

Fyzikální vlastnosti materiálů

FX001

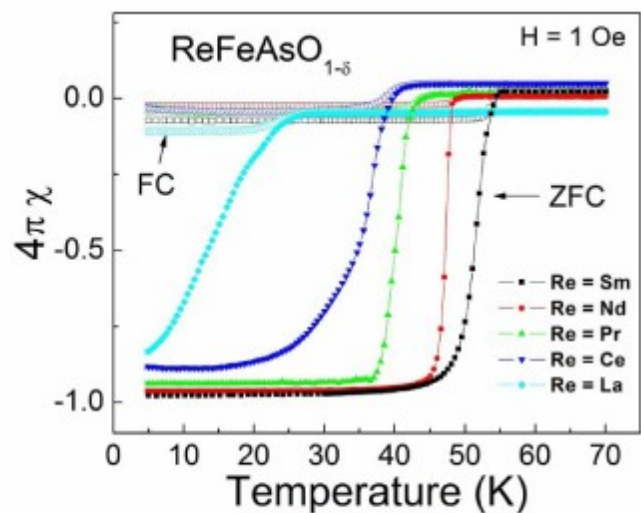
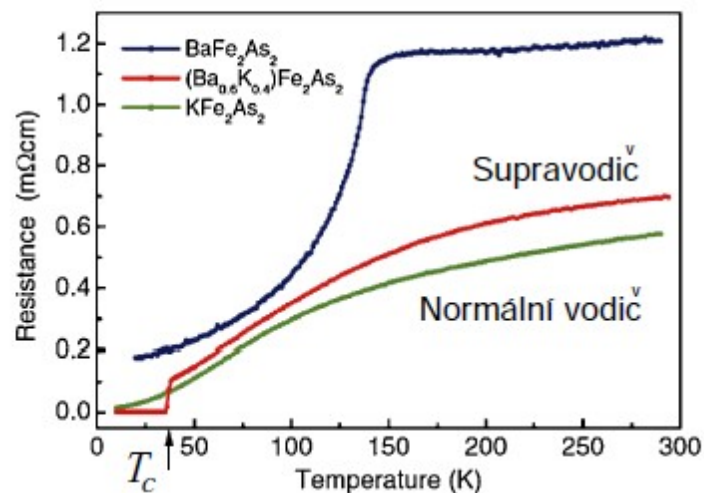
1. Vazba v pevné látce, elastické a tepelné vlastnosti materiálů
2. Elektrické vlastnosti materiálů
3. Optické vlastnosti materiálů
4. Magnetické vlastnosti materiálů
5. **Supravodiče a grafen**

Fyzikální vlastnosti materiálů

5. Supravodiče a grafen

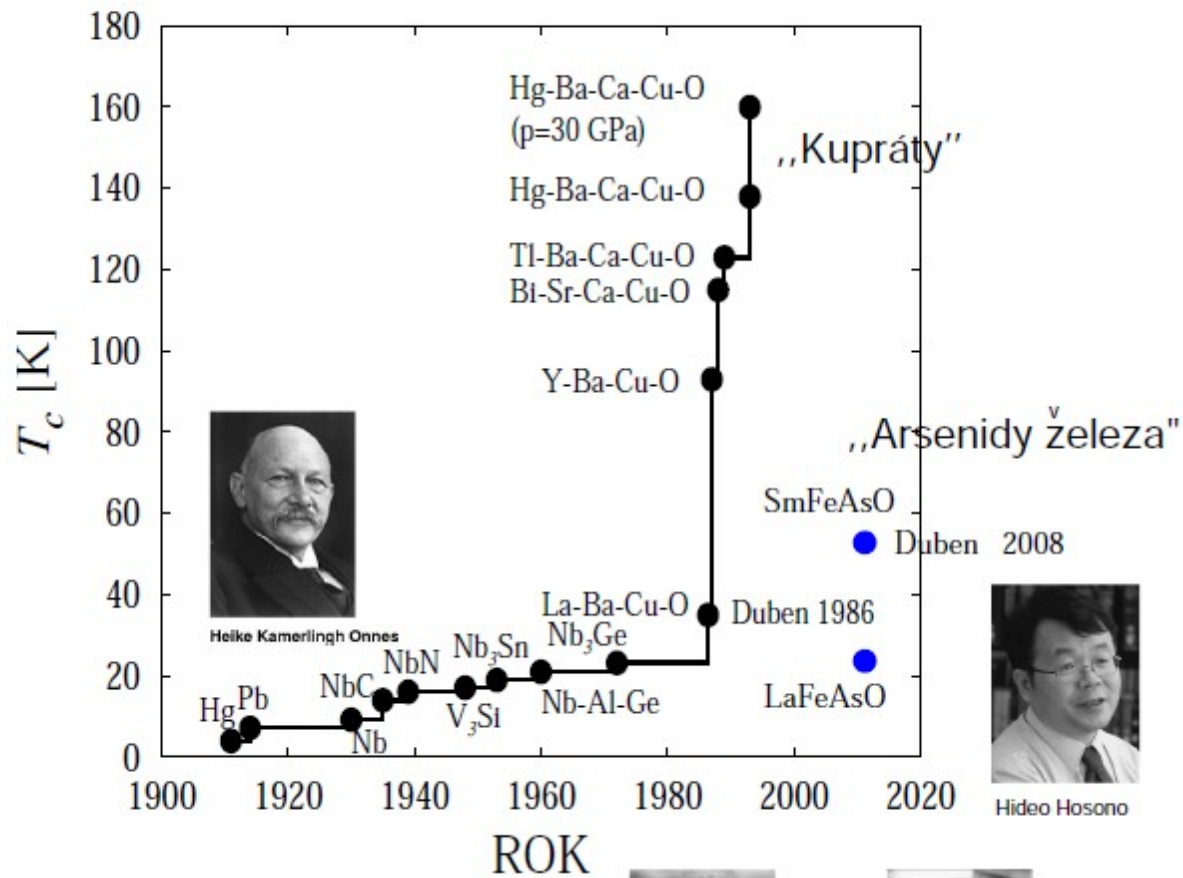
- a) Supravodivost – supravodivost kovů, Meissnerův jev, vysokoteplotní supravodiče.
- b) Grafen – Fermiho rychlost, elektrické, mechanické, optické a magnetické vlastnosti
- c) Jiné materiály s lineární disperzí – topologické izolátory

Definice supravodiče a kritická teplota



Rotter et al., PRL, 2008.

Ren et al., Europhys. Lett., 2008.



J. Georg Bednorz



K. Alexander Müller

Teplotní závislost měrného odporu Pt, Hg a Au (data K. Onnese)

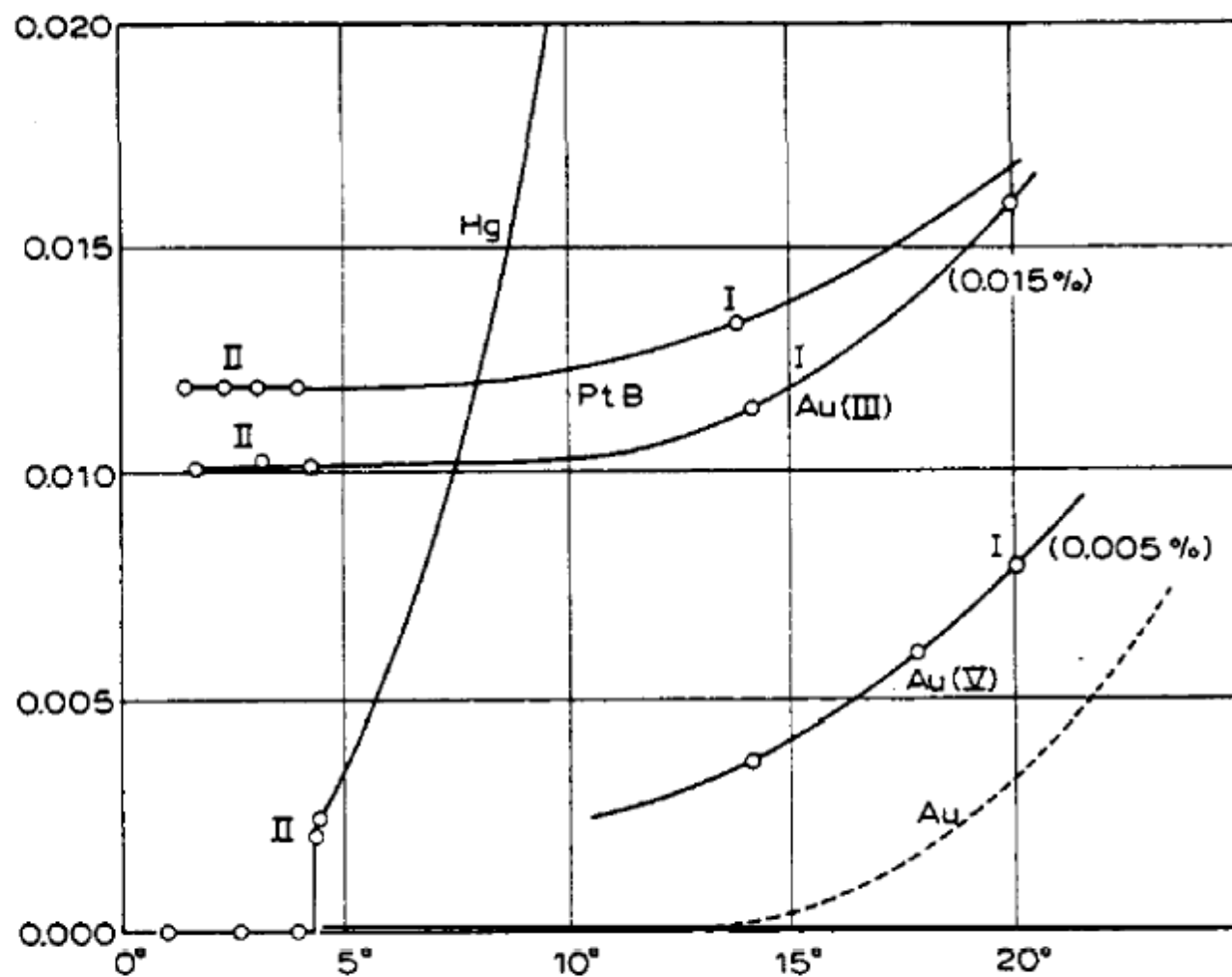


Fig. 14

Supravodivé prvky

KNOWN SUPERCONDUCTIVE ELEMENTS

■ BLUE = AT AMBIENT PRESSURE
■ GREEN = ONLY UNDER HIGH PRESSURE

	IA																			0
1	1	H											2	He						
2		3	4											5	6	7	8	9	10	
		Li	Be											B	C	N	O	F	Ne	
3		11	12											13	14	15	16	17	18	
		Na	Mg	III B	IV B	V B	VI B	VII B	VII		IB	II B	Al	Si	P	S	Cl	Ar		
4		19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
		K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr	
5		37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	
		Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe	
6		55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	
		Cs	Ba	*La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn	
7		87	88	89	104	105	106	107	108	109	110	111	112							
		Fr	Ra	+Ac	Rf	Ha	106	107	108	109	110	111	112							

SUPERCONDUCTORS.ORG

* Lanthanide Series

58	59	60	61	62	63	64	65	66	67	68	69	70	71
Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu

+ Actinide Series

90	91	92	93	94	95	96	97	98	99	100	101	102	103
Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

Supravodiče

	T_c (K)	H_c (gauss) T=0K
Al	1.196	99
Hg – alfa	4.15	411
La – hcp	4.9	798
La – fcc	6.06	1096
Nb	9.26	1980
Pb	7.19	803
Ta	4.48	830
Tc	7.77	1410
V	5.30	1020

Levitace

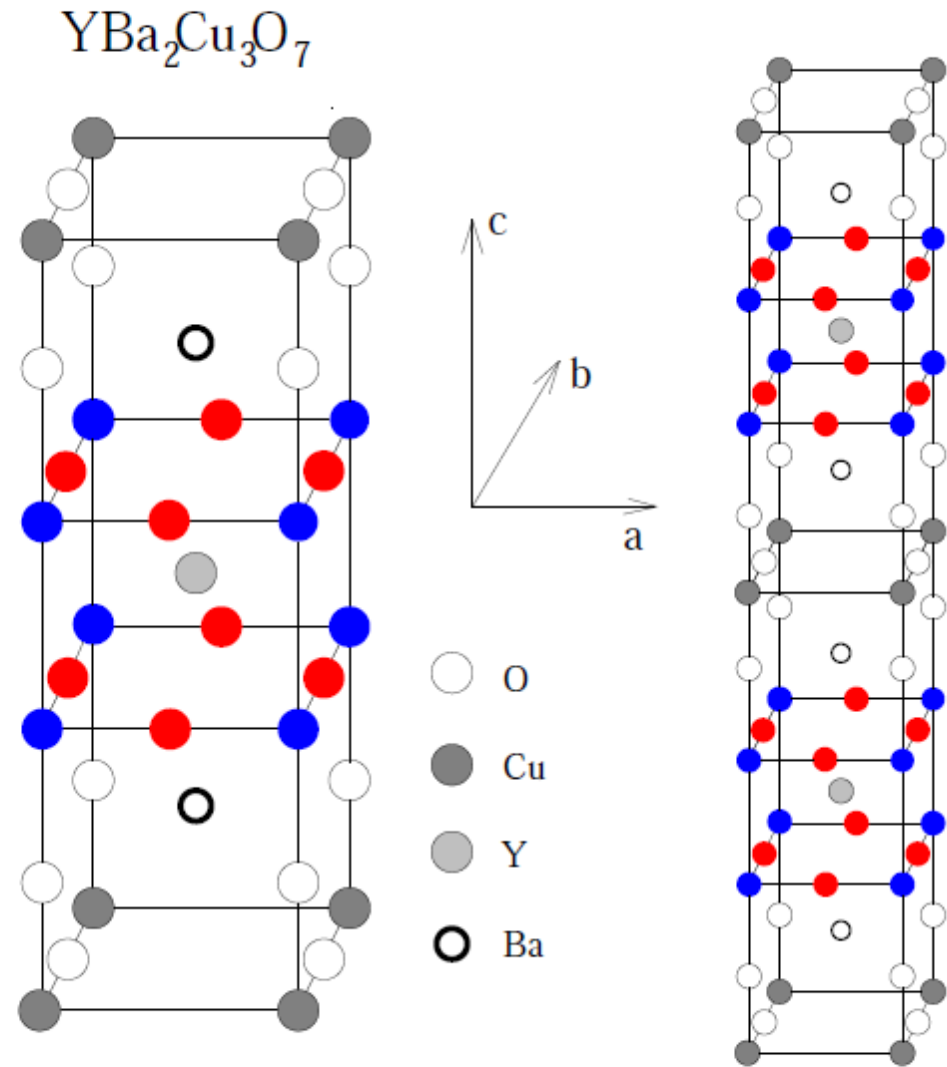
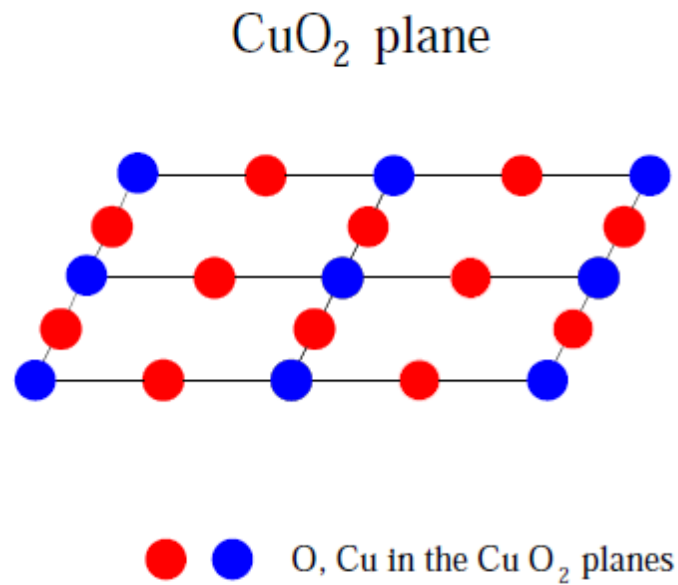


<http://www.fys.uio.no/super/levitation>
(Univerzita Oslo)



<http://www.ru.nl/hfml/research/levitation/diamagnetic>
(Nijmegen High Field Magnet Laboratory)

Kupráty: krystalová struktura supravodiče $\text{YBa}_2\text{Cu}_3\text{O}_7$



Supravodivost

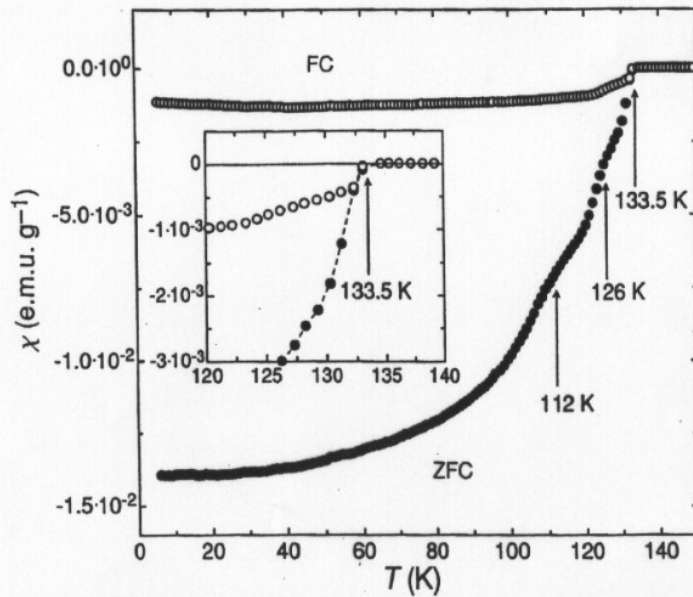


FIG. 2 Zero-field cooling (ZFC) and field cooling (FC) susceptibilities $\chi(T)$ of one of the investigated oxygen-annealed HBCCO samples, measured in $H=27$ Oe. The ZFC curve indicates the presence of several different superconducting phases.

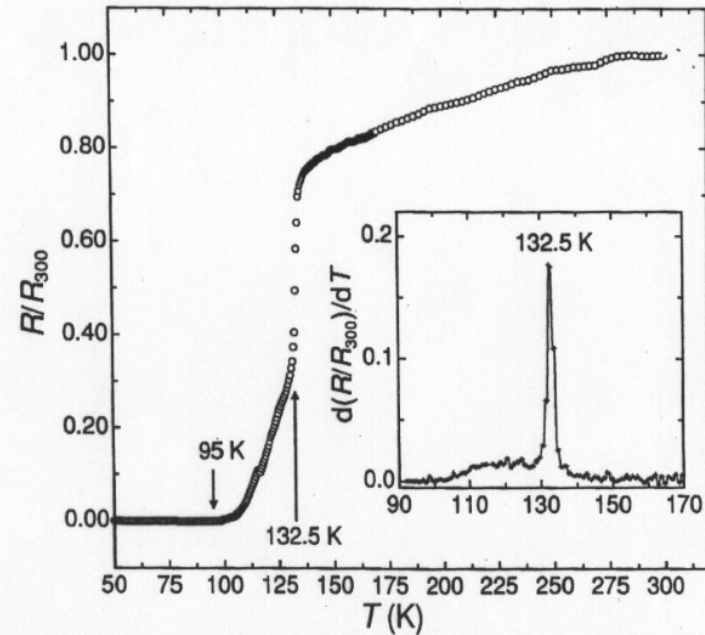
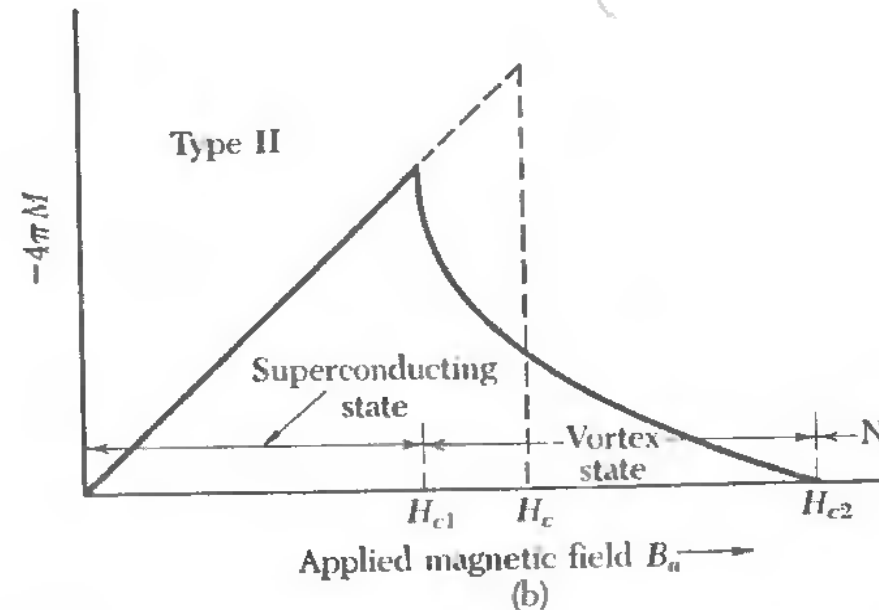
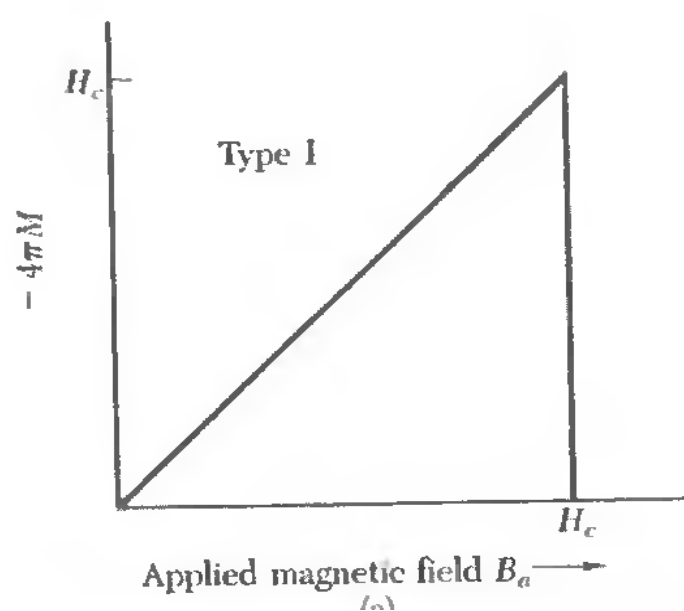


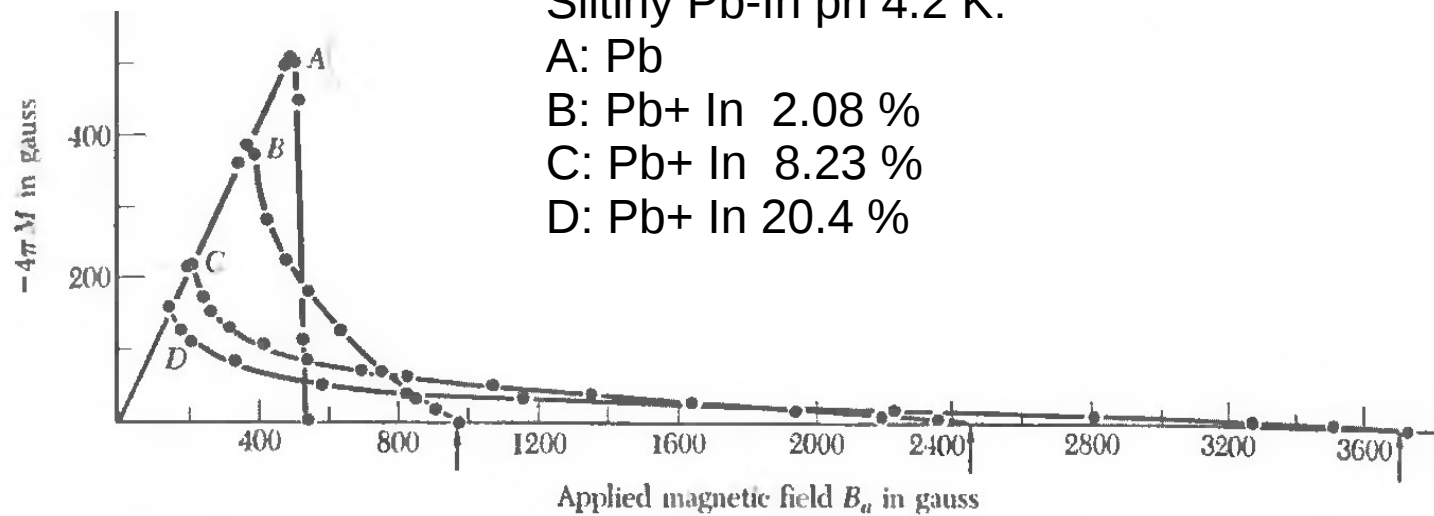
FIG. 3 Resistivity $R(T)$ of an annealed HBCCO specimen, normalized with respect to the resistance value $R(300) \approx 0.10 \Omega$. The inset displays the temperature derivative dR/dT to show the maximum resistivity drop at $T \approx 132.5$ K. Zero resistance is attained at $T=95$ K.

