Advanced Quantum Field Theory

Exam problems

1. Find the free massive propagator $\Delta(x-x')$ for a $(1+1)$-dimensional spacetime and study the behavior for $x-x'$ timelike (inside the lightcone) and $x-x'$ spacelike (outside the lightcone).

2. For a real scalar field with interaction $\lambda\varphi^4/4!$, draw all the connected Feynman diagram contributions to the 2-point function $G^{(2)}$ and the 4-point function $G^{(4)}$ up to order λ^3.

3. Consider a 4-dimensional real scalar field with the Lagrangian
$$\mathcal{L} = -\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{4!}\lambda\varphi^4.$$
Compute the lowest-order corrections to the propagator and compute the Z-factors of the quadratic terms in the Lagrangian in the $\overline{\text{MS}}$ scheme.

4. Yukawa theory is defined as a 4D theory of a Dirac fermion and real pseudoscalar field defined by the Lagrangian
$$\begin{aligned}\mathcal{L} = {} & i\bar\psi\partial\!\!\!/\psi - m\bar\psi\psi - \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}M^2\varphi^2 \\ & + ig\varphi\bar\psi\gamma_5\psi - \frac{1}{4!}\lambda\varphi^4.\end{aligned}$$
Derive the fermion-loop correction to the scalar propagator in the $\overline{\text{MS}}$ scheme. Show that there is an extra minus sign for a fermion loop as compared to a scalar loop.

5. In pure Yang-Mills theory fix the gauge using the axial gauge condition
$$n^\mu A^a_\mu = 0$$
for n^μ a fixed 4-vector. Find the gluon and ghost propagators and the ghost-gluon interaction vertices.

6. Consider performing the path integral for a scalar field theory in the presence of a background field $\bar\varphi(x)$. We define
$$e^{i\overline{W}[J,\bar\varphi]} = \int \mathcal{D}\varphi\ e^{iS[\varphi+\bar\varphi]+i\int d^dx J\varphi}.$$

Clearly $\overline{W}[J,0]$ is the original $W[J]$. We also define the quantum action in the presence of the background field

$$\overline{\Gamma}[\varphi,\bar{\varphi}] = \overline{W}[J,\bar{\varphi}] - \int d^d x \; J\varphi,$$

where now $J(x)$ is the solution of

$$\frac{\delta}{\delta J(x)}\overline{W}[J,\bar{\varphi}] = \varphi(x).$$

Show that $\overline{\Gamma}[\varphi,0]$ is equal to the original quantum action $\Gamma[\varphi]$ and that

$$\Gamma[\varphi+\bar{\varphi}] = \overline{\Gamma}[\varphi+\bar{\varphi},0] = \overline{\Gamma}[\varphi,\bar{\varphi}],$$

which means that we can calculate the original quantum action by calculating vacuum graphs in the background action

$$\Gamma[\bar{\varphi}] = \overline{\Gamma}[\bar{\varphi},0] = \overline{\Gamma}[0,\bar{\varphi}].$$

Confirm this by computing the 1-loop contribution to the 2-point function for a real scalar field in 4D with a $\frac{1}{4!}\varphi^4$ potential using this method and in the standard way and compare the results.