio, the relative sensitivity curve of the instrument is obtained by dividing the measured spectrum with the expected one. Although this technique seems to be close to “begging a question”, it does not differ much from the professional practice, if reliable standards and measuring procedures are used.

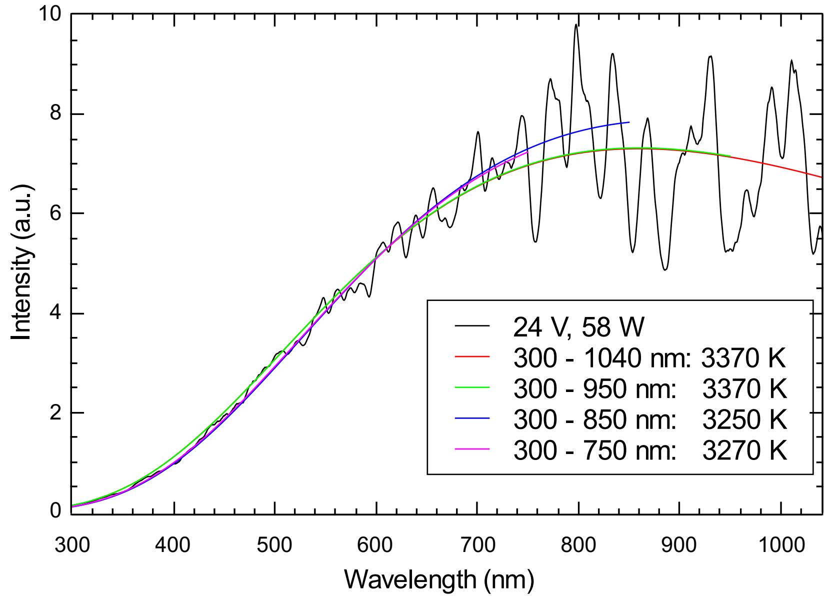

Figure 12: Comparison of filament temperatures determined from the fit of Planck law in spectral regions accessible with spectrometers available in school laboratories. Limiting the wavelength range for the fit, the determined values of temperature agree even more with the resistance measurement.

Acknowledgement

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