Supravodivost - závislost magnetizace na vnějším poli

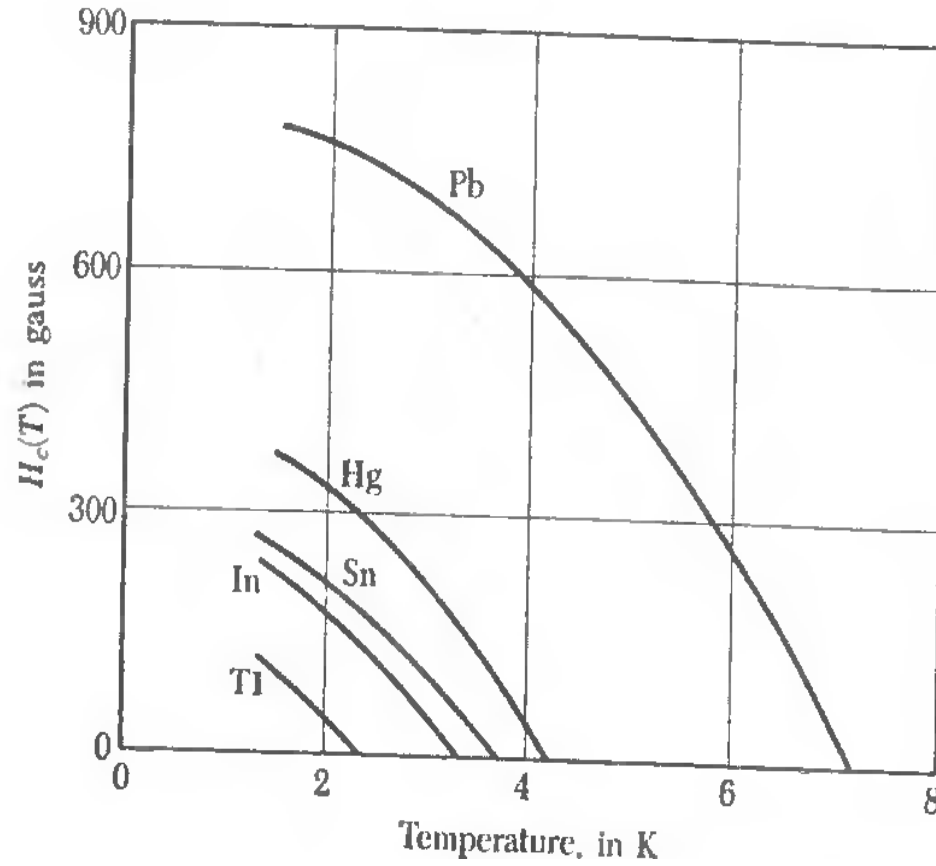


Slitiny Pb-In při 4.2 K:

- A: Pb
- B: Pb+ In 2.08 %
- C: Pb+ In 8.23 %
- D: Pb+ In 20.4 %



Supravodivost - Kritické pole supravodičů typu I



Supravodivost - Kritické pole supravodičů typu II

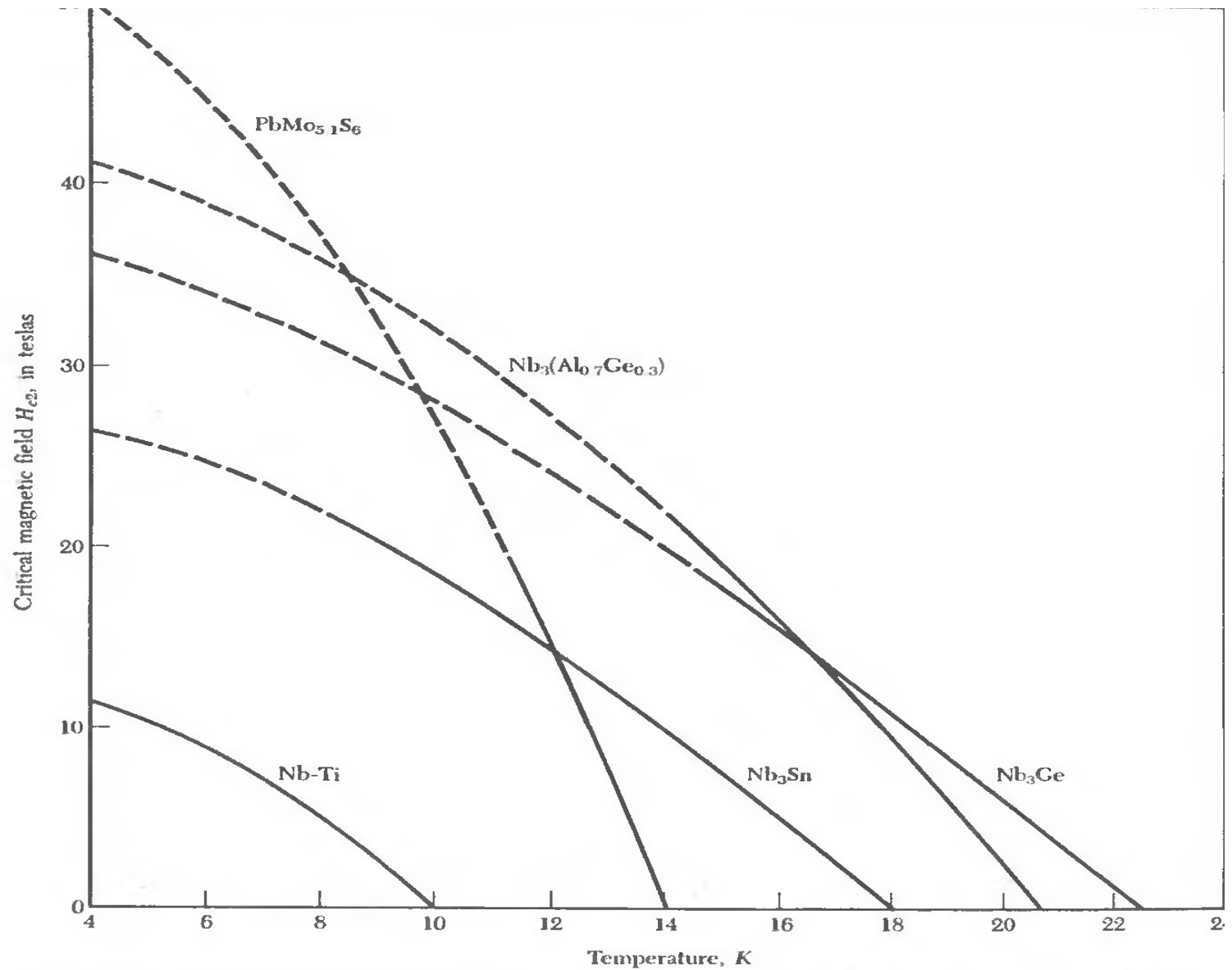


Figure 5b Stronger magnetic fields than any now contemplated in practical superconducting devices are within the capability of certain Type II materials. These materials cannot be exploited, however, until their critical current density can be raised and until they can be fabricated as finely divided conductors. (Magnetic fields of more than about 20 teslas can be generated only in pulses, and so portions of the curves shown as broken lines were measured in that way.)

Supravodivost - supravodiče typu II, aplikace

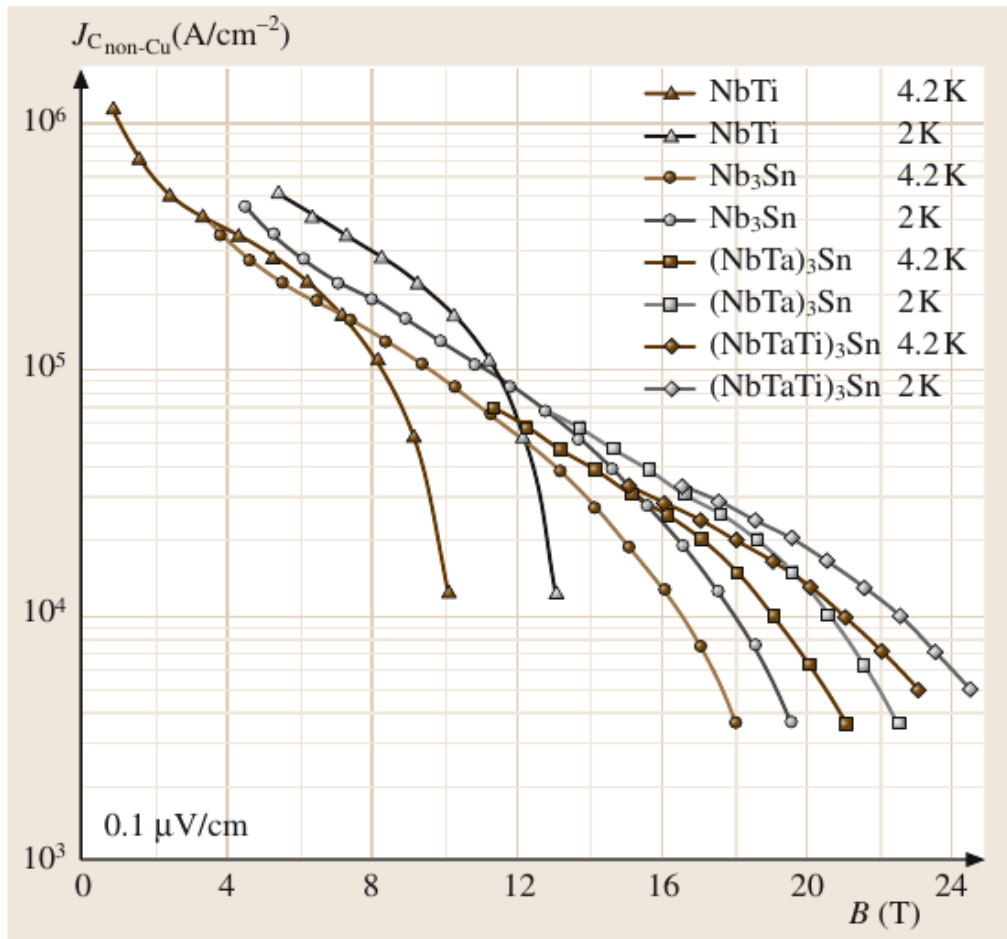
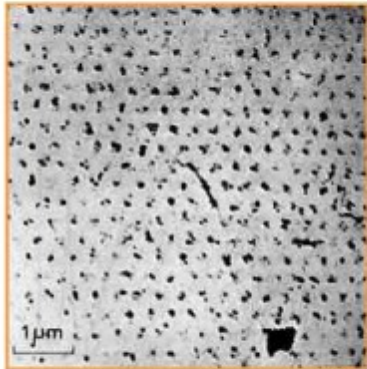


Fig. 4.2-6 Non-Cu J_c versus B characteristics of Nb–Ti, binary Nb₃Sn, alloyed (Nb, Ta)₃Sn, and alloyed (Nb, Ta, Ti)₃Sn multifilamentary superconductors [2.8]

Superconducting phase	Composition	T_c (K)	B_{c2} (T)	Magnetic field in application B (T)
Nb–Ti	46–52 wt% Ti, ≈ 47 wt% Ti optimal	≈ 10	≈ 10.5 (4.2 K)	≤ 9 (4.2 K) > 9 (1.8 K)
Nb ₃ Sn (Nb, Ta, Ti) ₃ Sn	25 at.% Sn, ≤ 7.5 wt% Ta, ≤ 0.2 wt% Ti	≈ 18	≈ 23 (4.2 K): ≈ 26–29 (4.2 K)	≤ 20 T (4.2 K) several T (< 4.2 K)

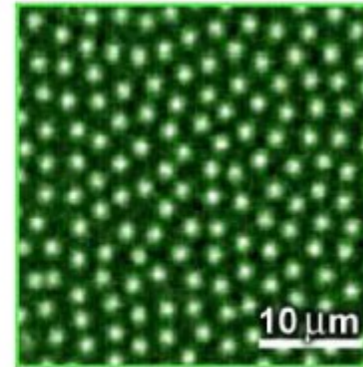
Vírové mřížky I



First image of Vortex lattice, 1967

Bitter Decoration
Pb-4at%In rod, 1.1K, 195G

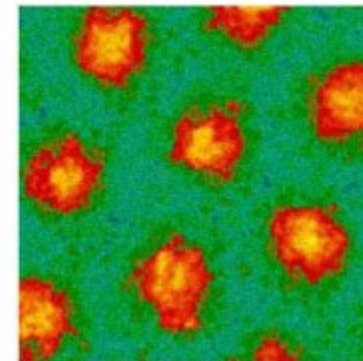
U. Essmann and H. Trauble
Max-Planck Institute, Stuttgart
[Physics Letters 24A, 526 \(1967\)](#)



Magneto-optical image of Vortex lattice, 2001

Magneto-Optical Imaging
NbSe₂ crystal, 4.3K, 3G

P.E. Goa et al.
University of Oslo
[Supercond. Sci. Technol. 14, 729 \(2001\)](#)



Vortices in MgB₂, 2002

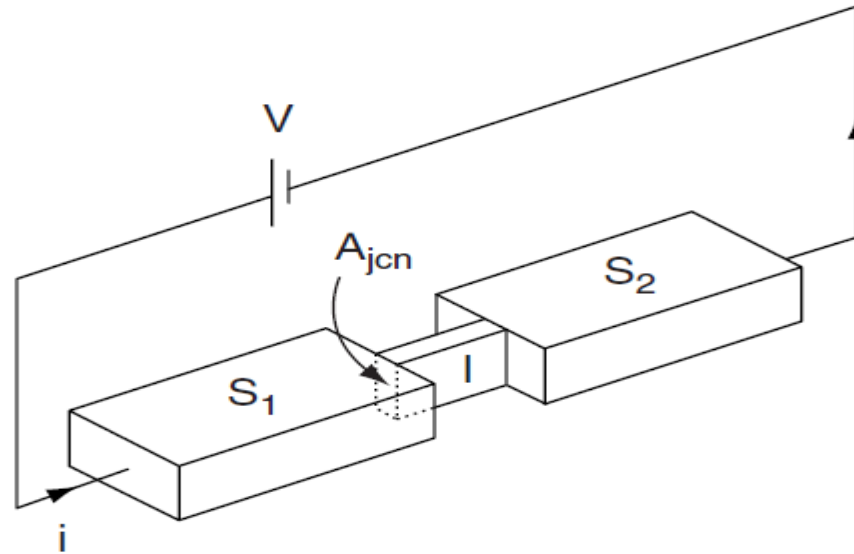
Scanning Tunnel Spectroscopy
MgB₂ crystal, 2K, 2000G

M. R. Eskildsen et al.
University of Geneva
[Phys. Rev. Lett. 89, 187003 \(2002\)](#)

Supravodiče

	Hloubka vniku (nm)	
Al	16	
Sn	34	
Pb	37	
Cd	110	
Nb	39	

Josephsonův přechod



$$\frac{d\theta}{dt} = \frac{2eV}{\hbar}$$

$$J(t) = J_c \sin \theta(t)$$

Josephsonův jev

DC Josephsonův jev

$$V = 0$$

$$J = J_c \sin \theta_0$$

AC Josephsonův jev

$$\theta(t) = \frac{2eVt}{\hbar} + \theta_0$$

$$\Phi_0 = 2\pi\hbar/2e$$

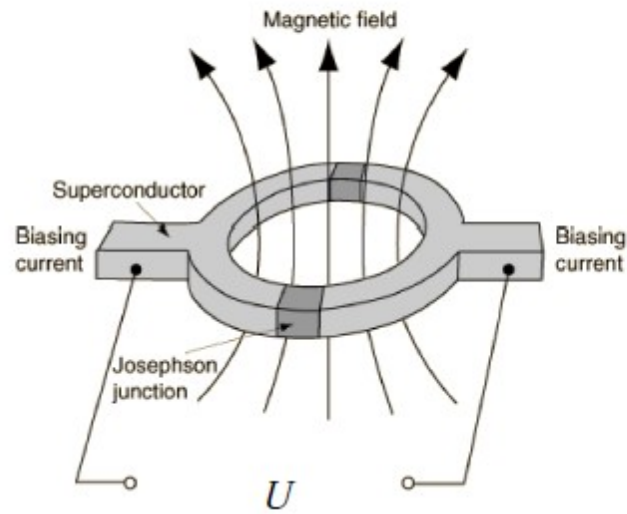
$$\Phi_0 = 2.0678 \times 10^{-15} \text{ Tm}^2$$

$$J(V, t) = J_c \sin \left(\frac{2eVt}{\hbar} + \theta_0 \right)$$

$$\omega_J = 2eV/\hbar = 2\pi V/\Phi_0$$

$$4.84 \times 10^{11} \text{ Hz/mV,}$$

Supravodivý kvantový interferometr



$$i_{\max}(\Phi) = 2i_c \left| \cos \frac{\pi\Phi}{\Phi_0} \right|$$

$$\Phi_0 = 2\pi\hbar/2e$$

$$\Phi_0 = 2.0678 \times 10^{-15} \text{ Tm}^2$$



Figure 2: Neuromag Ltd. 122 MEG system

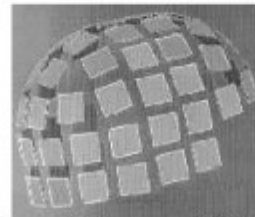


Figure 3: Neuromag Ltd. 122 sensor array

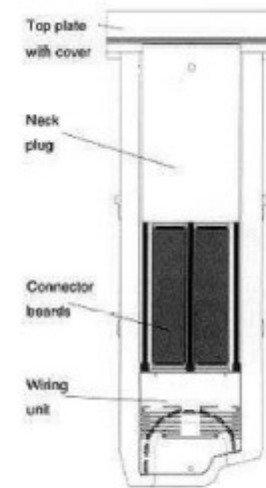
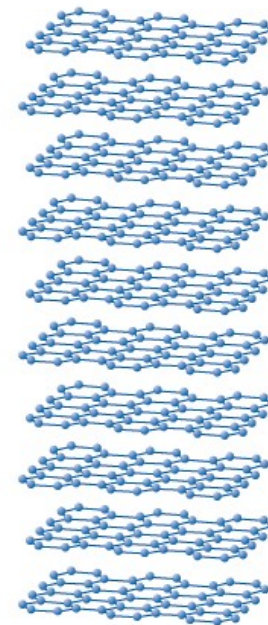
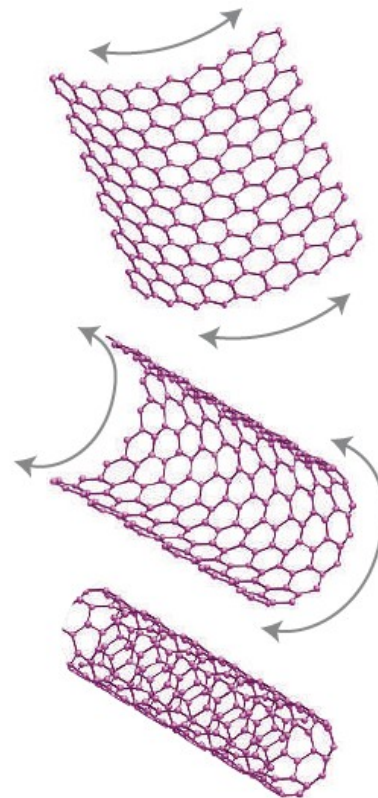
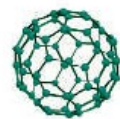
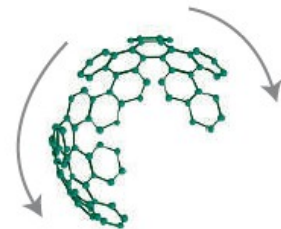
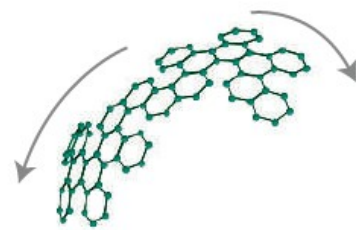
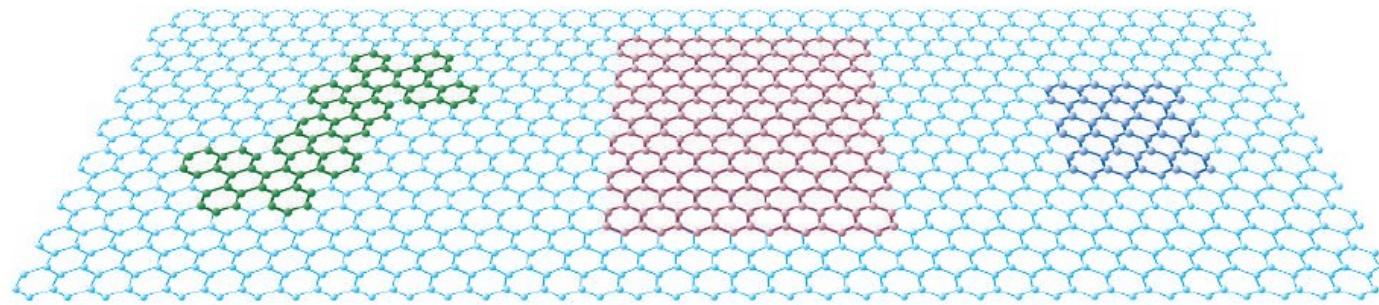


Figure 4: Neuromag Ltd. 122 probe

Grafen je dvojdimenzionální krystal uhlíku

A. Geim & K. Novoselov, Nature (2006)

grafen



Nobelova
cena
1996

fullerenes

nanotrúbky

grafit

nobelova cena za fyziku 2010

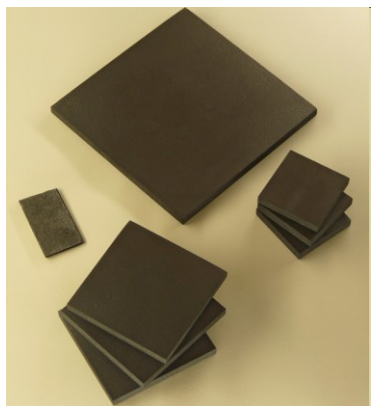


Andre K. Geim a Konstantin S. Novoselov



za zásadní objevy ohledně dvojdimenzionálního materiálu
grafenu

Jak by grafen objeven?

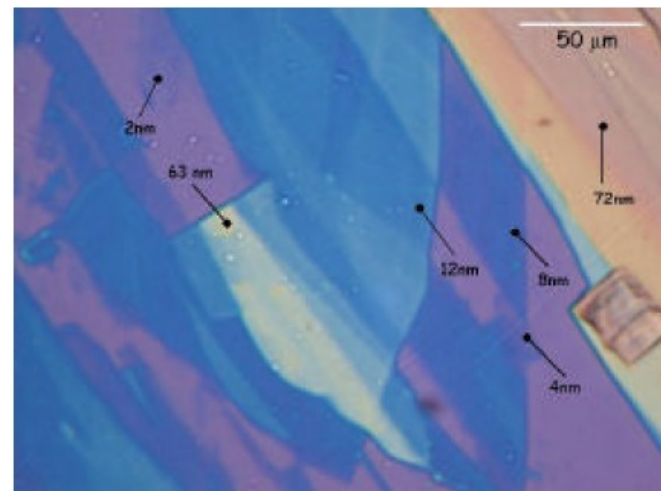


grafit

+



technika „izolepa“



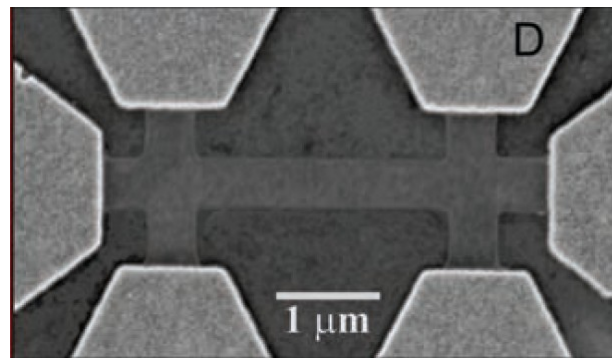
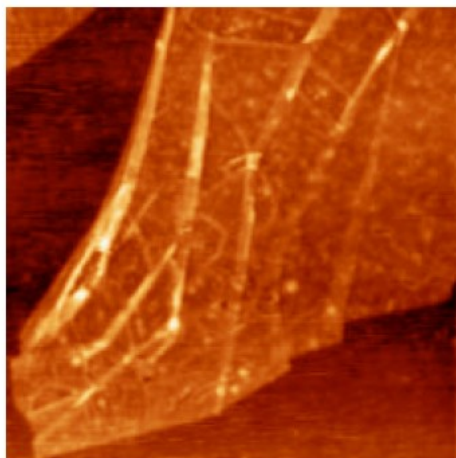
optická identifikace na SiO₂/Si substrátu



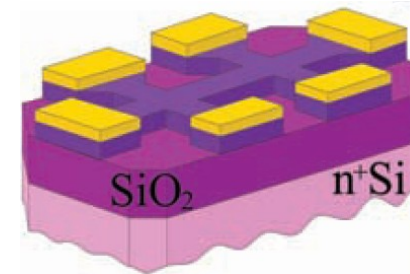
výška přehybu ~ 8 Å



AFM

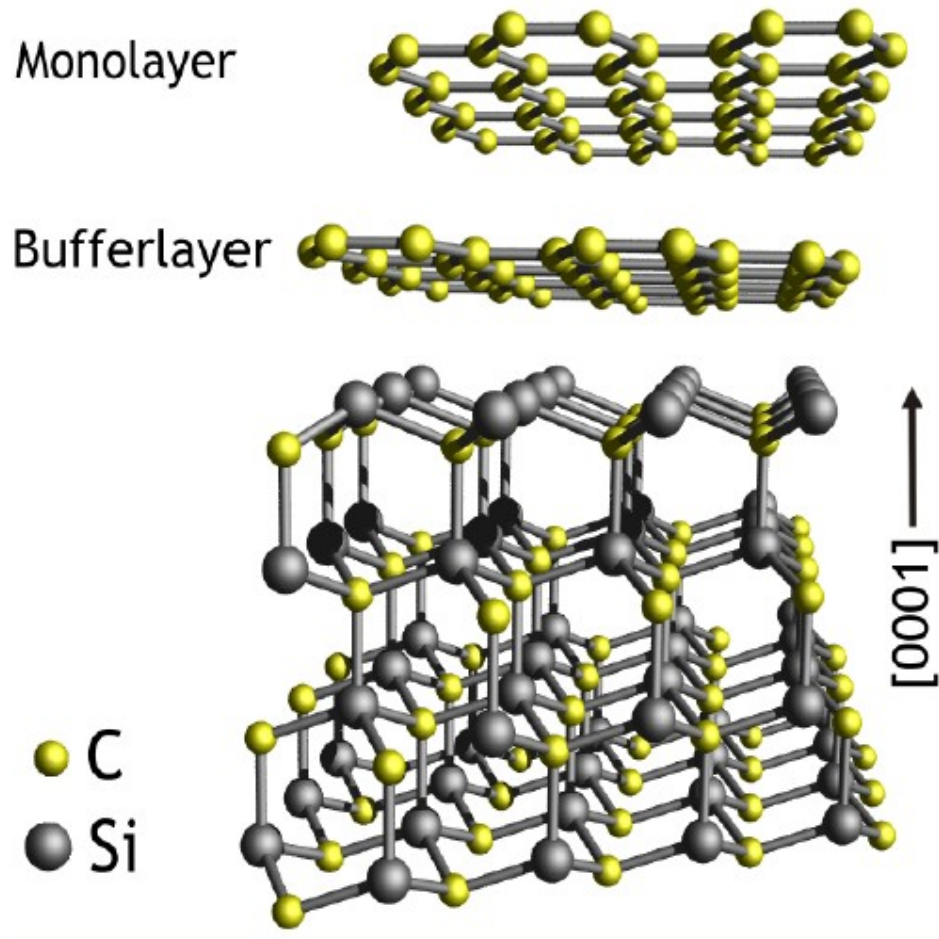


měření odporu atd.



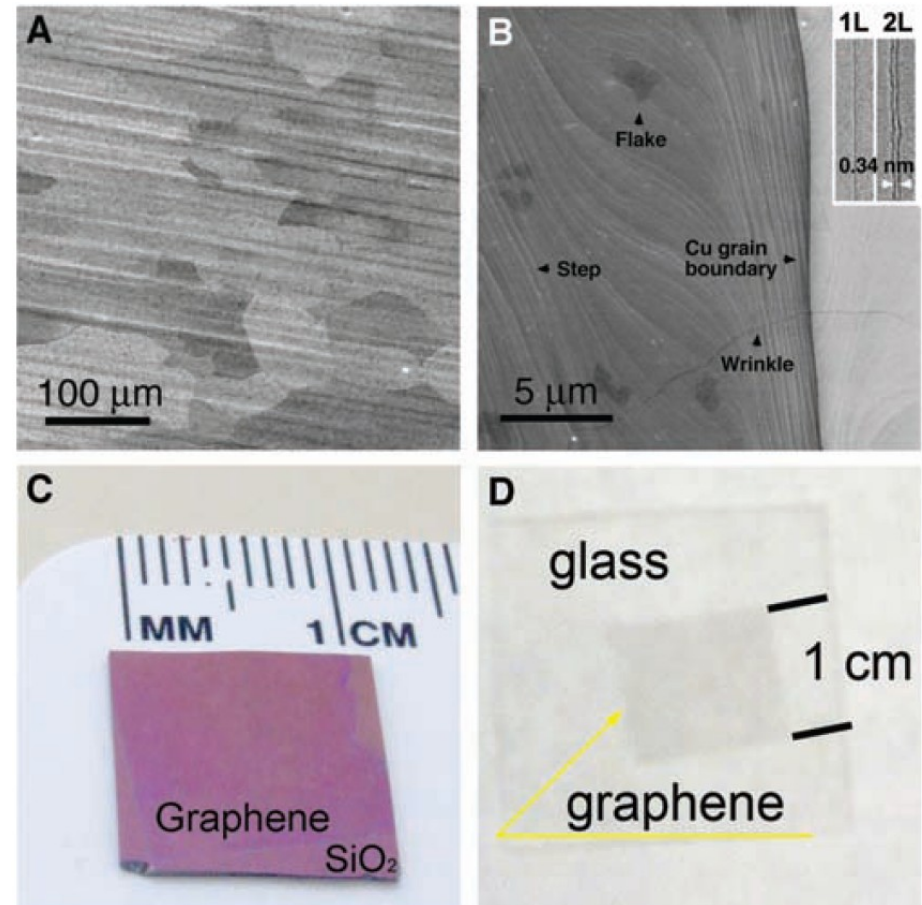
Alternativní metody přípravy grafenu

zahřívání karbidu křemíku



C. Berger *et al.*, J. Phys. Chem. B (2004)

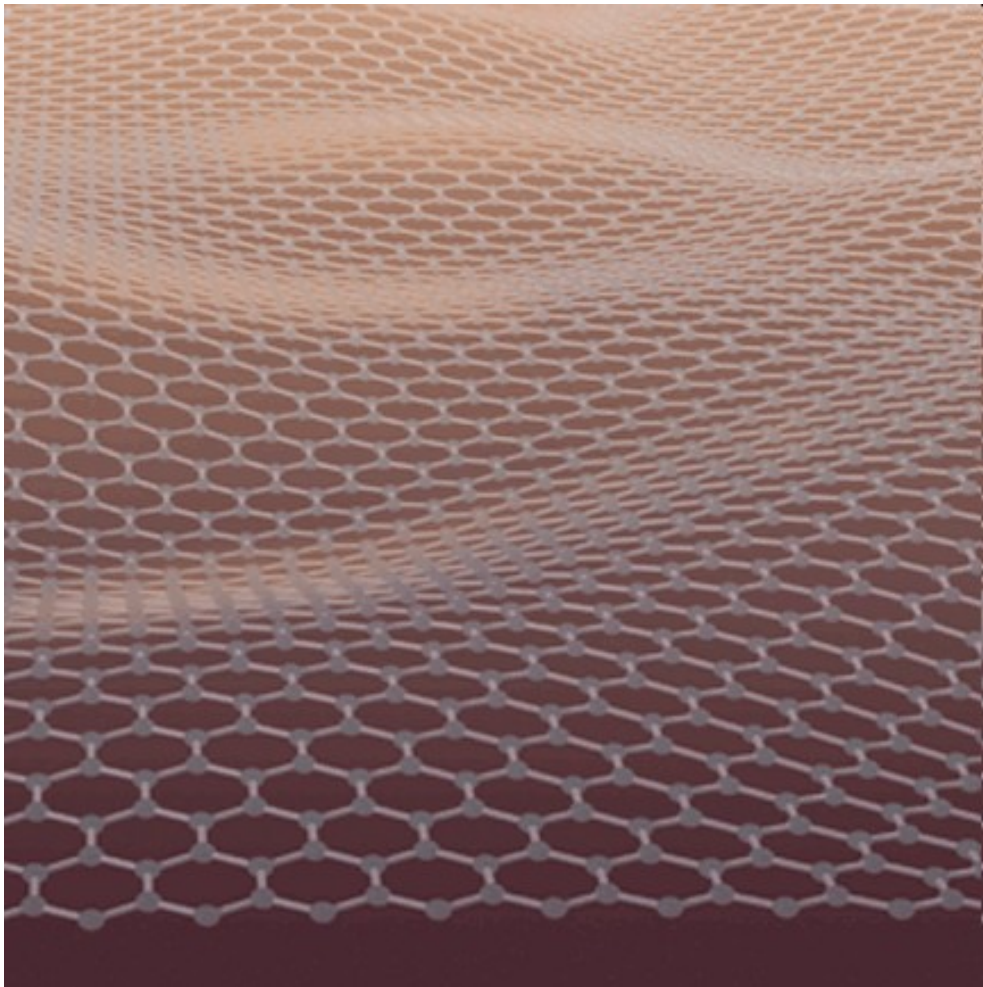
chemická depozice z plynné fáze na mědi



Li *et al.* Science (2009)

Grafen – materiál, který neměl existovat

- 1960 – teoretikové David Mermin a Herbert Wagner vypočítali že dvojdimenzionální krystal je nestabilní díky termickým fluktuacím

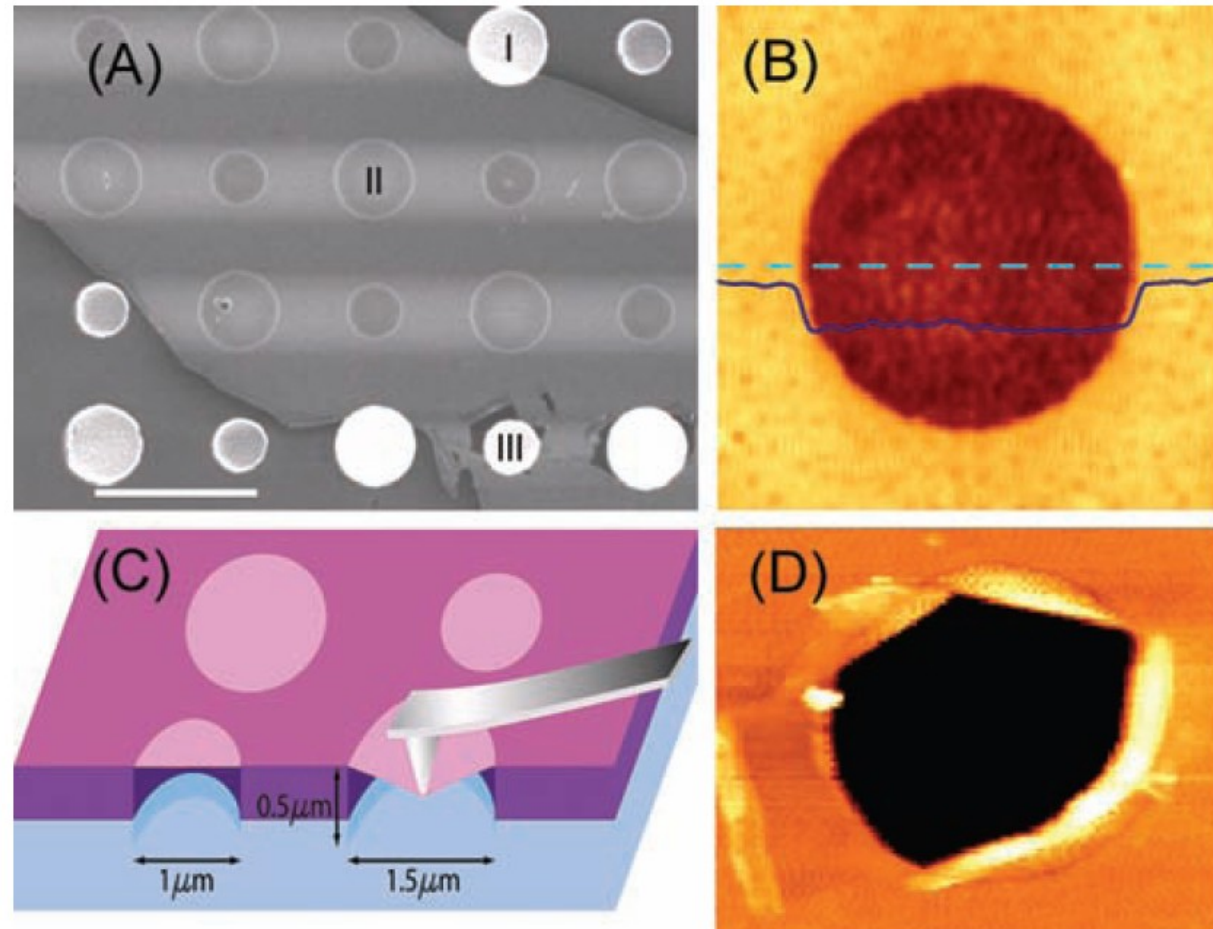


- Jejich teorie byla úspěšně testována mnohým pozorováním až do objevu grafenu ...
- malé zvlnění grafenu pravděpodobně krystal stabilizuje

umělecké zobrazení grafenu Davida Shanda,
obálka Nature Materials, Vol 6 (2007).

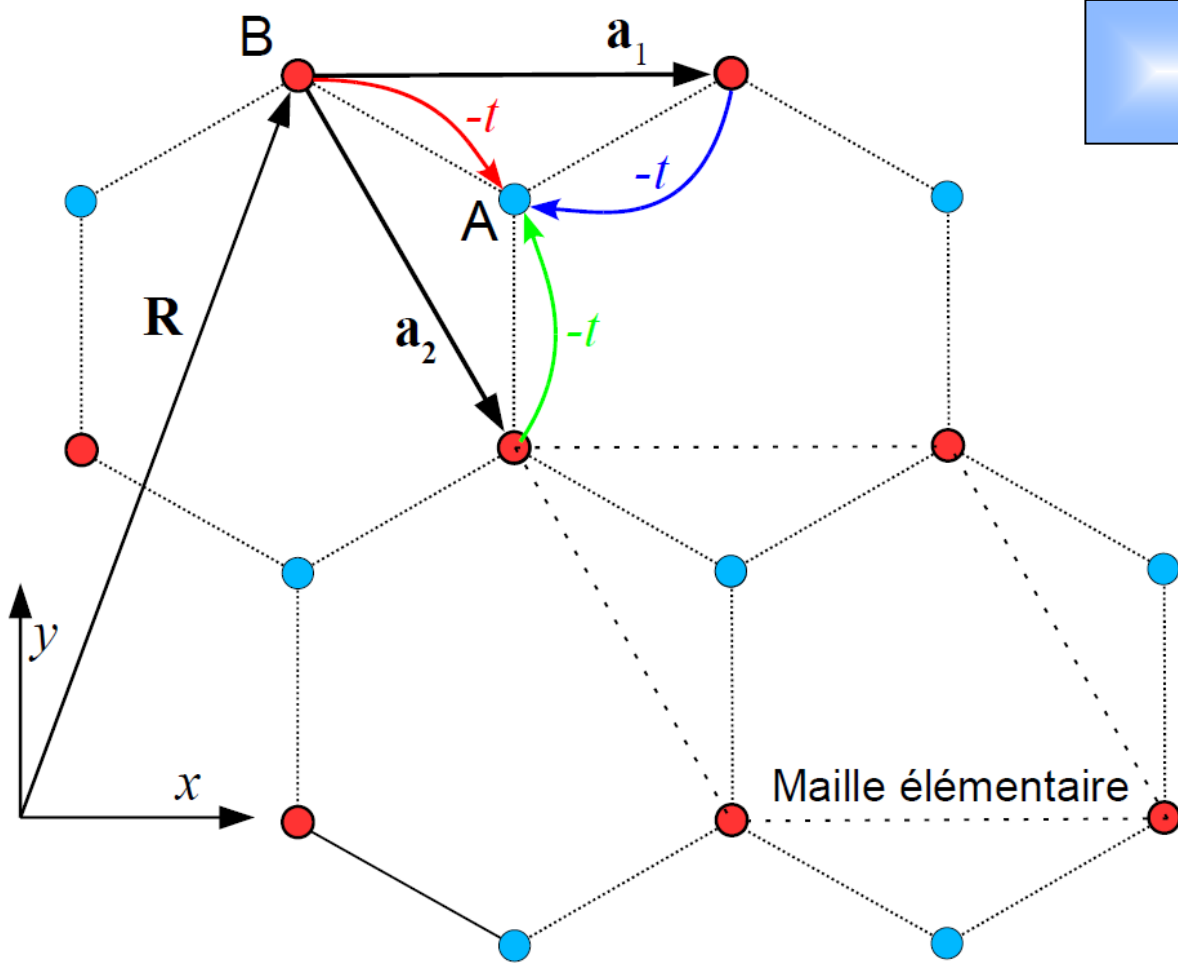
grafen má několik rekordních vlastností: *rekordní tuhost*

- nejpevnější materiál na světě
- mez pevnosti: 42 Nm^{-1}
- 200 krát silnější než ocel



Ch. Lee *et al.* Science (2008)

Model těsné vazby



- hexagonální mříž
- dvě trojúhelníkové podmříže - A a B
- přechody mezi nejbližšími sousedy

$$H = -t \sum_{\mathbf{R}, \sigma} [a_{\sigma}^{+}(\mathbf{R})b_{\sigma}(\mathbf{R}) + a_{\sigma}^{+}(\mathbf{R})b_{\sigma}(\mathbf{R} + \mathbf{a}_1) + a_{\sigma}^{+}(\mathbf{R})b_{\sigma}(\mathbf{R} + \mathbf{a}_2) + \text{c.c.}]$$

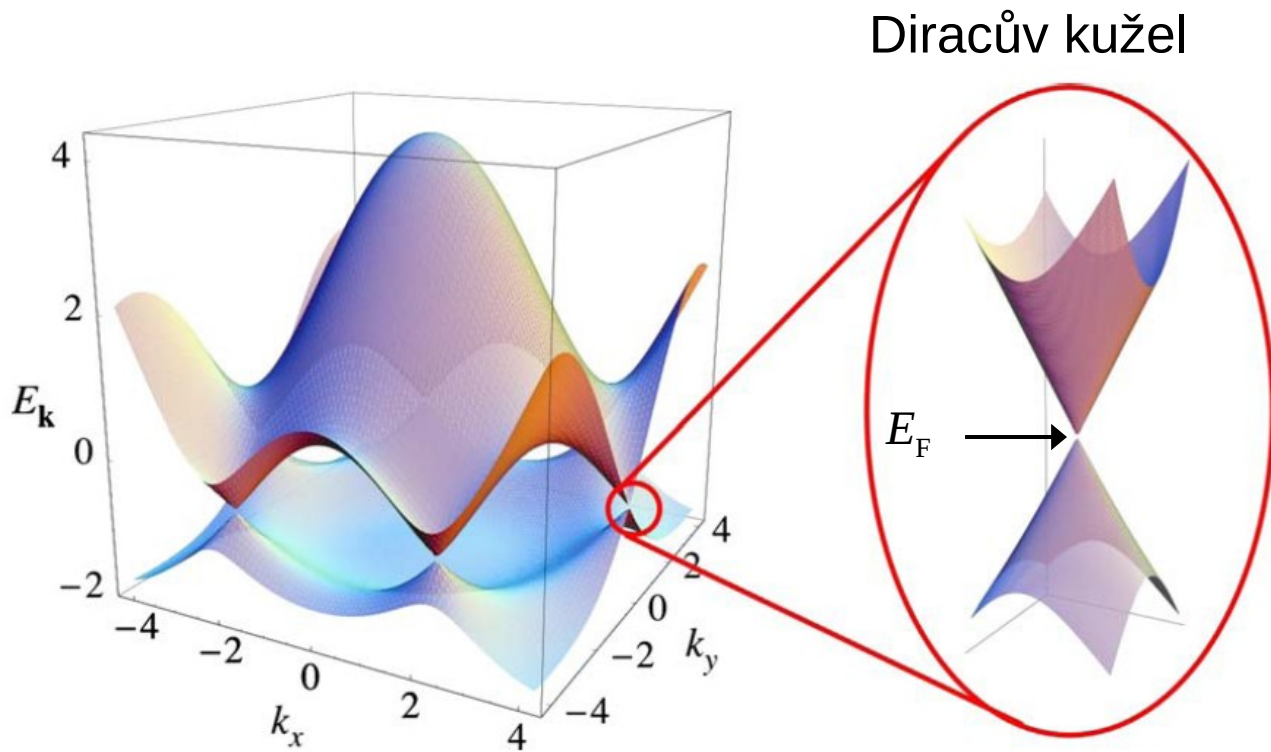
$$b_{\sigma}(\mathbf{R} + \mathbf{a}_1) = \int d\mathbf{k} b_{\sigma, \mathbf{k}} e^{-i\mathbf{k}(\mathbf{R} + \mathbf{a}_1)} = \int d\mathbf{k} b_{\sigma, \mathbf{k}} e^{-i\mathbf{k}\mathbf{R}} e^{-i\mathbf{k}\mathbf{a}_1}$$

$$H = -t \sum_{\mathbf{k}, \sigma} [a_{\sigma, \mathbf{k}}^{+} b_{\sigma, \mathbf{k}} \underbrace{(1 + e^{-i\mathbf{k}\mathbf{a}_1} + e^{-i\mathbf{k}\mathbf{a}_2})}_{s_{\mathbf{k}}} + b_{\sigma, \mathbf{k}}^{+} a_{\sigma, \mathbf{k}} s_{\mathbf{k}}^{*}]$$

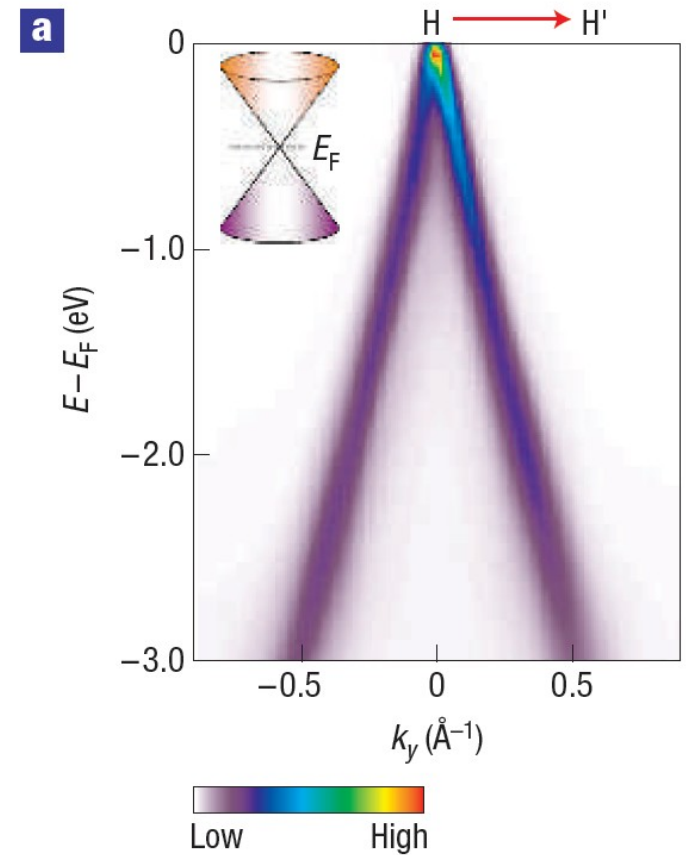
$$H = \sum_{\mathbf{k}, \sigma} \left(a_{\sigma, \mathbf{k}}^+, b_{\sigma, \mathbf{k}}^+ \right) \underbrace{(-t) \begin{pmatrix} 0 & s_{\mathbf{k}} \\ s_{\mathbf{k}}^* & 0 \end{pmatrix}}_{H_{\mathbf{k}}} \begin{pmatrix} a_{\sigma, \mathbf{k}} \\ b_{\sigma, \mathbf{k}} \end{pmatrix}, \quad H_{\mathbf{k}} |\psi\rangle = E_{\mathbf{k}} |\psi\rangle$$

P. R. Wallace Phys. Rev. (1947)

$$\Rightarrow E_{\mathbf{k}} = \pm t |s_{\mathbf{k}}| = \pm t \sqrt{3 + 2 \cos(ak_x) + 4 \cos(ak_x/2) \cos(ak_y \sqrt{3}/2)}$$



Castro Neto *et al.* Rev. Mod. Phys. (2009)



S. Y. Zhou *et al.* Nat. Phys. (2006)

- lineární závislost energie na (kvazi) impulzu blízko Fermiho meze

$$E_{\mathbf{q}} = \pm v_F \hbar \mathbf{q}, \quad \mathbf{q} = \mathbf{k} - \mathbf{K}$$

elektrony s nulovou klidovou hmotností

Kinetická energie volné hmotné nerelativistické částice:

$$E = \frac{1}{2}mv^2 = \frac{1}{2m}p^2$$

lineární energiová závislost
elektronů v grafenu:

$$E_{\mathbf{q}} = \pm v_F \hbar \mathbf{q} = \pm v_F \mathbf{p}$$

cos tím?

Energie relativistické hmotné
částice:

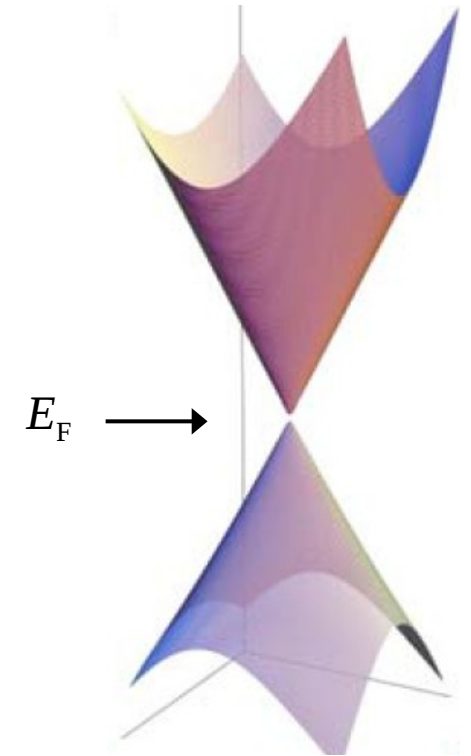
$$E^2 = m_0^2 c^4 + |c\mathbf{p}|^2$$

Srovnáním s grafenem vidíme, že
klidová hmotnost

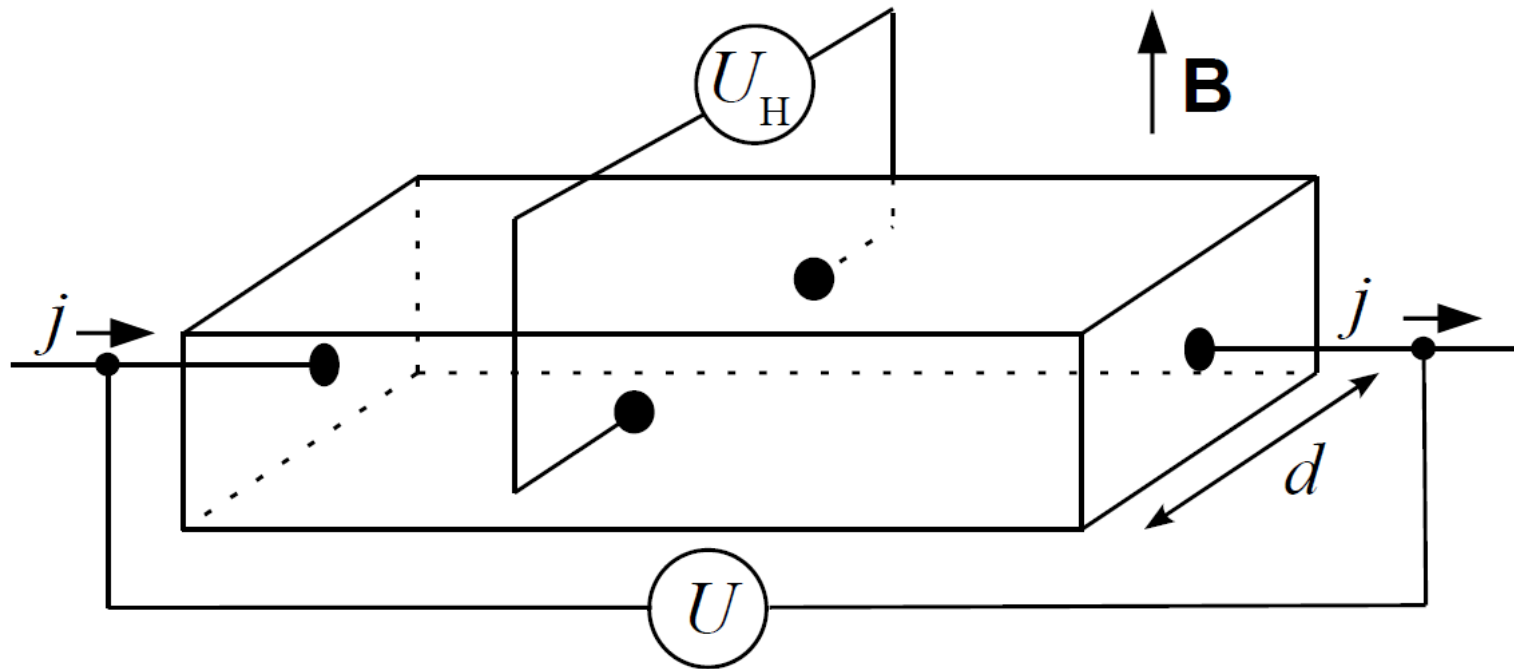
$$\implies m_0 = 0$$

Elektrony blízko Fermiho meze se *efektivně* chovají jako relativistické částice s nulovou klidovou hmotností, které se pohybují (Fermiho) rychlostí

$$v_F \approx \frac{c}{300}$$



Hallův efekt



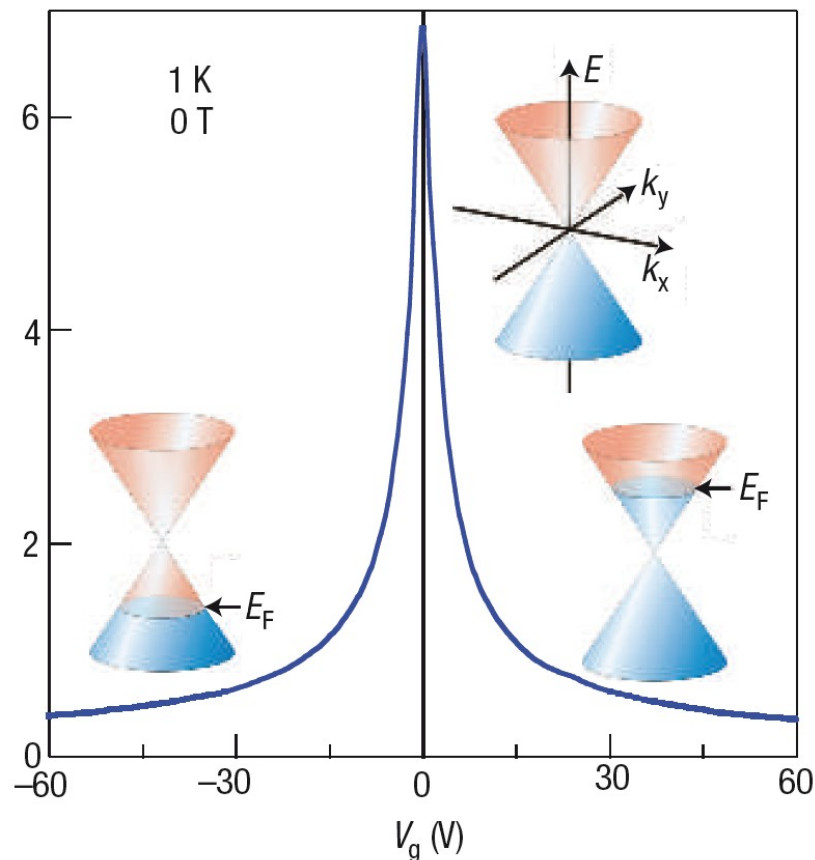
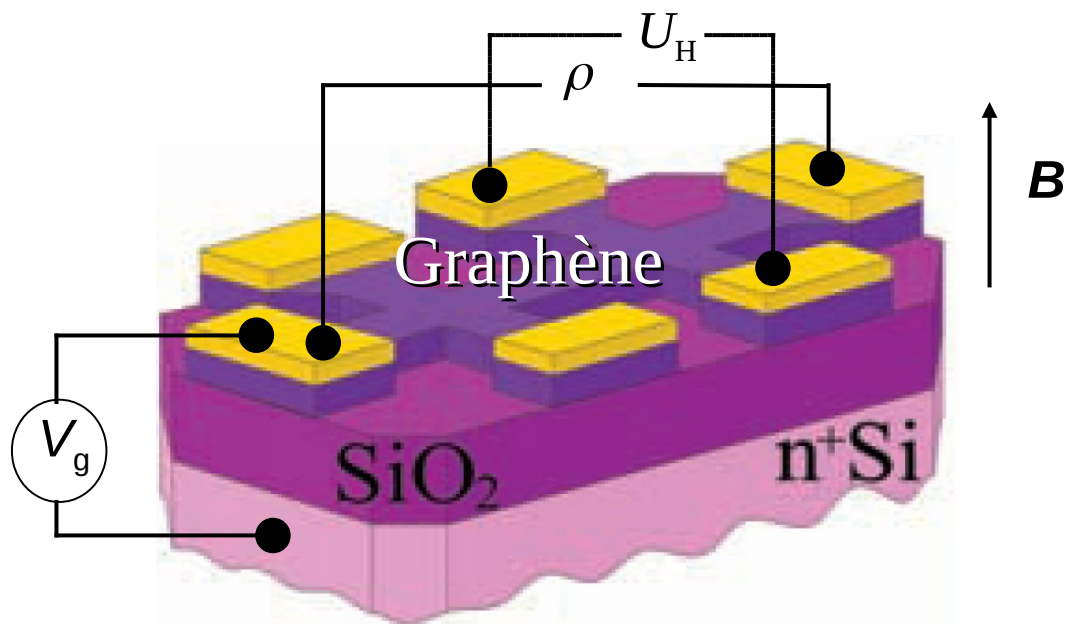
$$R_H \equiv \frac{dU_H}{jB} = \frac{1}{nq}$$

$$\rho_{xx} = \frac{U}{j}, \quad \rho_{xy} = \frac{U_H}{j}$$

$$\mu = \frac{R_H}{\rho_{xx}}$$

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 \rho_{yy}^2}, \quad \sigma_{xy} = -\frac{\rho_{xy}}{\rho_{xx}^2 \rho_{yy}^2}$$

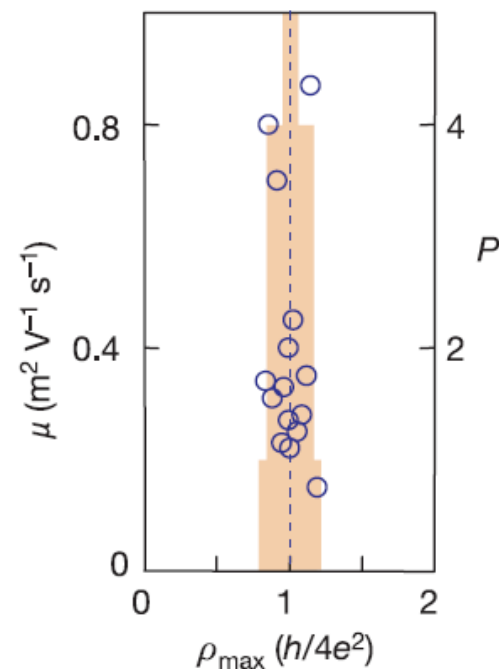
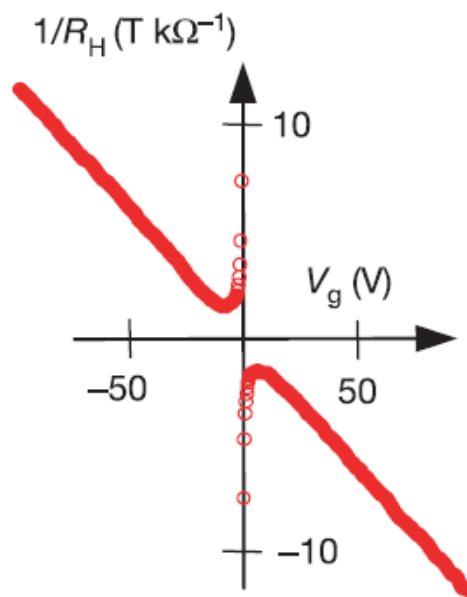
Řízení elektrickým polem



- elektrické pole mění koncentraci elektronů

- inverzní Hallův koeficient $1/R_H = nq$ je lineární a mění znaménko při $V_g = 0 \Rightarrow$ vytváříme buď elektrony nebo díry

- kvantovaný odpor $\rho_{\max} = h/4e^2$



K.S. Novoselov, A. K. Geim, *et al.* Nature (2005)

A. K. Geim and K.S. Novoselov Nature (2006)

Grafen

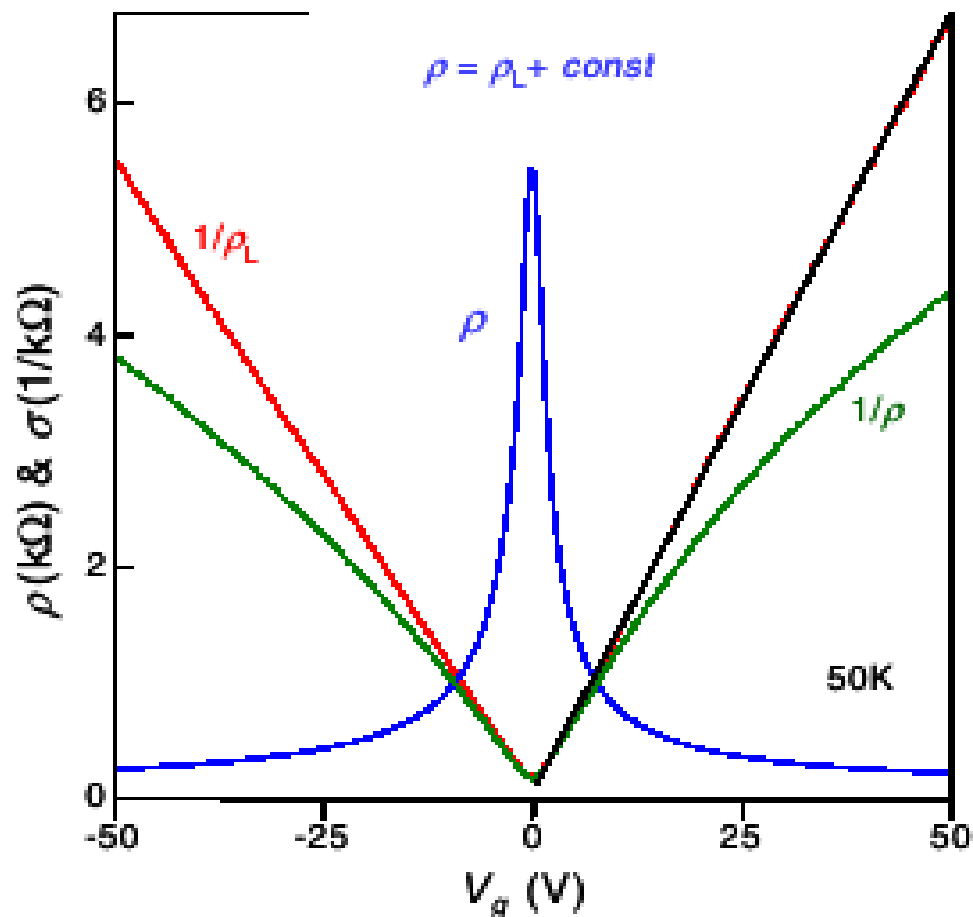
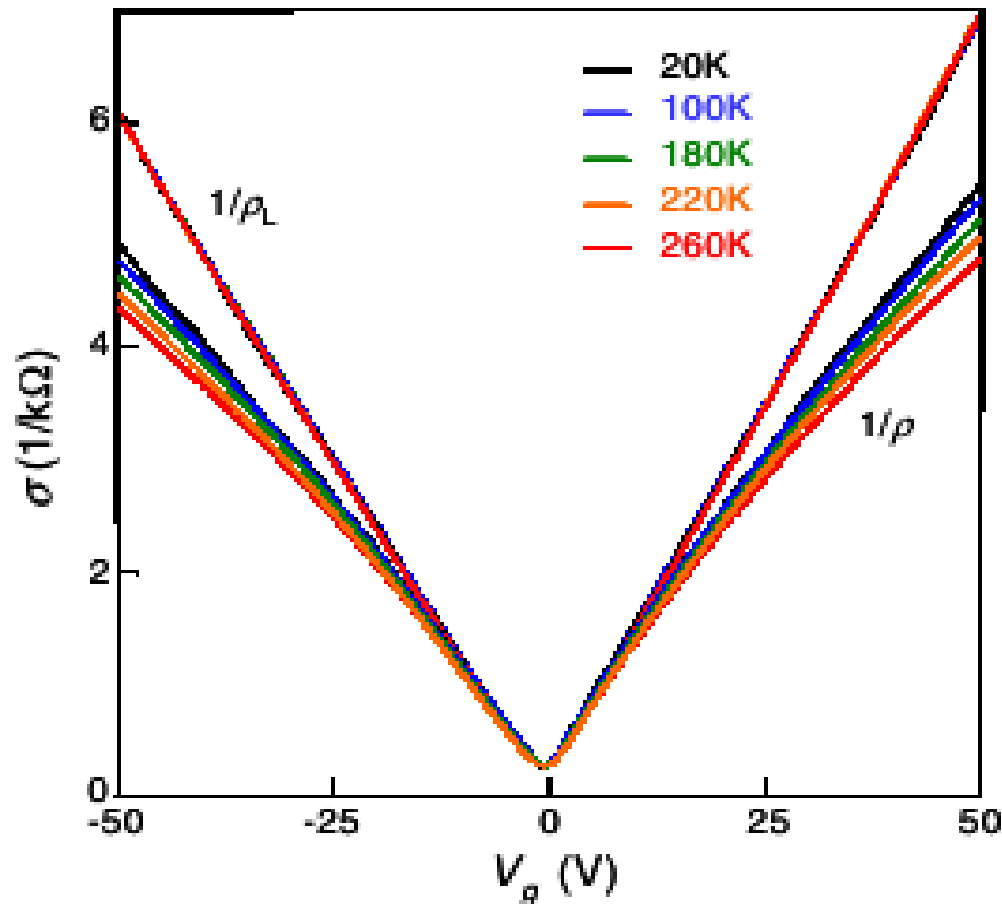


FIG. 1 (color online). Resistivity ρ (blue curve) and conductivity $\sigma = 1/\rho$ (green curve) of SLG as a function of gate voltage. If we subtract a constant of $\approx 100 \Omega$ (used here as a fitting parameter), the remaining part $\rho_L(V_g)$ of resistivity becomes inversely proportional to V_g (red curve). The thin black line (on top of the red curve for $V_g > 0$) is to emphasize the linearity (the red curve is equally straight for negative V_g). The particular device was $1 \mu\text{m}$ wide, and $T = 50 \text{ K}$ was chosen to be high enough to suppress universal conductance fluctuations, still visible on the curves.

Grafen

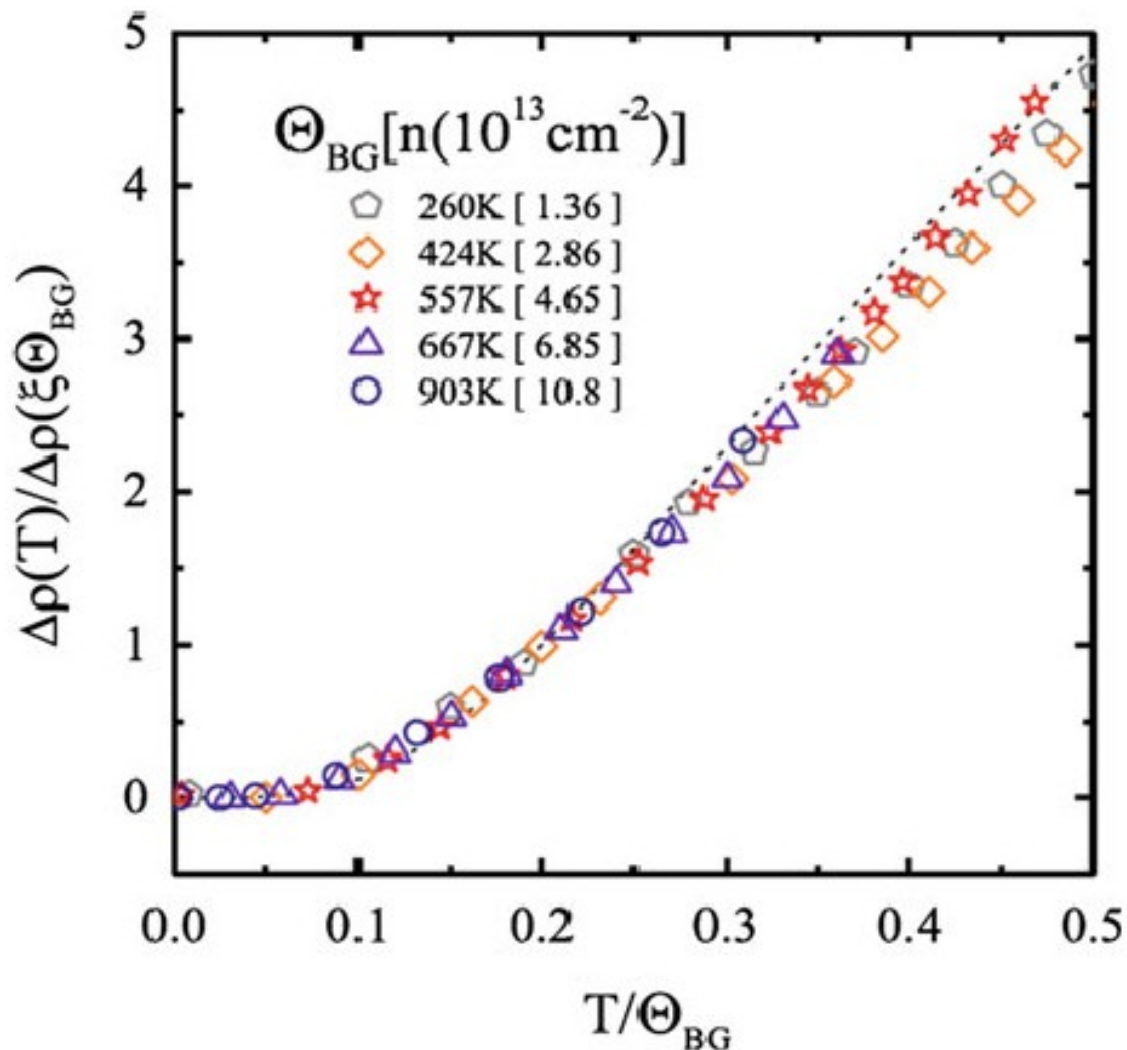


S. V. Morozov,
K. S. Novoselov,
M. I. Katsnelson,
F. Schedin, D. C. Elias,
J. A. Jaszczak,
and A. K. Geim,
PRL **100**, 016602 (2008).

FIG. 2 (color online). Electron transport in graphene below 300 K can be described by the empirical expression $\rho(V_g, T) = \rho_L(V_g) + \rho_S(T)$ where ρ_S is independent of V_g but varies with T . After subtracting ρ_S that for this sample changed from $\approx 40 \Omega$ at low T to $\approx 70 \Omega$ at 260 K, the resulting curves $\sigma_L(V_g) = 1/\rho_L(V_g)$ became indistinguishable (the cluster marked $1/\rho_L$ consists of 5 such curves). The experiments were carried out in a field of 0.5 T to ensure that weak localization corrections (rather small [1,22] but still noticeable) do not contribute into the reported T dependences.

Grafen

Fig. 1.5 Normalized dependence of graphene resistivity on temperature, dominated by 2D acoustic phonon scattering. (Reprinted figure with permission from Efetov and Kim, Fig. 4. Copyright (2010) by the American Physical Society)



Bloch-Grüneisen temperature

$$\theta_{BG} = 2\hbar v_S k_F / k_B < \theta_D$$

$$v_F = 10^6 \text{ m/s}, v_S = (2.6 \pm 0.4) \times 10^4 \text{ m/s}$$

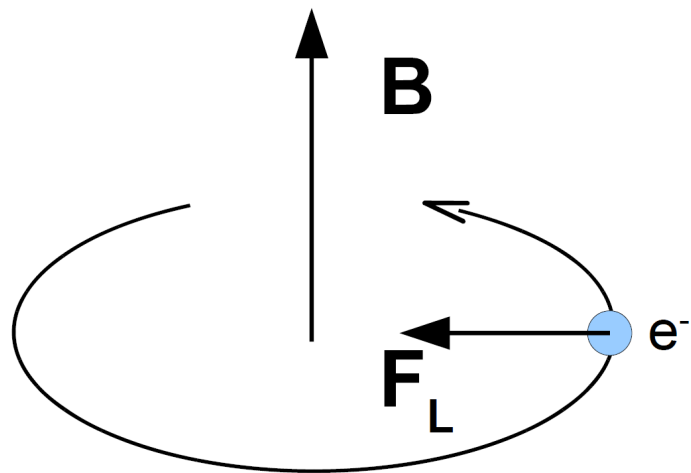
Grafen

Material	Remarks	Carrier mobility (cm ² /V.s)		
		~ 4.2K	~ 77K	300K
Copper			~ 200	32
Si (bulk)	For electrons (holes = 450)			1,450
GaAs	Heterostructure -2DEG	36.10 ⁶	3.10 ⁵	5.10 ³
Bismuth	Single crystal, large size, high purity	43.10 ⁶	6.4 10 ⁵	3.2 10 ⁴
HOPG	Heat treated above 3,000 C		6.7 10 ⁴	1.2 10 ⁴
Graphene	Highest values claimed*	2. 10 ⁵ -10 ⁶		5.10 ⁴

Aoki, Dresselhaus (eds.): Physics of graphene, Springer (2014).

Cyklotronová frekvence

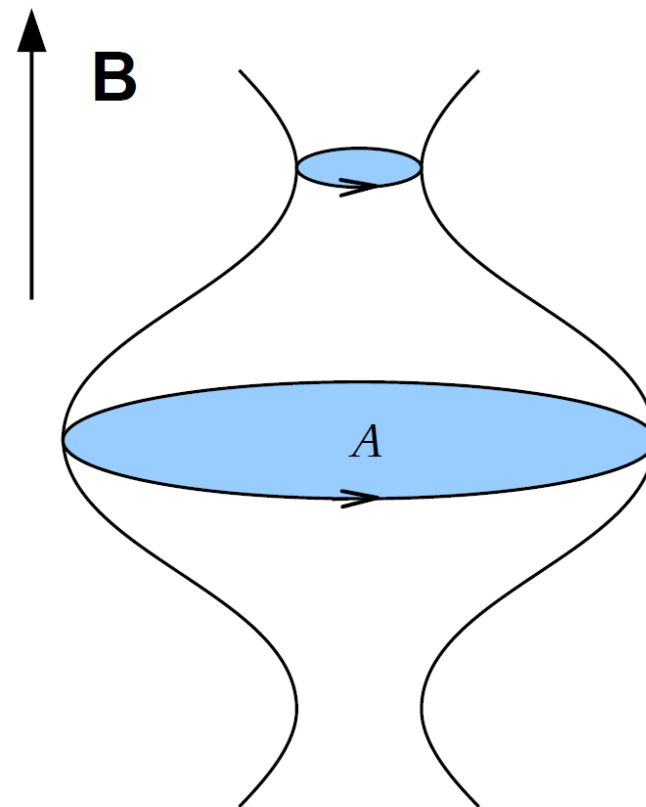
volný elektron v magnetickém poli



rotuje s (cyklotronovou) frekvencí

$$\omega_c = \frac{eB}{m}$$

elektron v pásové struktuře

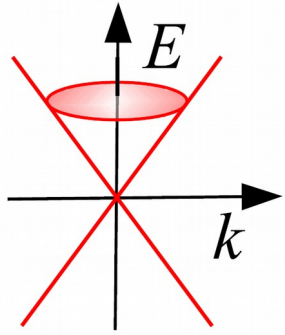


$$\omega_c = \frac{eB}{m_c}, \quad m_c = \frac{\hbar^2}{2\pi} \frac{\partial A(E)}{\partial E}$$

A = plocha orbity

m_c = cyklotronová hmotnost

Cyklotronová efektivní hmotnost



Lineární disperze

$$E = \pm v_F \hbar k$$

$$A(E) = \pi k^2 = \frac{\pi}{\hbar^2} \frac{E^2}{v_F}$$

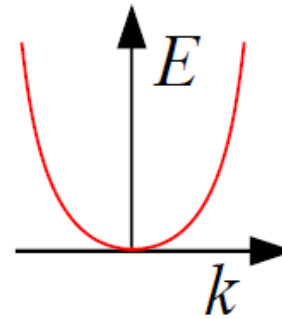
$$m_c = \frac{\hbar^2}{2\pi} \frac{\partial A(E)}{\partial E} = \frac{E}{v_F^2}$$

$$E = m_c v_F^2$$

“ $E = mc^2$ ”

$$n = \frac{k_F^2}{\pi} \implies m_c = \frac{\hbar^2 \sqrt{\pi}}{v_F} \sqrt{n}$$

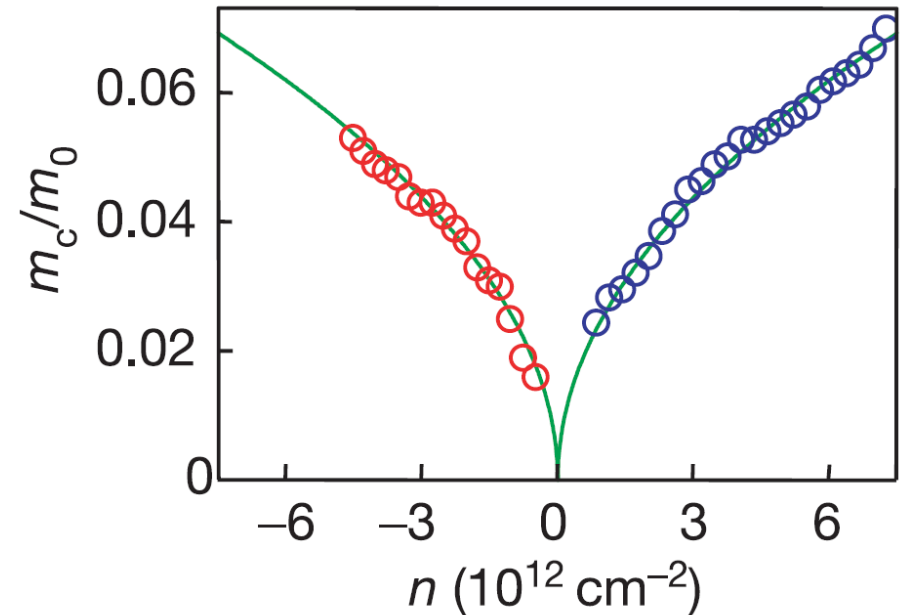
Parabolická disperze



$$E = \frac{1}{2} \frac{\hbar^2}{m^*} k^2$$

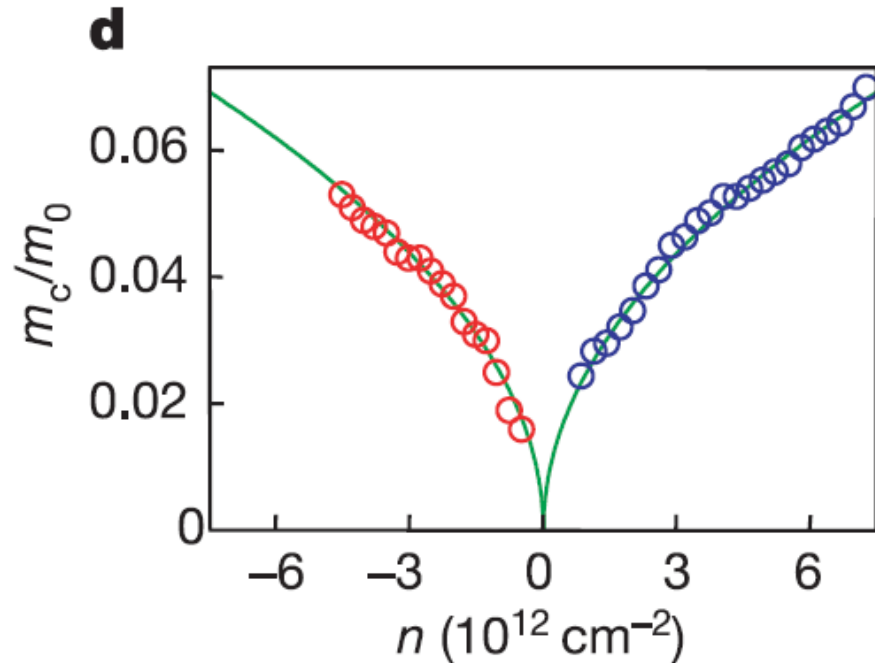
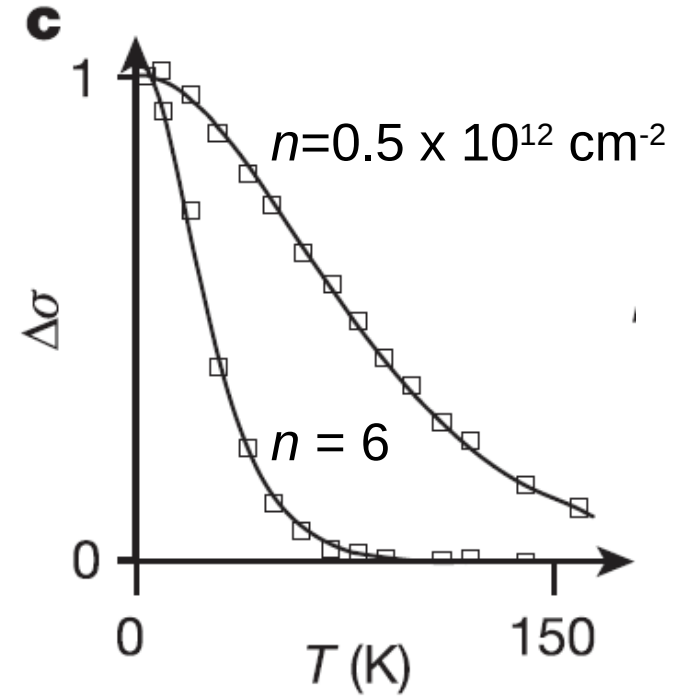
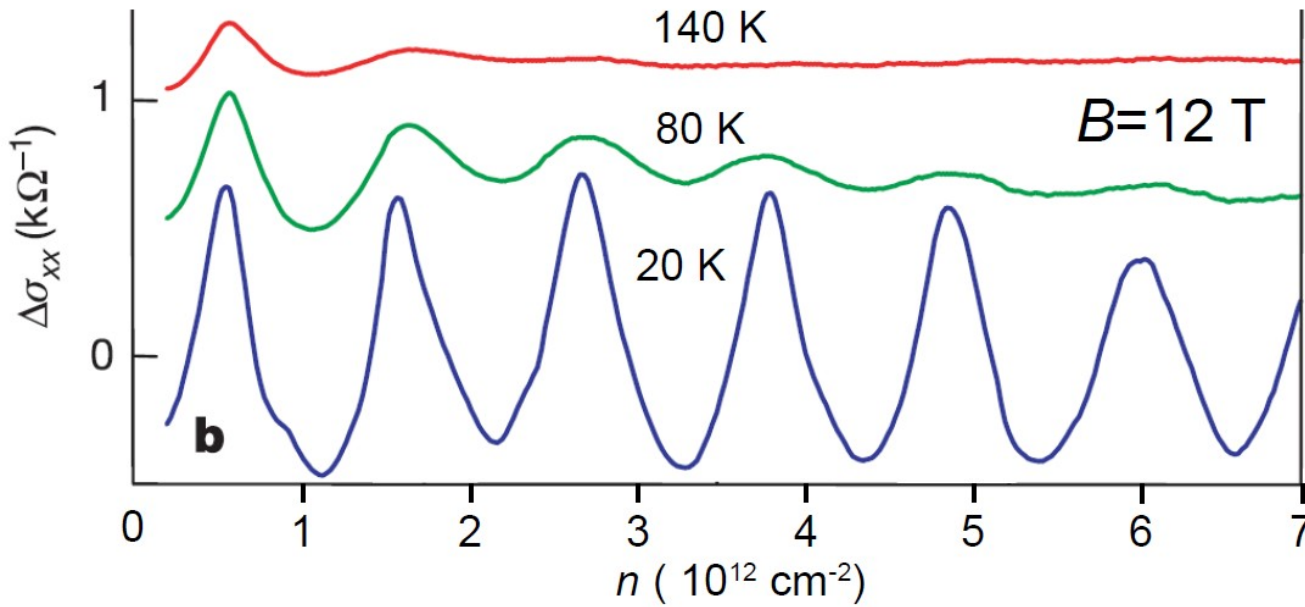
$$m_c = m^*$$

d



Oscillation quantiques de la résistance (osc. de Shubnikov-de Haas)

K.S. Novoselov, A. K. Geim, et al. Nature (2005)

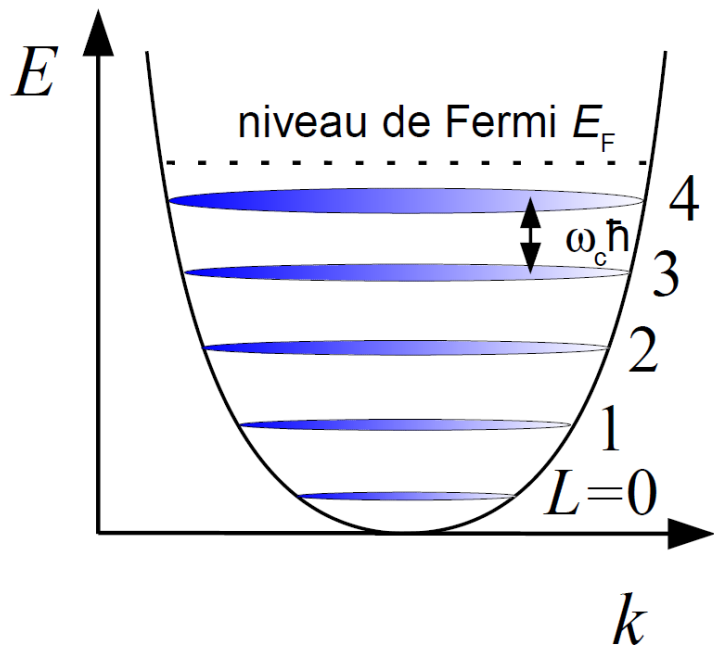
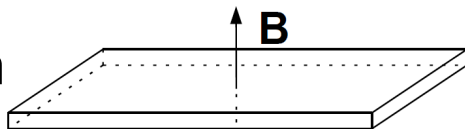


$$\Delta\sigma \propto \frac{x}{\sinh x}$$

$$x = \frac{2\pi^2 k_B T}{\hbar\omega_c} = \frac{2\pi^2 k_B T m_c}{\hbar e B}$$

Kvantování stavů v magnetickém poli: Landauovy hladiny

nerelativistické elektron
ve 2D



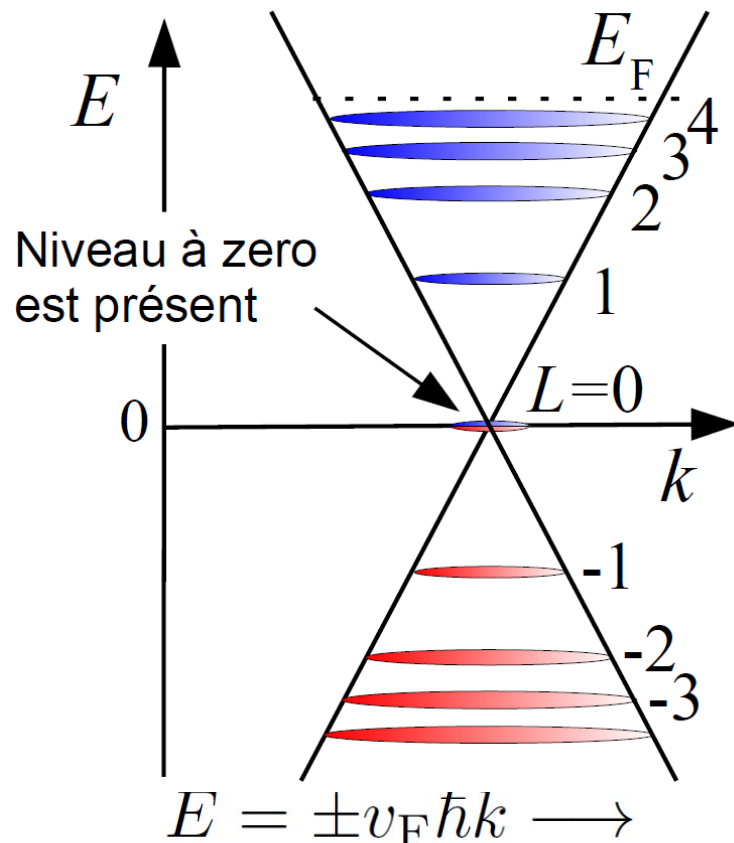
$$E(k) = \frac{\hbar^2 k^2}{2m} \longrightarrow$$

$$E_L = \hbar\omega_c \left(L + \frac{1}{2}\right)$$

$$L = 0, 1, 2 \dots, \quad \omega_c = \frac{eB}{m_c}$$

L.D. Landau, Z. Phys. 64, 629 (1930)

relativistické částice s $m_0=0$

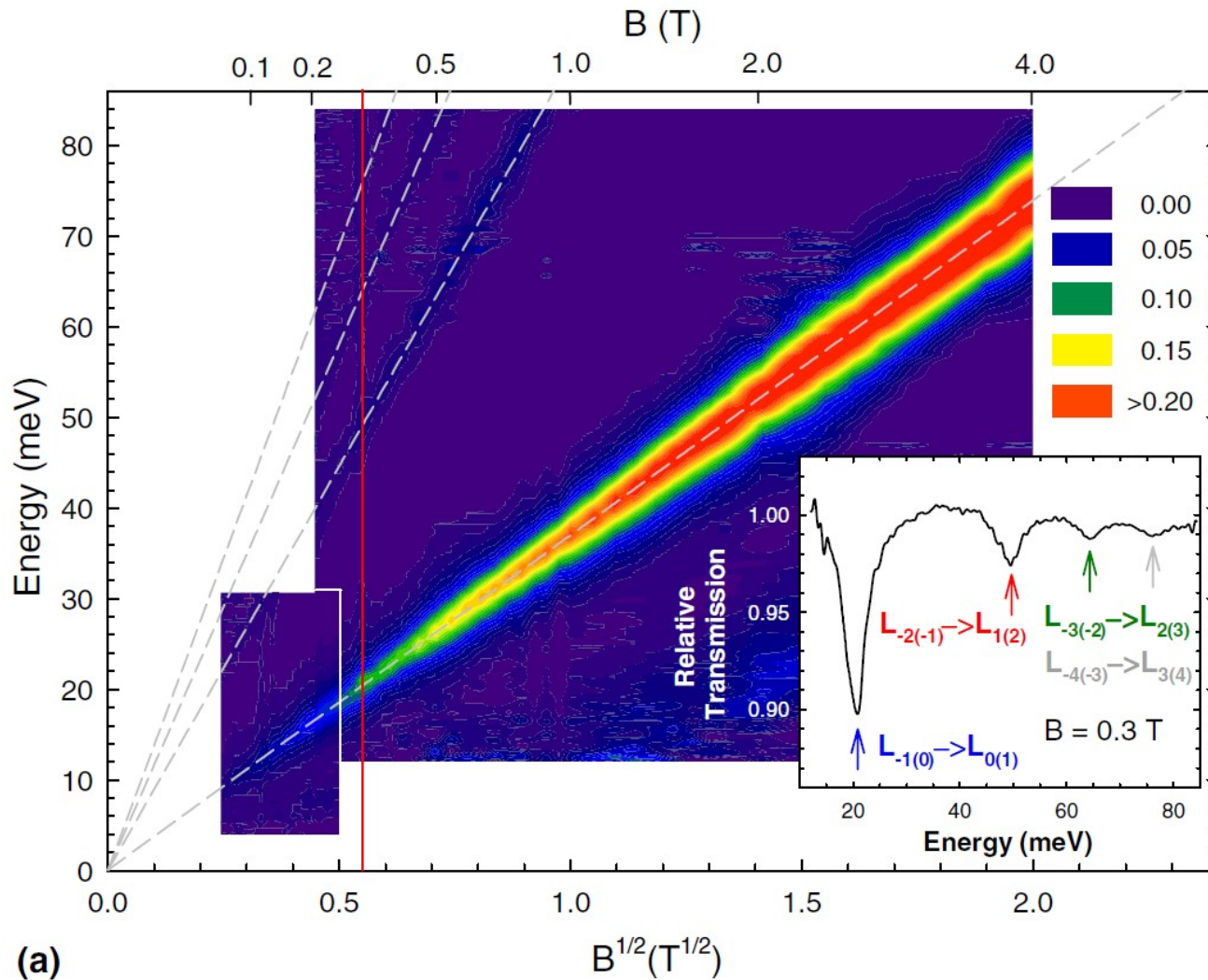


$$E_L = \text{sgn}(L) \sqrt{2e\hbar v_F^2 |LB|}$$

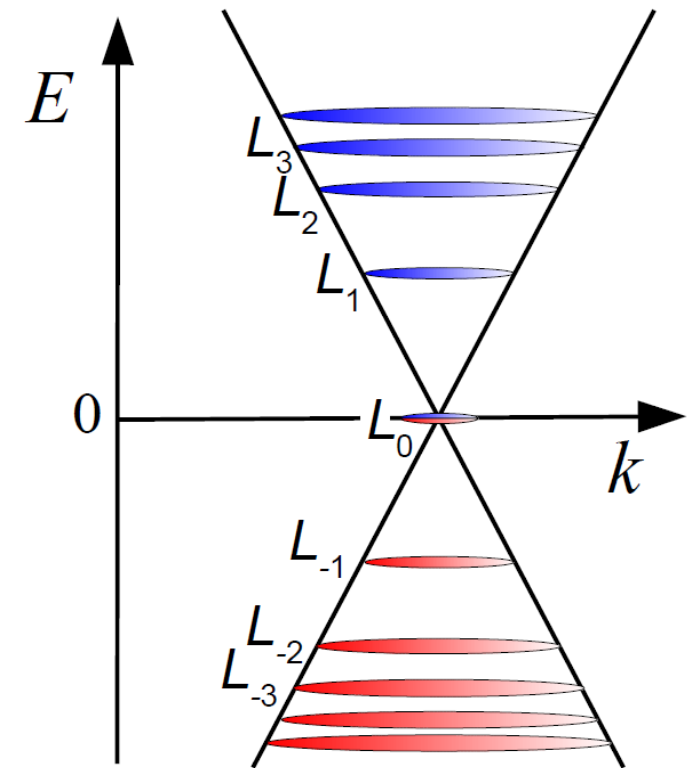
$$L = 0, \pm 1, \pm 2, \dots$$

I.I. Rabi, Z. Phys. 49, 507 (1928)

Infračervená spektroskopie v magnetickém poli



(a) M. Orlita et al. PRL (2008)

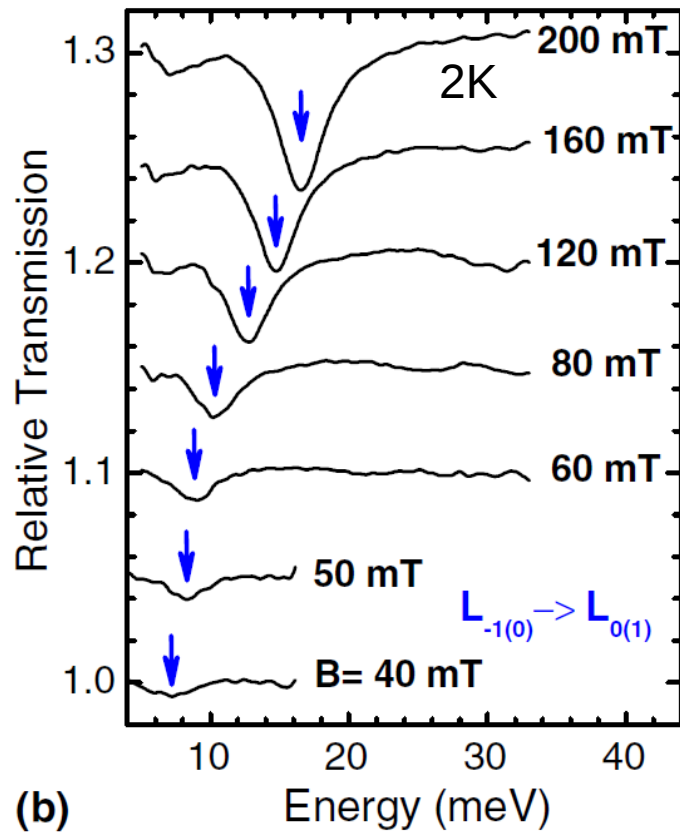


$$E = \pm v_F \hbar k \longrightarrow$$

$$E_L = \text{sgn}(L) \sqrt{2e\hbar v_F^2 |LB|}$$

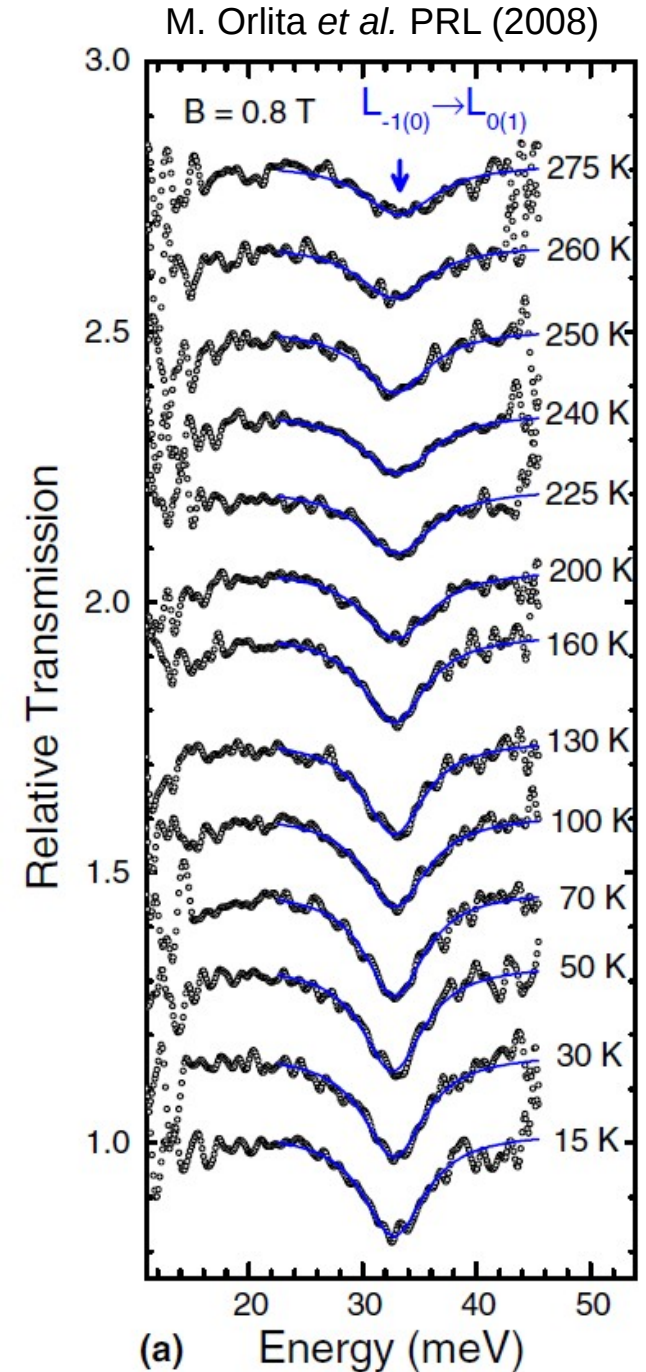
$$L = 0, \pm 1, \pm 2, \dots$$

Infračervená spektroskopie v magnetickém poli

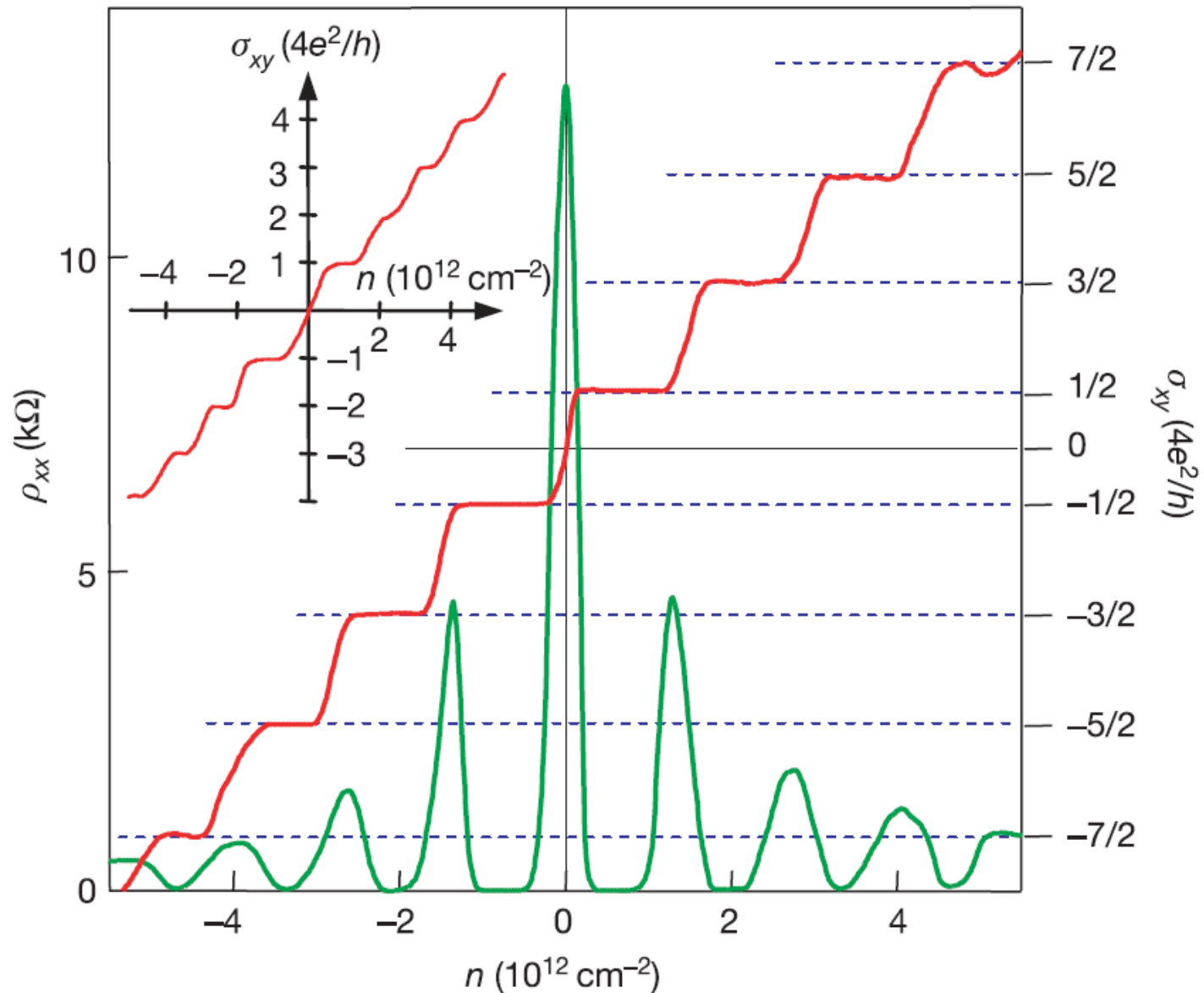


pro $B = 40 \text{ mT} \Rightarrow \mu > 250\,000 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ na 2 K

μ se výrazně nemění až do 300 K



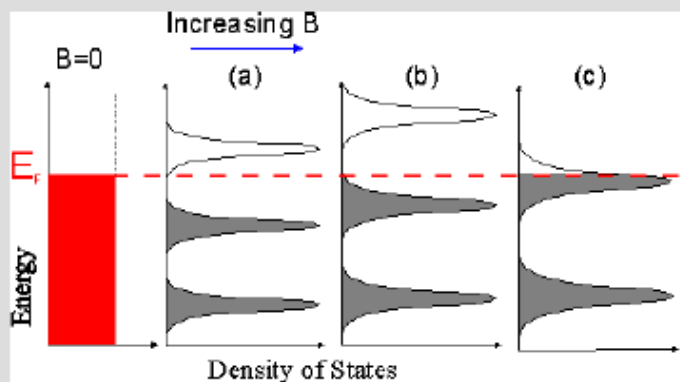
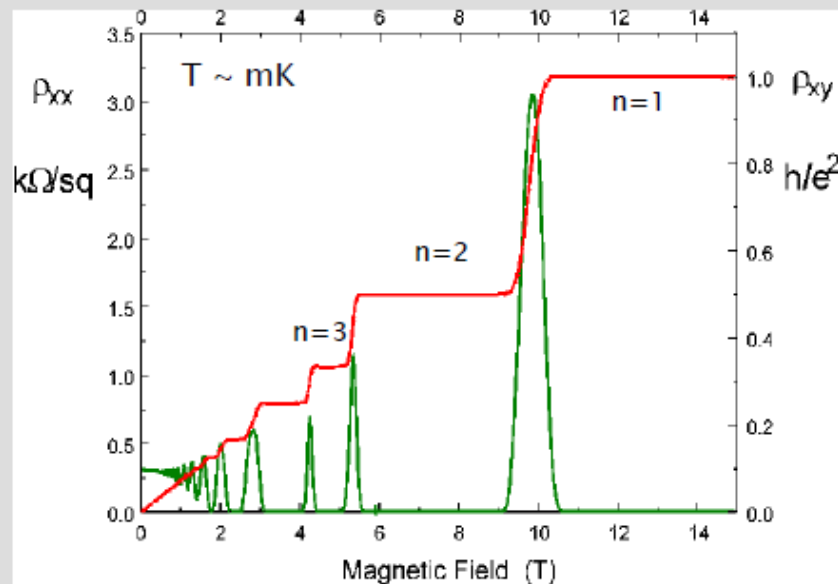
Kvantový Hallův jev



Novoselov *et al.* Nature (2005)

Kvantový Hallův jev

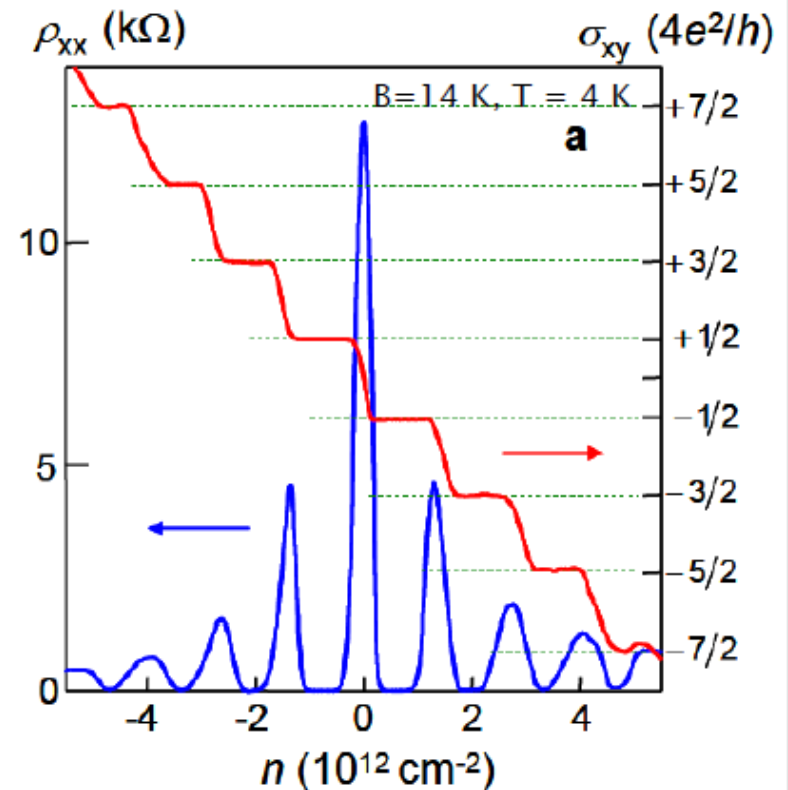
2D electron gases (GaAs-AlGaAs)



$$\sigma_{xy} = \frac{e^2}{h} n$$

von Klitzing, Pepper and Dorda, PRL (1980)

Monolayer graphene



$$\sigma_{xy} = -\frac{4e^2}{h} \left(n + \frac{1}{2} \right)$$

K. Novoselov et al, Nature (2005)

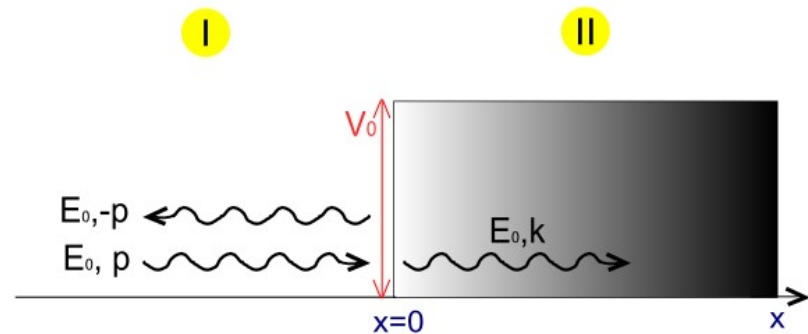
Y. Zhang et al. Nature (2005)

V. Gusynin and S. Sharapov PRL (2005)

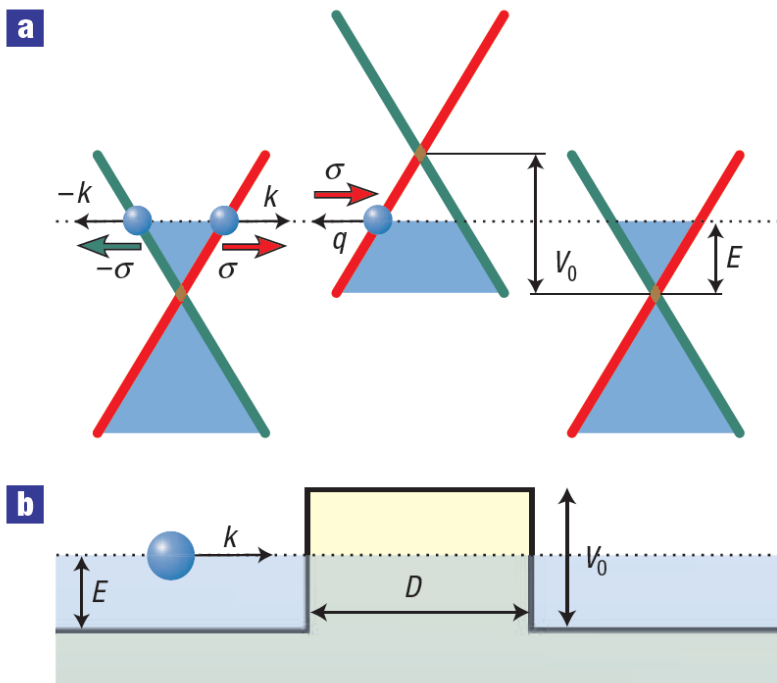
T. Ando J. Phys. Soc. Japan (2002)

První demonstrace Kleinova paradoxu

O. Klein vypočetl (1929) na základě Diracovy rovnice, že pokud bariéra při tunelování částice je větší než její klidová hmotnost, pak je pravděpodobnost tunelování prakticky 100%.

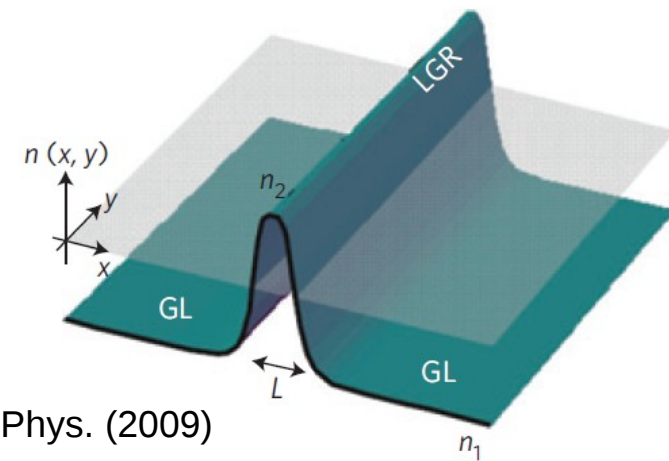
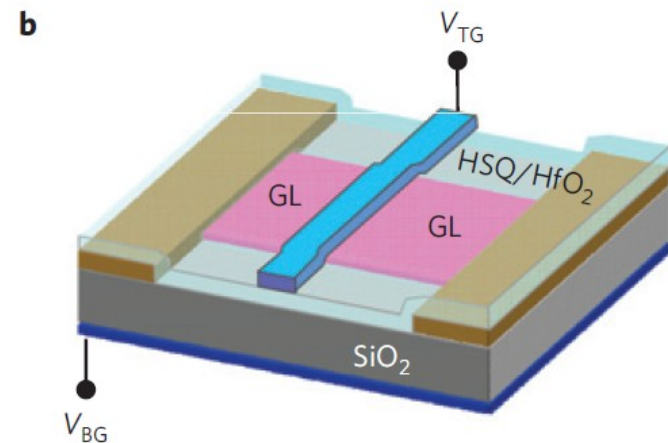


Katsnelson, Geim, Novoselov, Nature (2006)



transmission $T=100\%$

První experimentální demonstrace:



Young and Kim, Nat. Phys. (2009)

VIDĚT konstantu jemné struktury

- Konstanta jemné struktury α charakterizuje sílu elektromagnetické interakce. Je bezrozměrná, nezávislá na volbě jednotek. Fyzika ani po víc než 90 letech nemá pro její hodnotu teoretické vysvětlení.

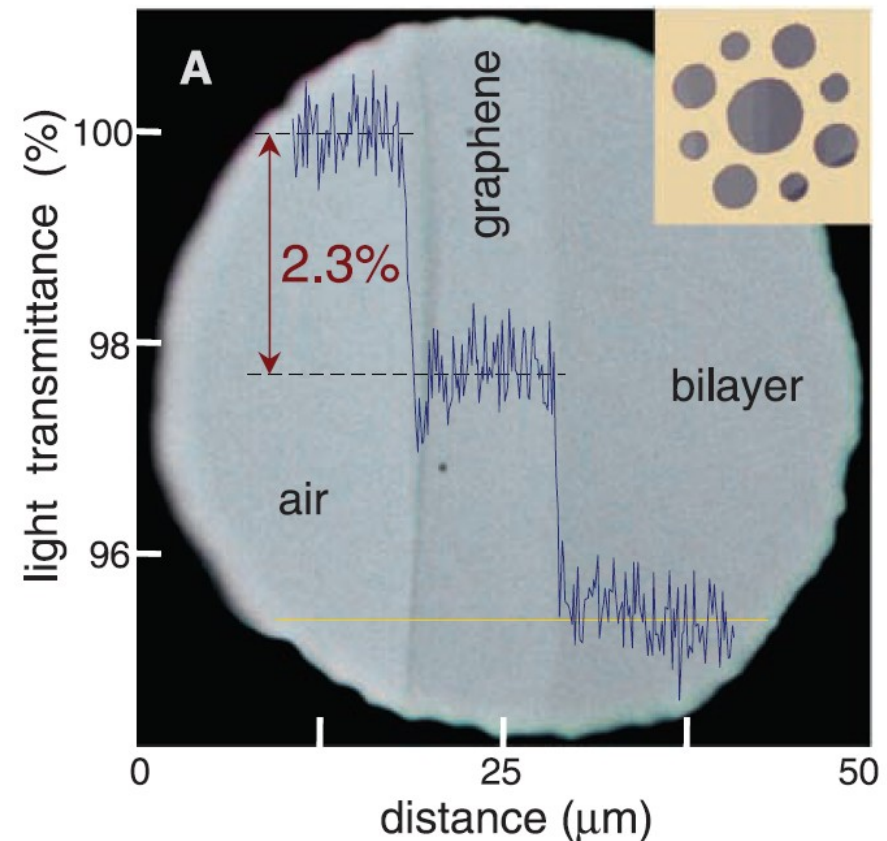
- konstanta jemné struktury je dána základními fyzikálními konstantami:

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

- optická propustnost grafenu je nezávislá na frekvenci a má hodnotu

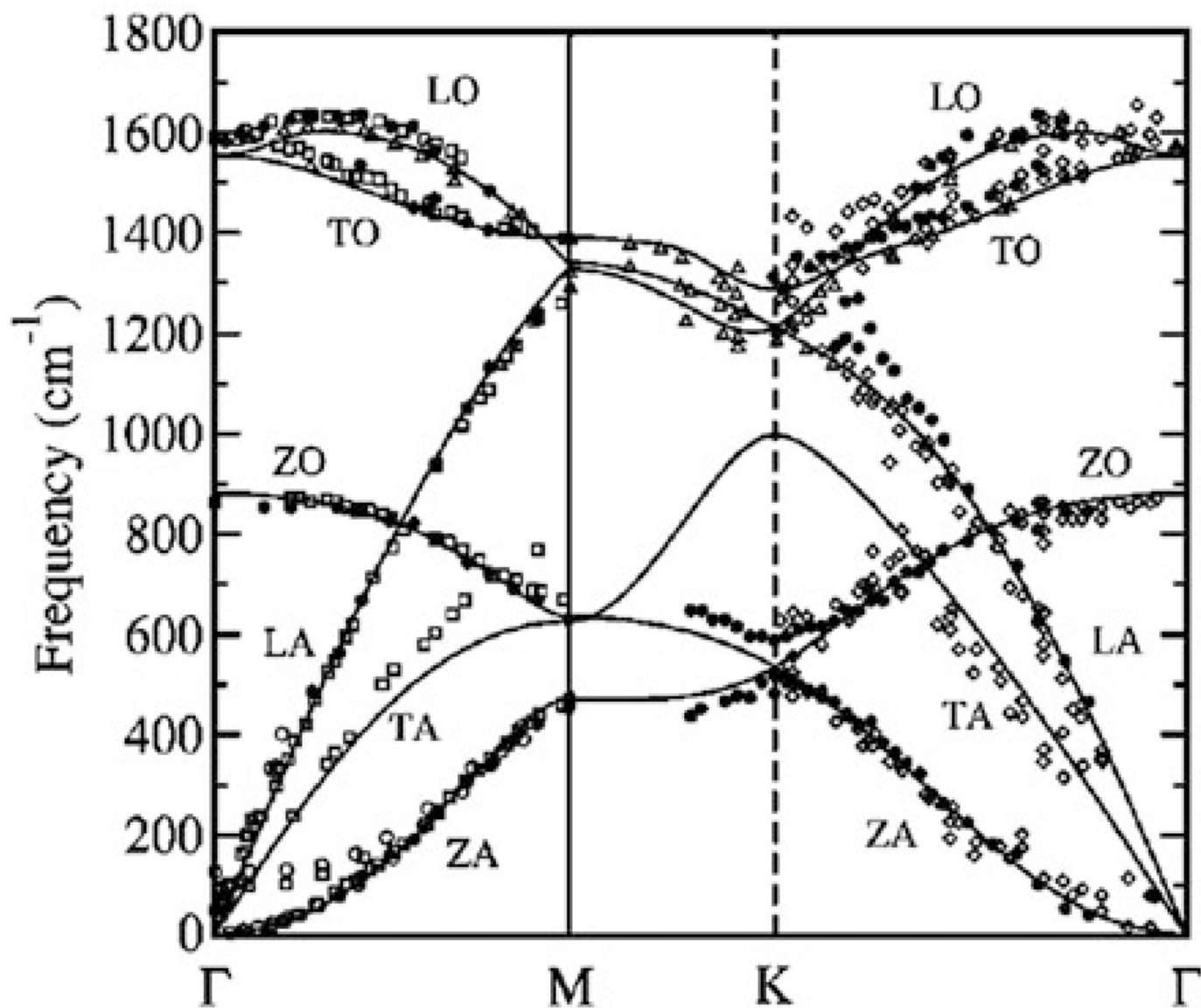
$$T_{\text{opt}} = \left(1 + \frac{\pi\alpha}{2}\right)^{-2} \approx 1 - \pi\alpha = 97.7\%$$

Kuzmenko *et al.* PRL (2008)



Nair *et al.* Science (2008)

Grafen



ZA, ZO – vibrace mimo 2D rovinu. ZA – kvadratická závislost v okolí bodu Gama.

Grafen

Thermal Conductivity $\text{Wm}^{-1}\text{K}^{-1}$	Form of Carbon	Heat Treatment Temperature
> 3,000	Graphene	
> 3,000	Nanotubes	
2,000 – 2,500	Diamond	
~ 2,000	HOPG in-plane Vapor deposited fibers	> 3,000°C > 3,000°C
100 – 1,000	Pitch-derived fibers	> 3,000°C
450	Pure copper	
10	PAN-derived fibers	
< 10	Amorphous carbons	
0.1	Isotropic polymeric materials	

Aoki, Dresselhaus (eds.): Physics of graphene, Springer (2014).

Grafen

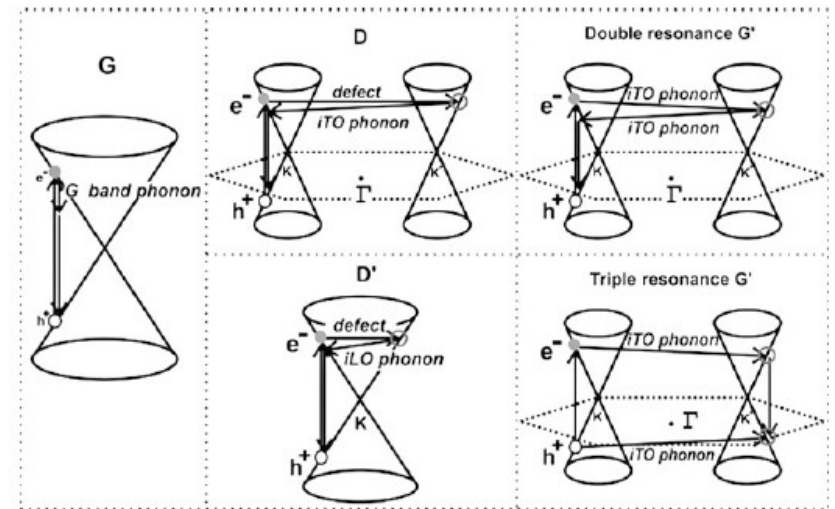
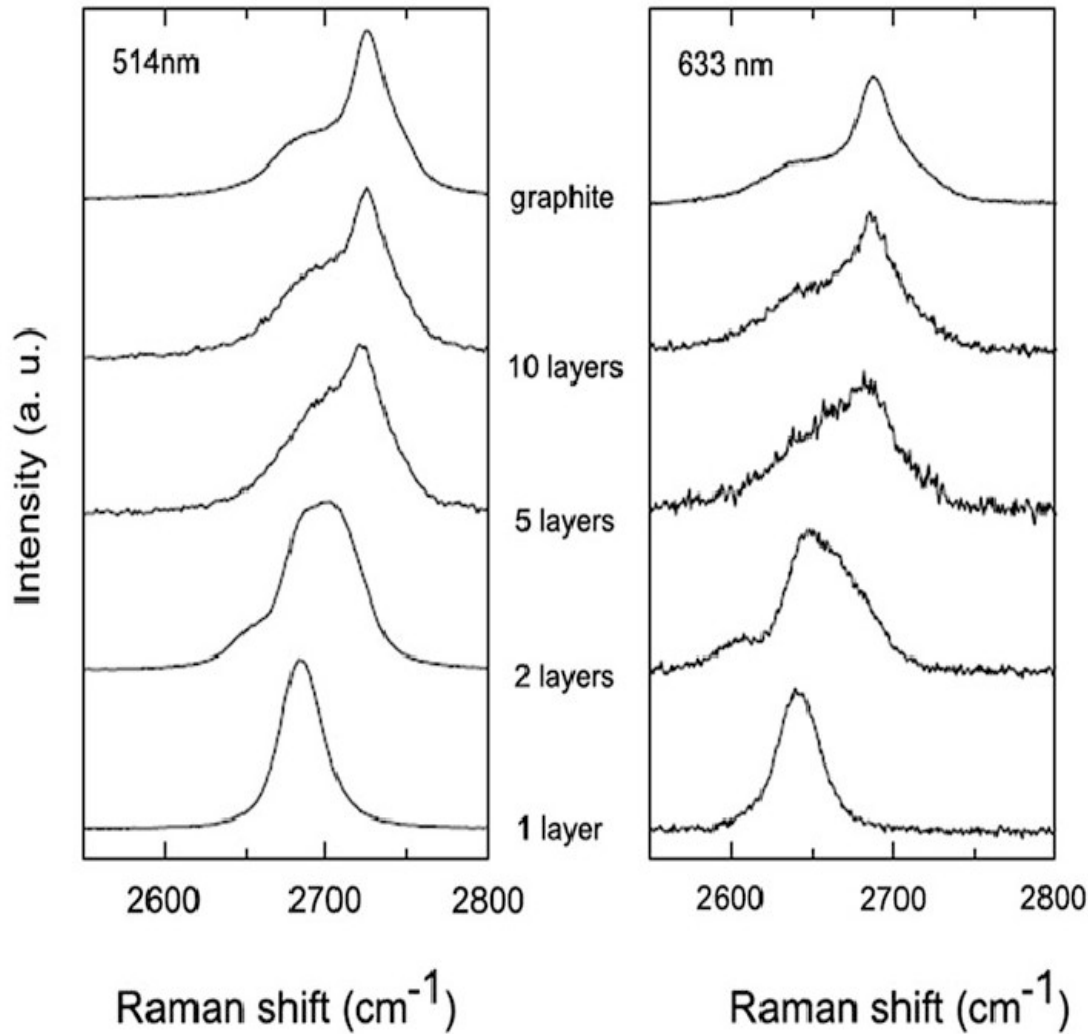
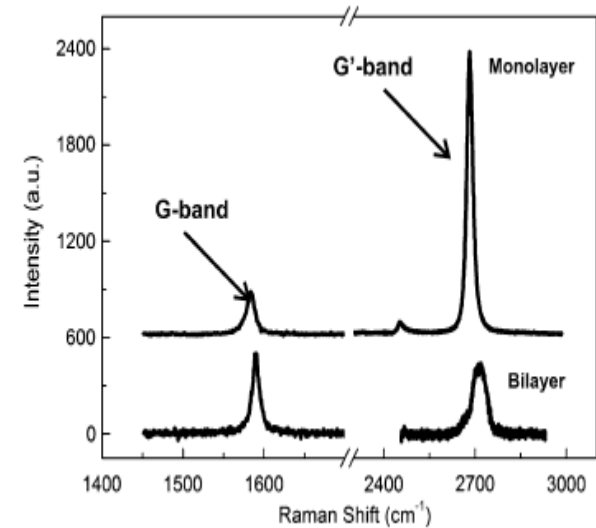
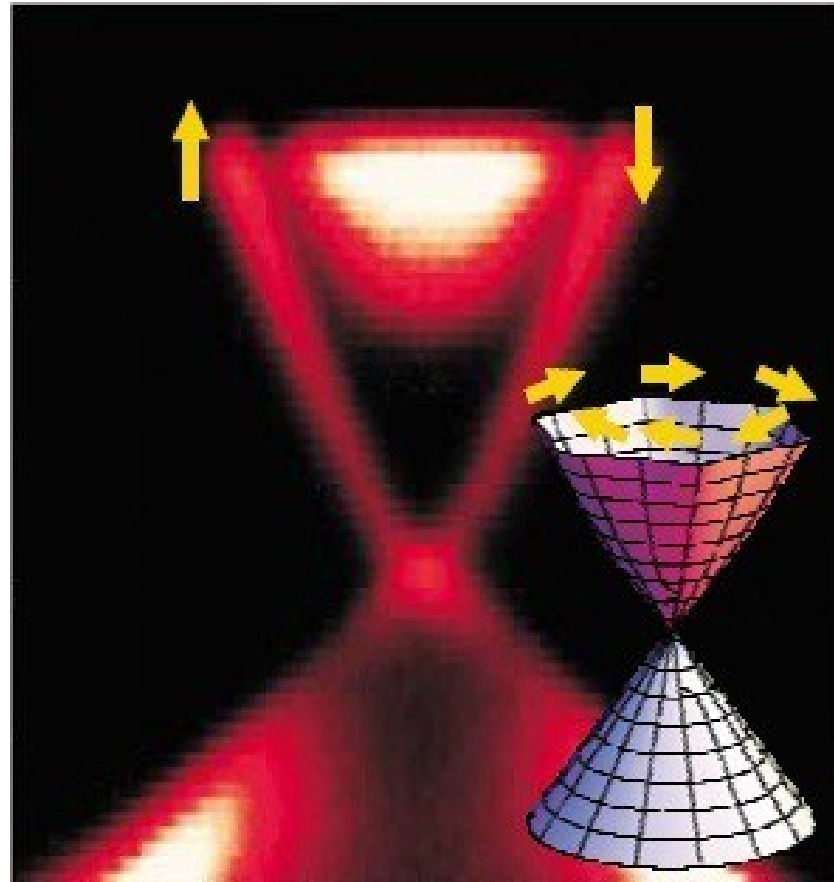
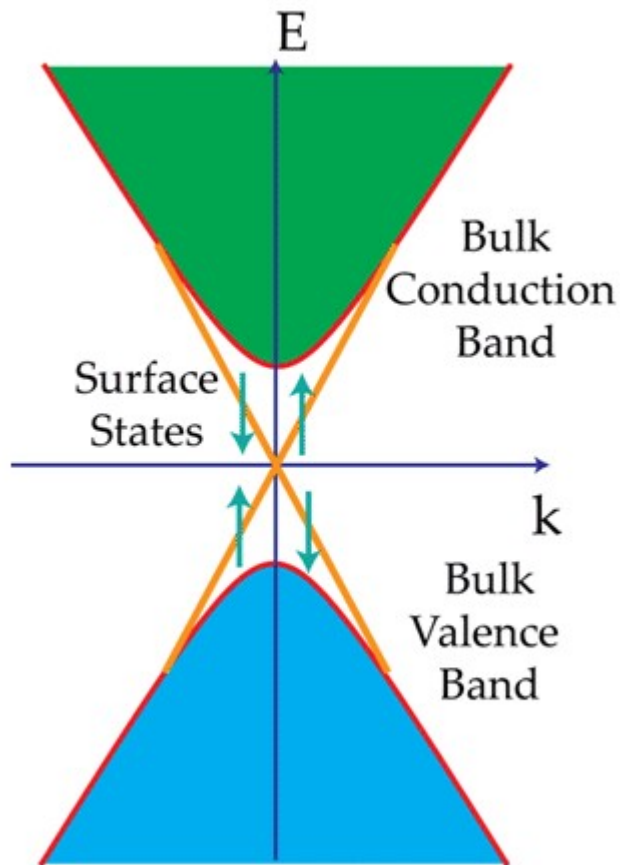


Fig. 3.21 (Top) the Raman spectrum showing the G- and G' (or 2D)-bands. As experimentally observed, the G-band occurs around 1583 cm⁻¹ while the G'-band occurs around 2670 cm⁻¹. (Bottom) the Raman spectrum showing the G- and G'-bands for Bi-layer graphene. The spectra were measured with a 532 nm laser line and the power density used was 1 mW/μm² [91, 98]



Povrchové stavy v topologických izolátorech

Diracův kužel povrchových stavů díky silným relativistickým vlivům v pásové struktuře – spin-orbitální interakce



Topologické izolátory – materiály s velkým počtem elektronů
Dosud známé $\text{Bi}_x\text{Sb}_{1-x}$, Bi_2Se_3 , Bi_2Te_3 , Sb_2Te_3

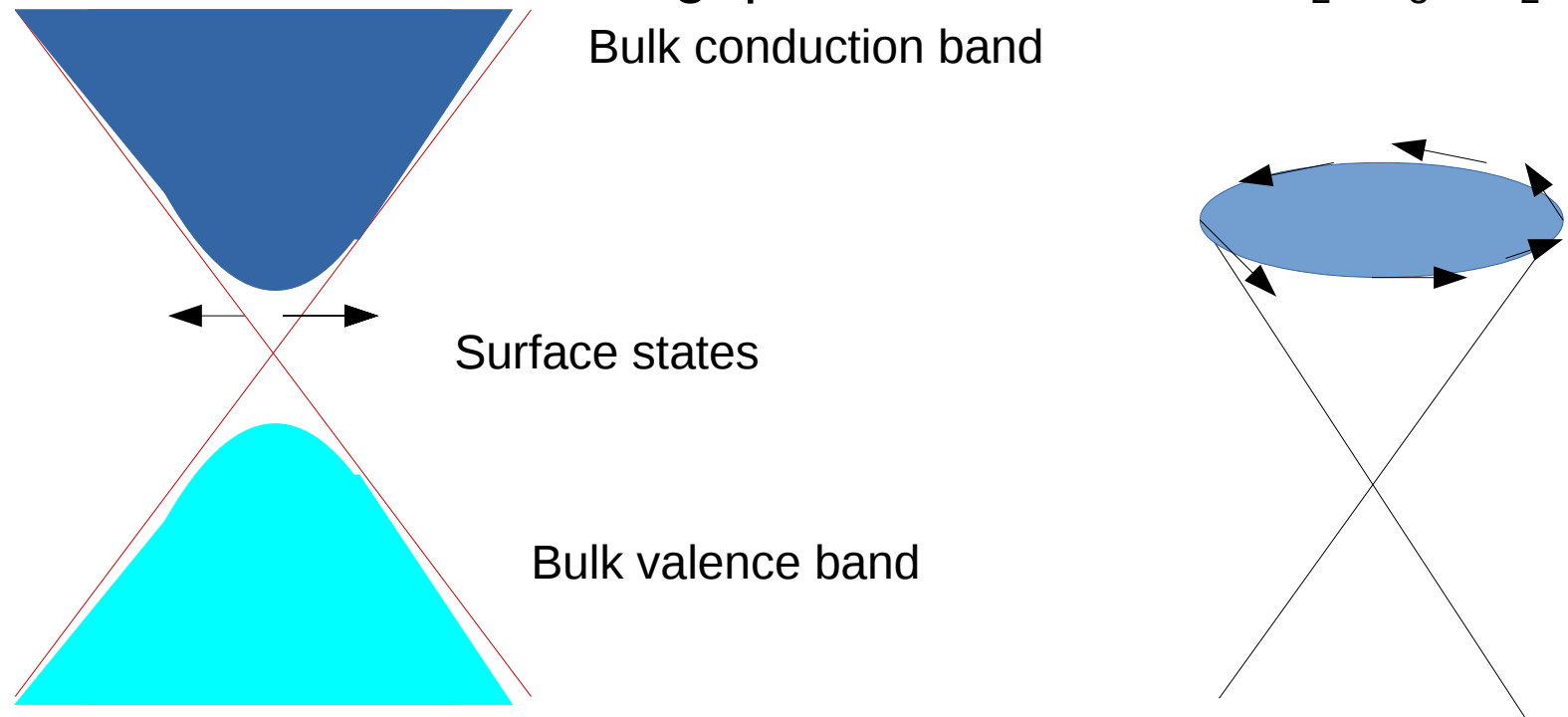
Topological insulators

Band structure of topological insulator:

Large spin orbit splitting and time reversal symmetry

→ spin polarized surface states with Dirac-cone dispersion

Prototypical materials: narrow band gap semiconductors Bi_2Se_3 , Bi_2Te_3



Ferromagnetic ordering brakes time reversal symmetry

→ band gap within surface states, Quantum anomalous Hall effect

Topologické izolátory

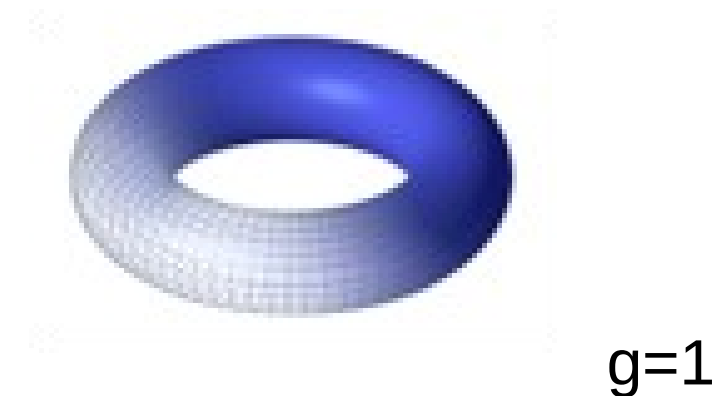
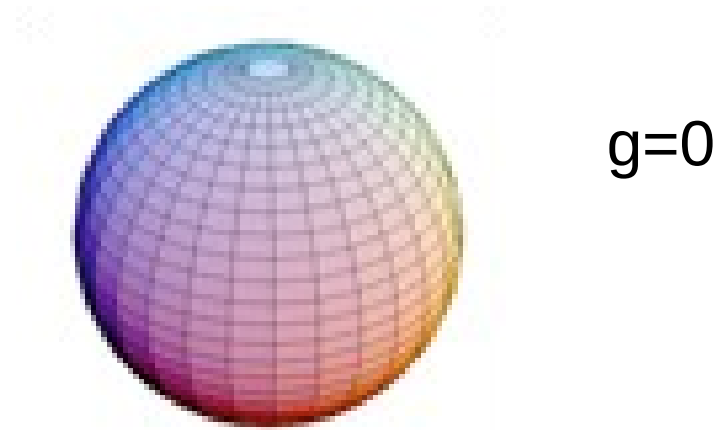
Topologie

Gaussův-Bonnetův teorém

$$\kappa = (r_1 r_2)^{-1}$$

$$\int_M \kappa dA = 2\pi\chi = 2\pi(2 - 2g)$$

g je genus - "počet děr"



Topological insulators

Topology in solid state physics

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r})$$

$$\mathbf{A} = \langle u_{\mathbf{k}} | -i\nabla_{\mathbf{k}} | u_{\mathbf{k}} \rangle$$

$$\mathbf{F} = \nabla \times \mathbf{A} \quad \text{Berry curvature}$$

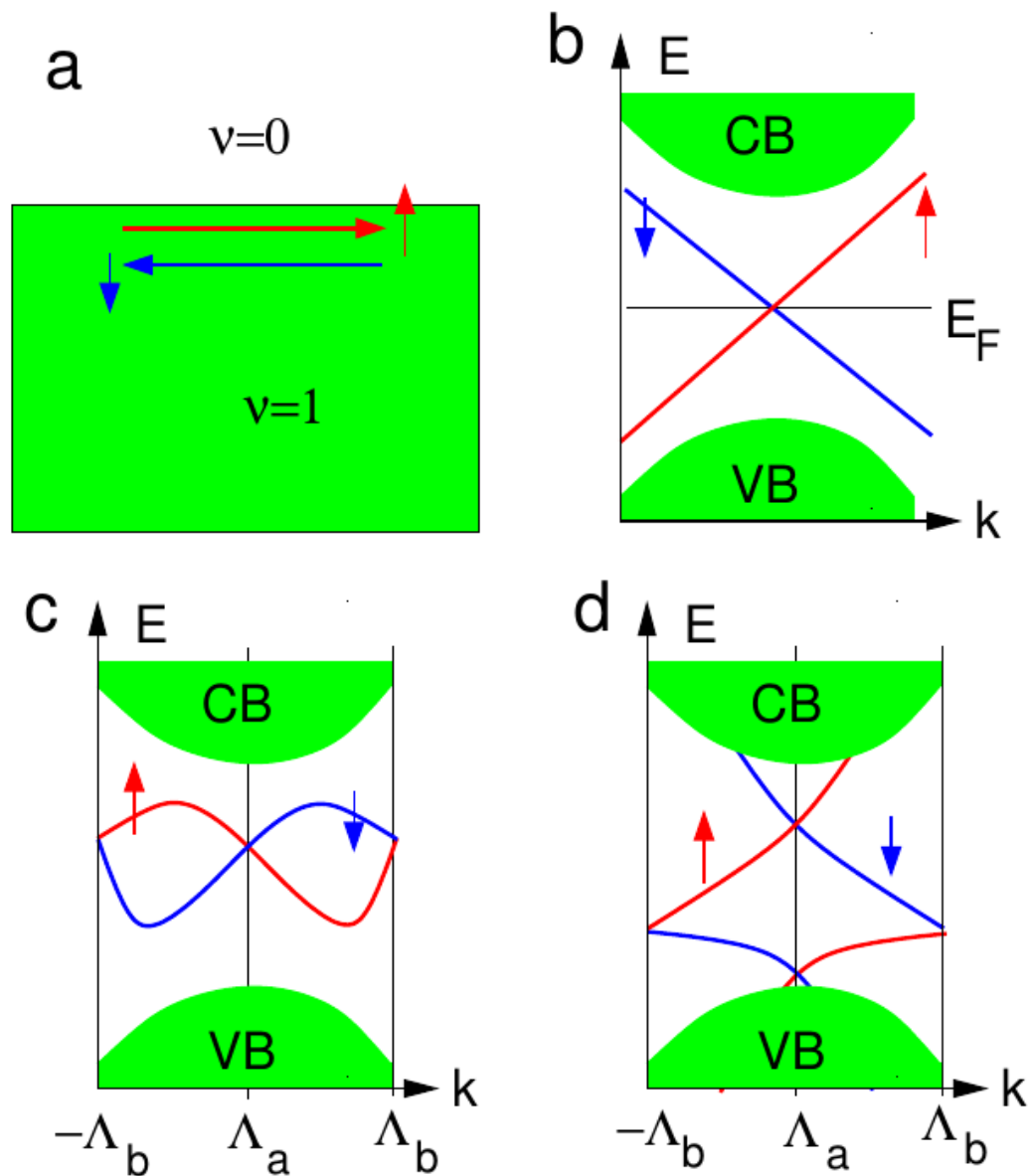
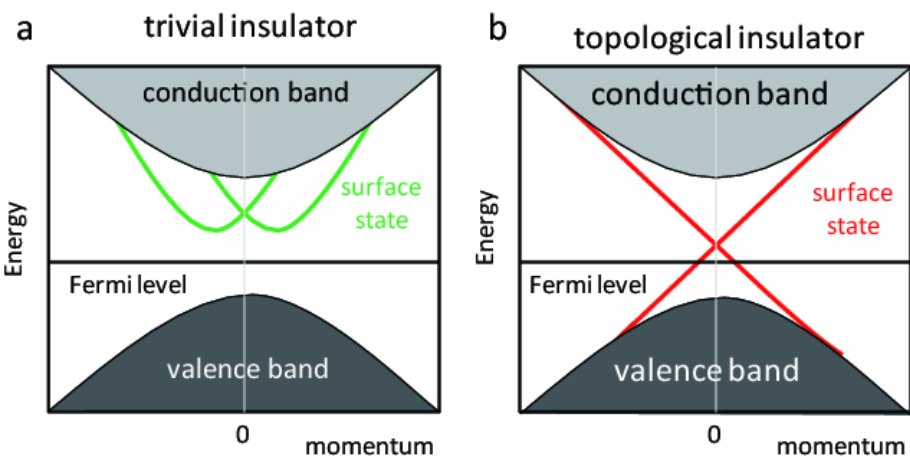
Interface of two regions of different Chern number – existence of topologically protected interface states

Integral of Berry curvature over certain path: Chern numbers

If system has time-reversal symmetry four invariants can be constructed

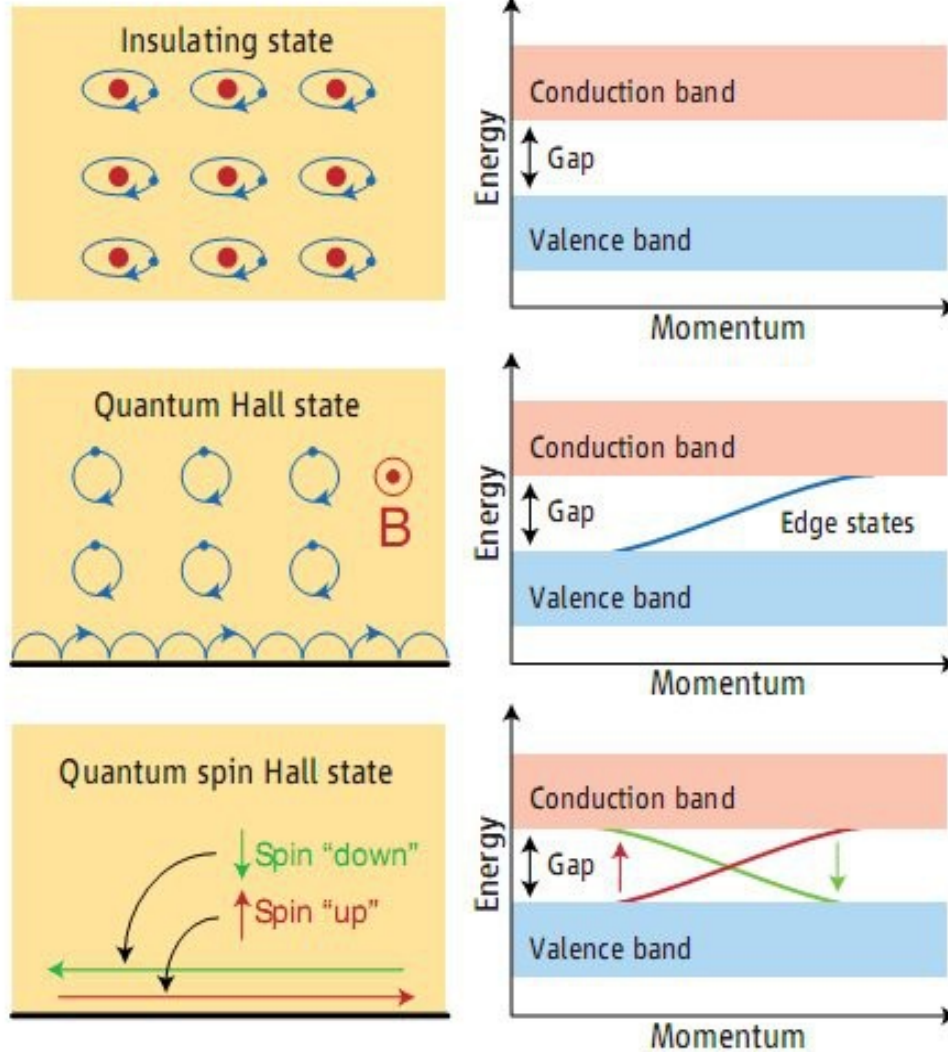
Topological insulators

Povrchové stavy polarizovaného spinu



Topological insulators

Schematic band structure of insulator, Quantum Hall state and topological insulator

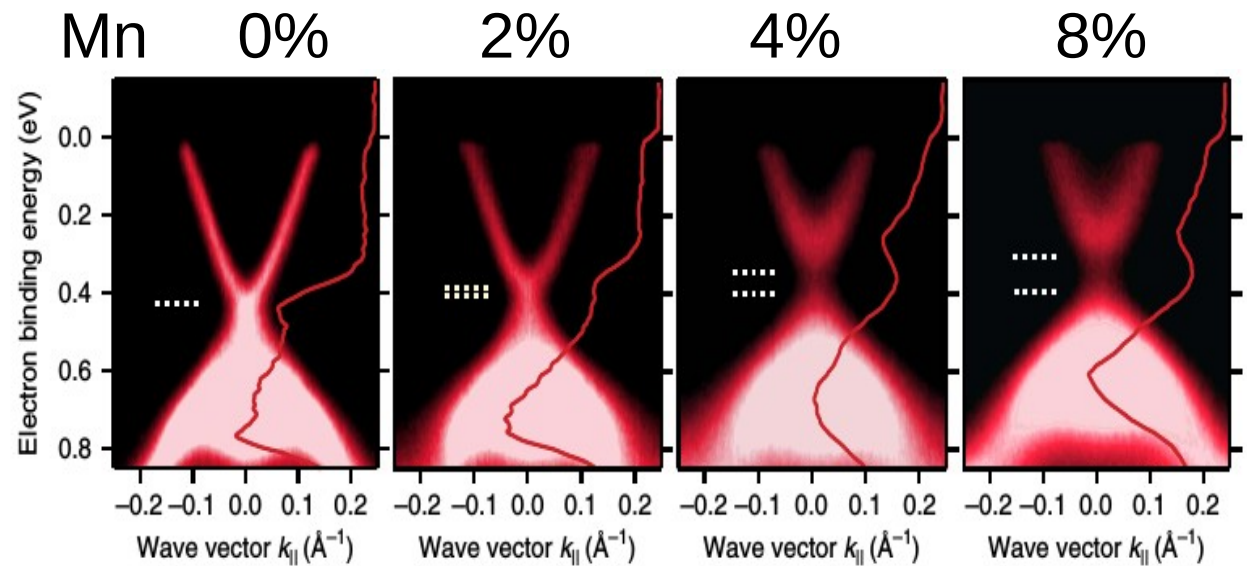
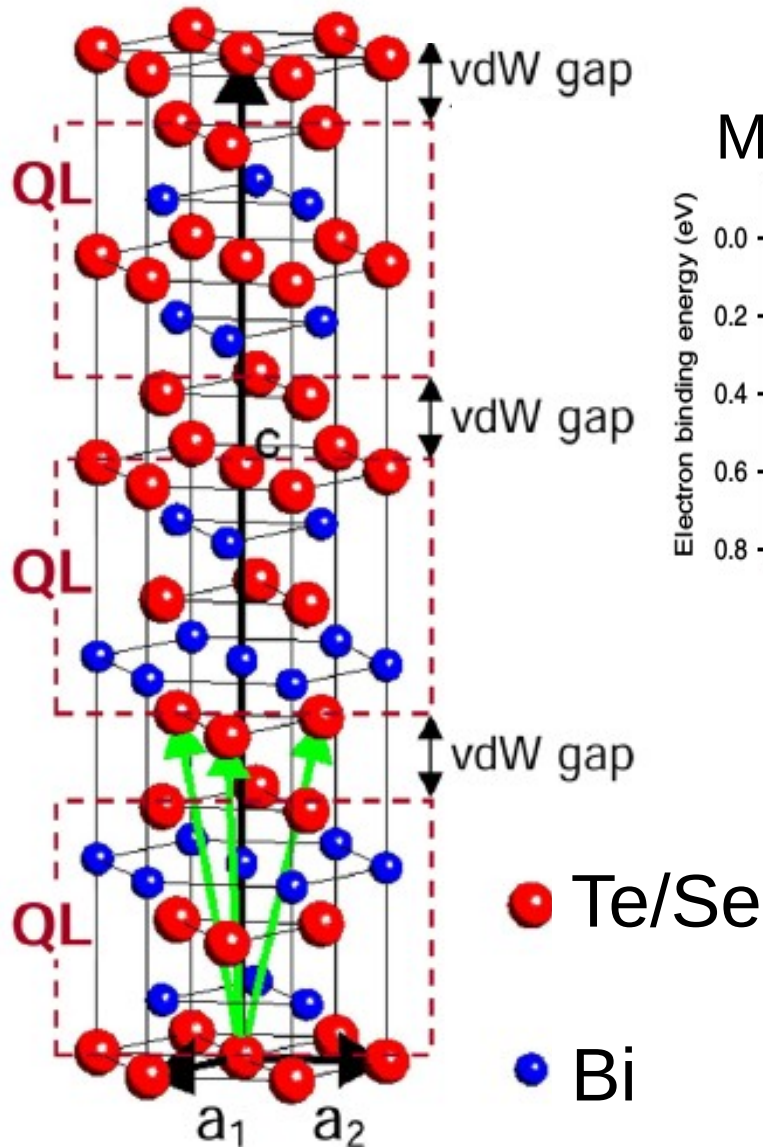


C. L. Kane, E. J. Mele, Science 314, 1692 (2006).

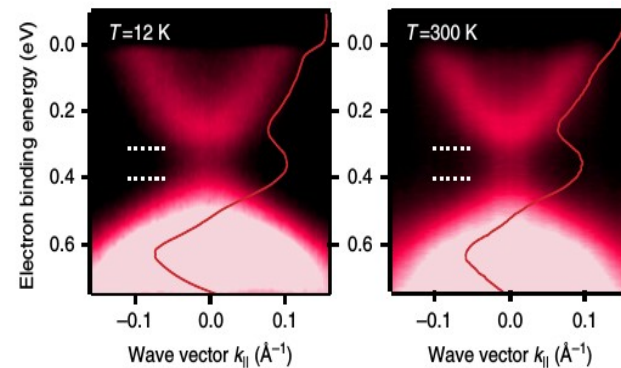
Povrchové stavy v topologických izolátorech

Bi_2Se_3 , Brillouinova zóna, pásová struktura

Velký vliv relativistických korekcí – spin-orbitální vazba

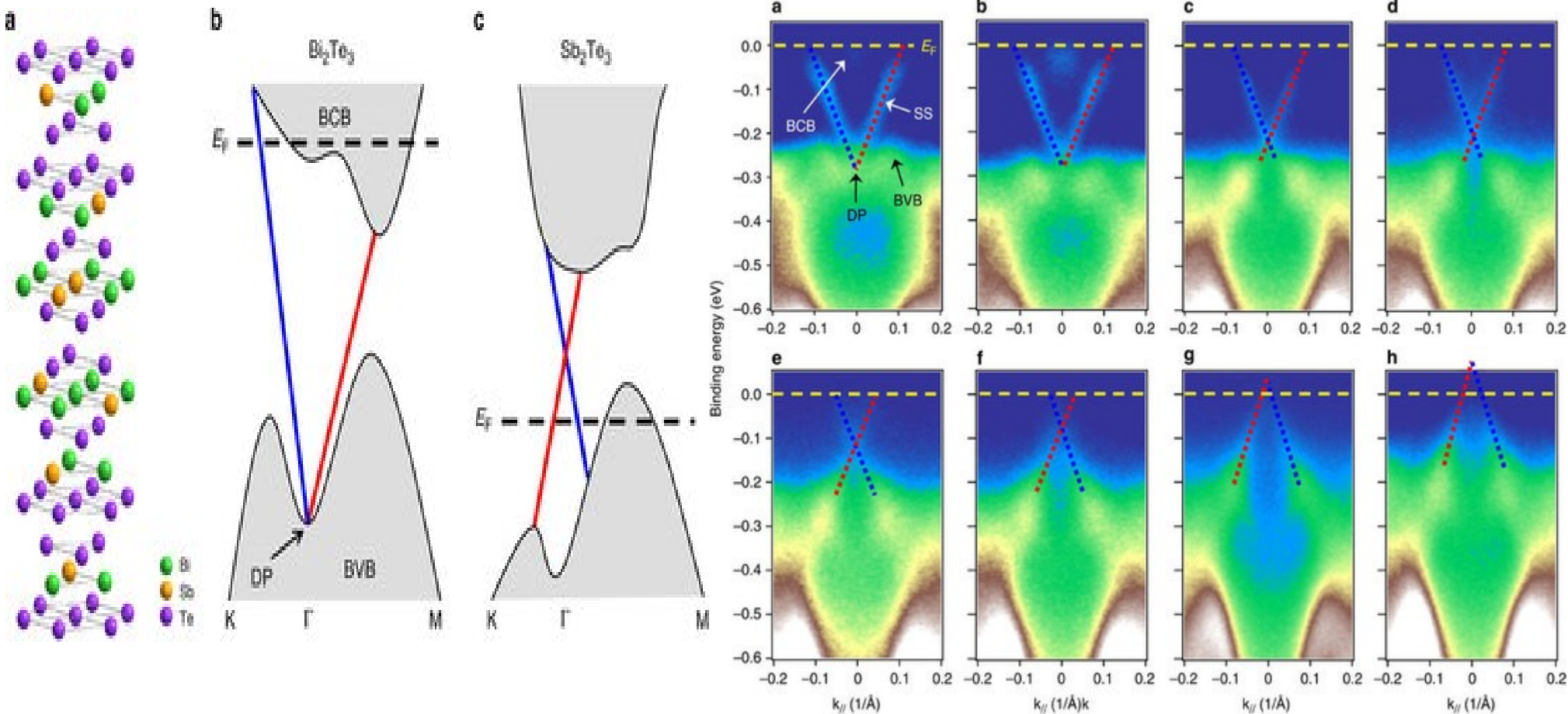


Mn
8%



J. Sanchez-Barriga et al.,
Nature
Comm.
7:10559
(2015).

Pásová struktura topologických izolátorů



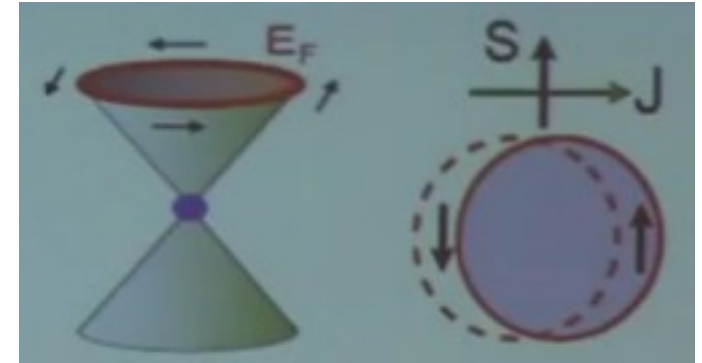
Jinsong Zhang et al, Nature Communications 2, 574 (2011).

X=0, 0.25, 0.62, 0.75,
 0.88, 0.94, 0.96 1.0

Topologické izolátory

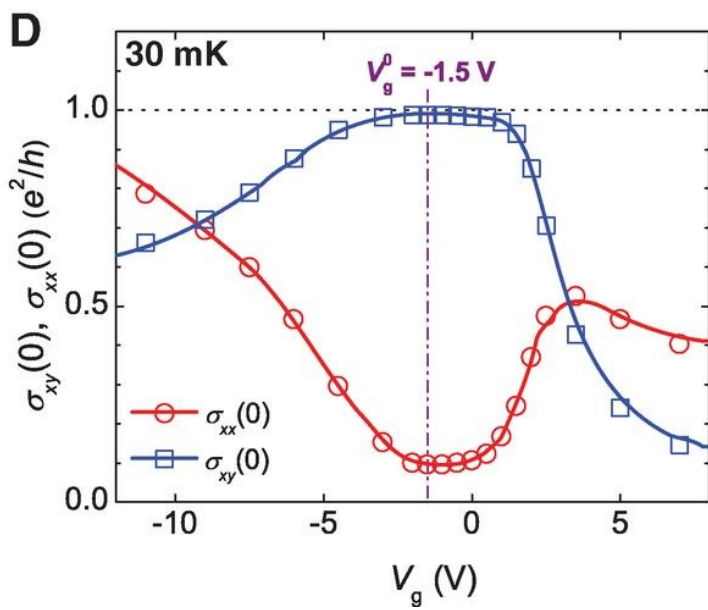
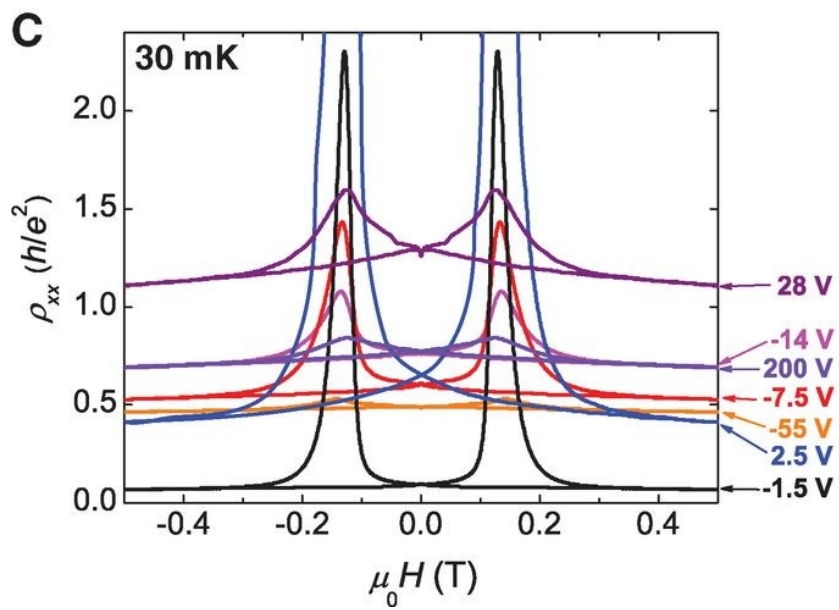
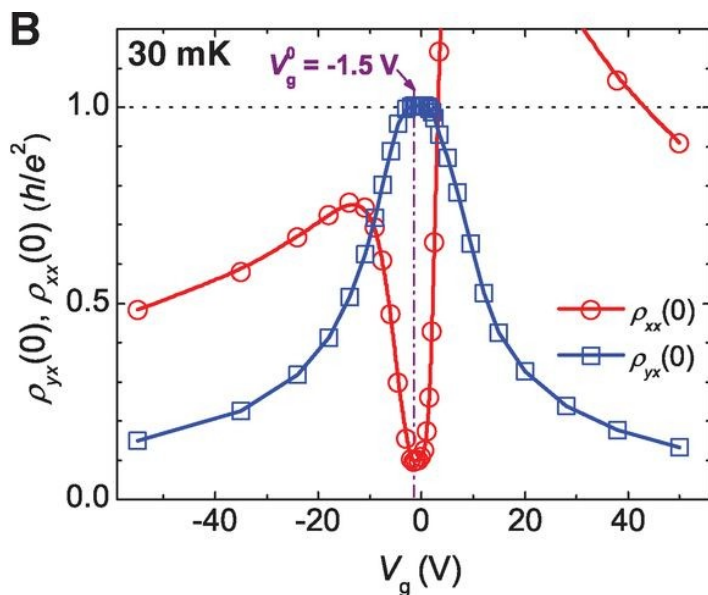
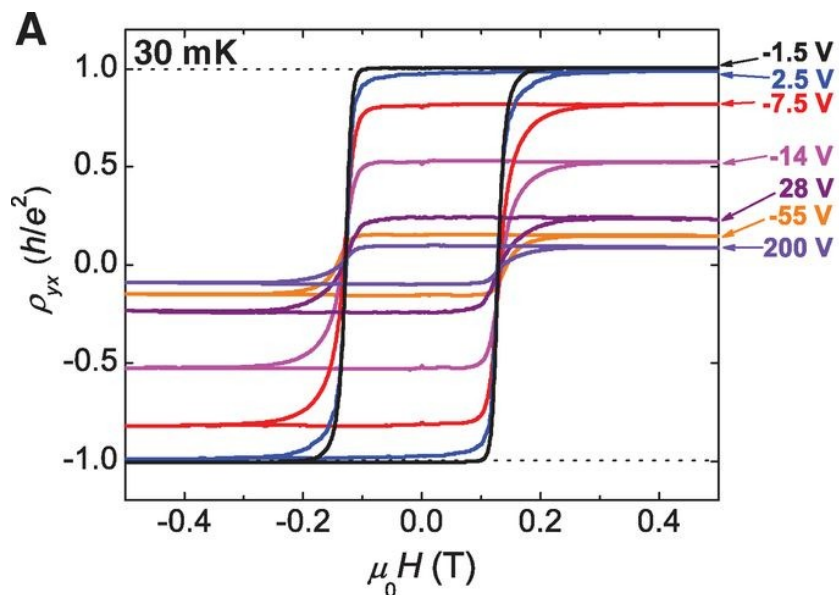
Vlastnosti topologických izolátorů:

- proud je úměrný magnetizaci
 - nové aplikační možnosti
- Absence zpětného rozptylu elektronů
 - vysoké pohyblivosti až $10000\text{cm}^2/\text{Vs}$, elektronika s malým příkonem
- Robustní vůči nemagnetickým nečistotám
 - povrchové stavy jsou důsledkem symetrie

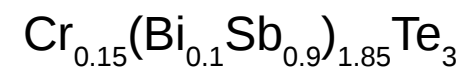


Topologické izolátory

Kvantový anomální Hallův jev - feromagnetismus + topologický izolátor bezdisipační vedení proudu



Experimental
Observation of the
Quantum
Anomalous Hall
Effect in a Magnetic
Topological Insulator
Cui-Zu Chang et al.
Science 12 Apr
2013:
Vol. 340, Issue
6129, pp. 167-170
DOI:
10.1126/science.123
4414

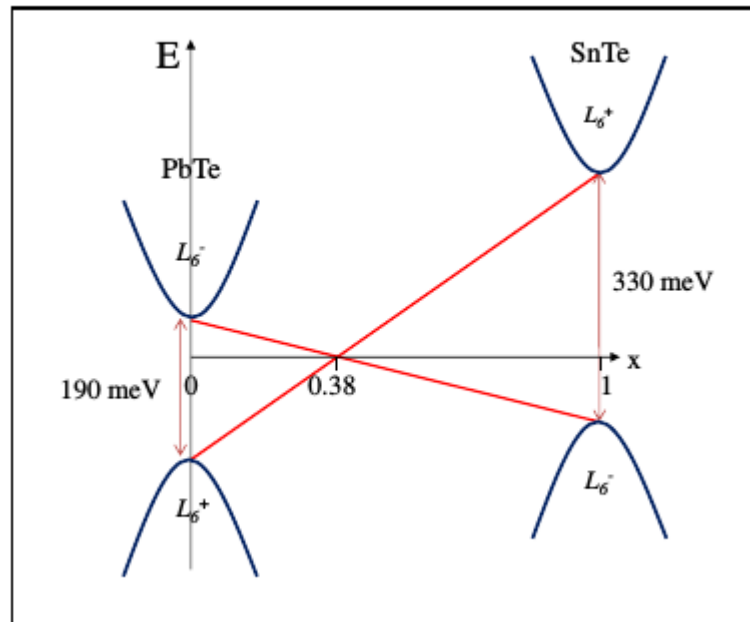


Topologické izolátory

Další možné aplikace:

- kvantové počítače
- spintronické součástky
- ...

Topologické krystalové izolátory



Zde je příčinou zrcadlová prostorová symetrie.

Výsledek je sudý počet Diracových kuželů. Citlivé na poruchy – narušení zrcadlové symetrie.

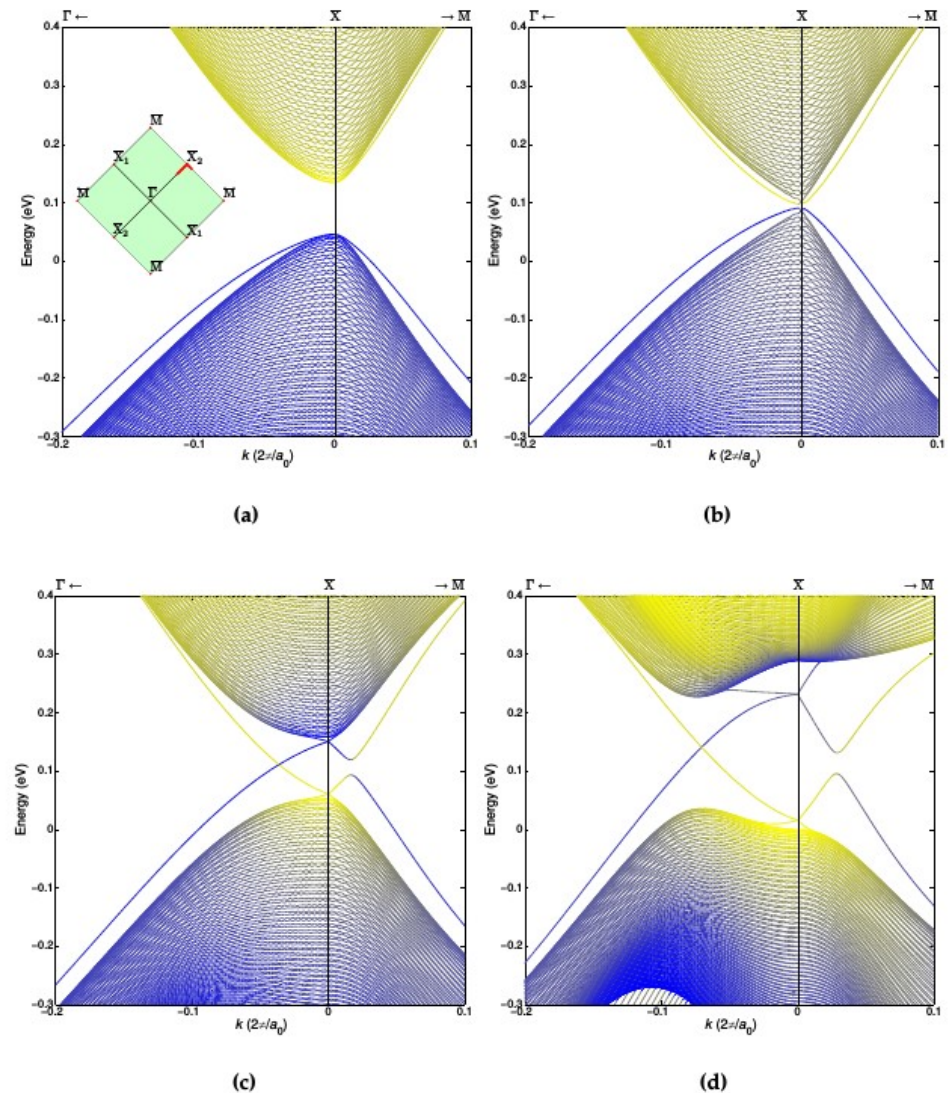
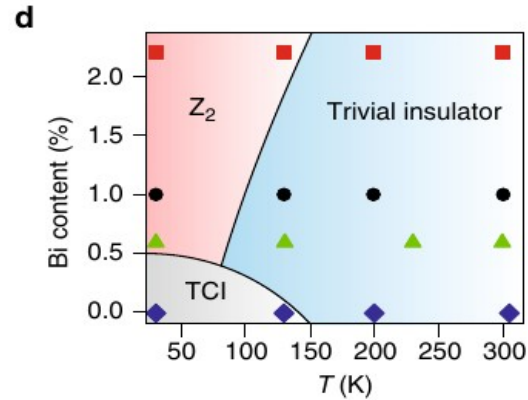
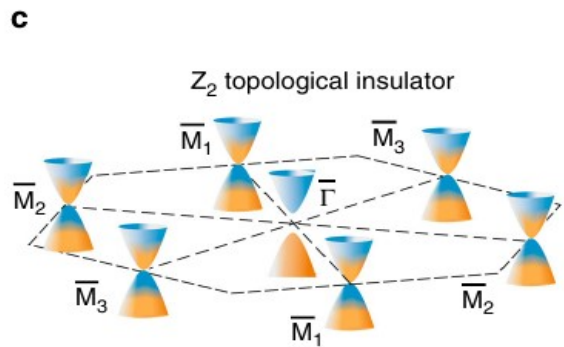
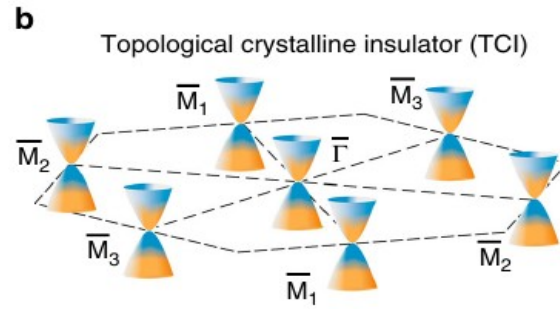
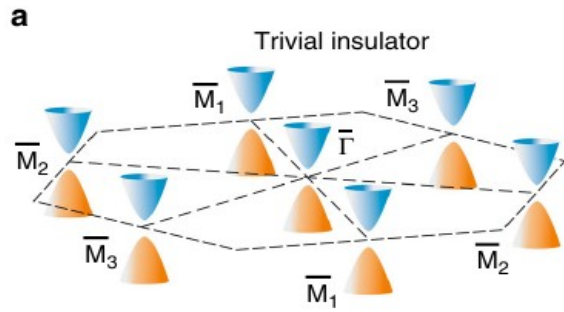
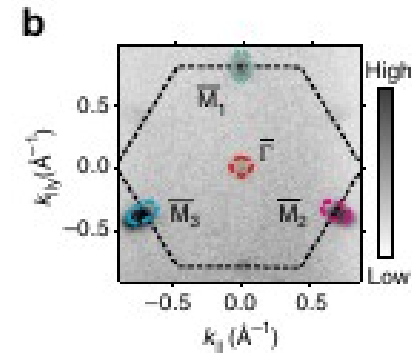
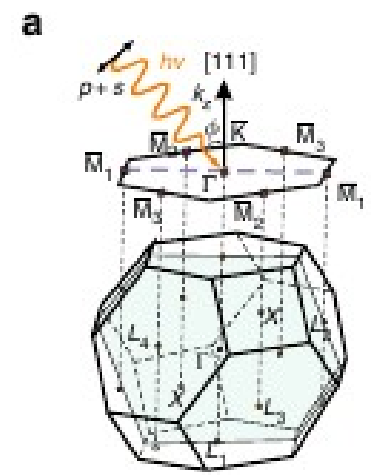


Figure 4.6: The band structure of a 280 monolayers (~ 90 nm) (001)-oriented slab of $\text{Pb}_{1-x}\text{Sn}_x\text{Te}$ for different compositions; (a) $x = 0.2$ trivial insulator, (b) $x = 0.38$ closed bulk band gap, (c) $x = 0.6$ TCI and (d) $x = 1.0$ pure SnTe TCI. Yellow and blue lines respectively represent the cation and anion p-type orbitals dominant contribution to the state's wave function, which is exchanged for $x > x_c$ due to the inversion of band gap. $k = 0$ corresponds to L points projection (shown in Fig. 4.4). Dense packed lines show the bulk states, whereas surface states are the ones that are significantly separated from bulk states. Appearing of the gapless surface states for $x > x_c$ indicates occurrence of the topological phase transition.

Topologické krystalové izolátory



System
(Pb,Sn)Se (111)
Sn 28%, Bi
dopovany



Mandal et al., Nature communications 2017

