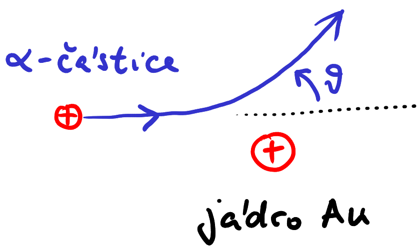
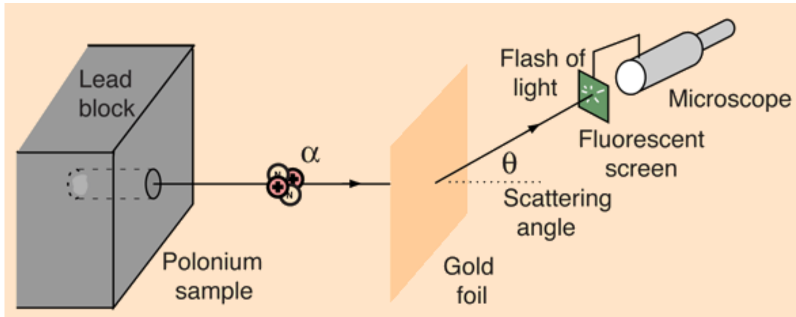
Rutherfordův experiment

zkoumání atomů rozptylem α částic





kvantitativně popisováno dif. účinným průřezem:

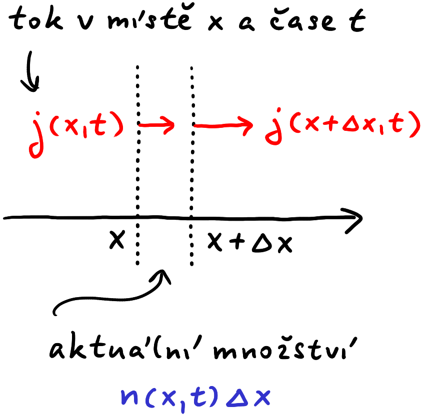
$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{tok částic rozptýlených do určitého směru vztažený na prostorový úhel}}{\text{plošný tok dopadajících částic}}$$

pro coulombovský potenciál jádra $\left(\frac{d\sigma}{d\Omega}\right) \sim \frac{1}{\sin^4\frac{\vartheta}{2}}$

Připomínka difúze a rovnice kontinuity

– koncentrační profil difundující substance

– odvození rovnice kontinuity:



změna množství substance $n\Delta x$ za čas Δt

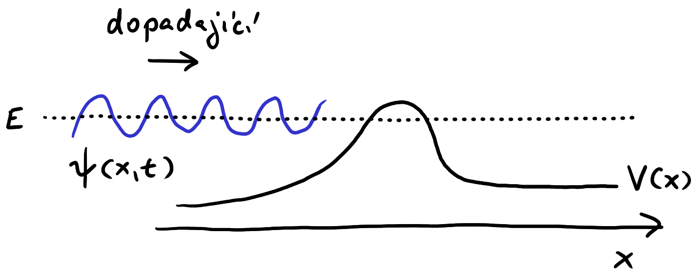
$$n(x,t+\Delta t)\Delta x - n(x,t)\Delta x = \underbrace{j(x,t)\Delta t}_{\text{přiteče}} - \underbrace{j(x+\Delta x,t)\Delta t}_{\text{odteče}}$$

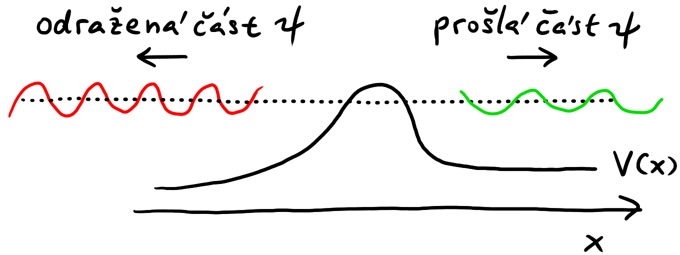
$$\downarrow$$

$$\left(\frac{\partial n}{\partial t}\Delta t\right)\Delta x = \left(-\frac{\partial j}{\partial x}\Delta x\right)\Delta t \quad \rightarrow \quad \frac{\partial n}{\partial t} + \frac{\partial j}{\partial x} = 0$$

(1D rovnice kontinuity)

– získali jsme 1D rovnici kontinuity, její 3D verze zní $\dfrac{\partial n}{\partial t} + \nabla\cdot\vec{j} = 0$



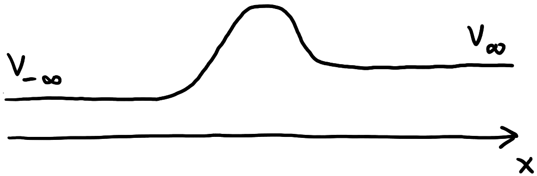


① pomocí stacionárního stavu $\psi(x,t) = \Psi(x)\, e^{-\frac{i}{\hbar}Et}$

dopadající + odražená

$$\Psi(x) \to e^{ikx} + R e^{-ikx}$$

$$k = \frac{\sqrt{2m(E-V_{-\infty})}}{\hbar}$$

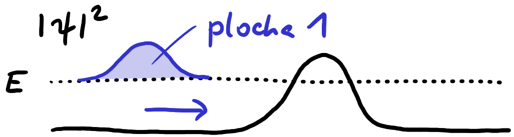


prošlá

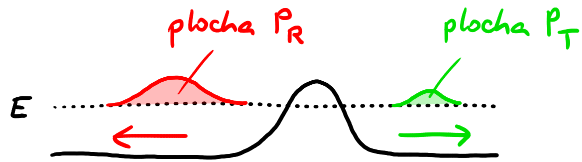
$$\Psi(x) \to T e^{ik'x}$$

$$k' = \frac{\sqrt{2m(E-V_{\infty})}}{\hbar}$$

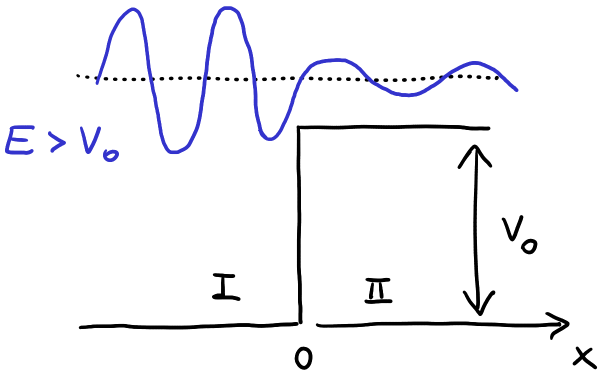
② pomocí balíků



pozdější čas :

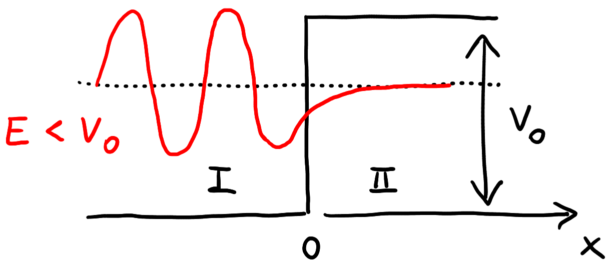


"Propustnost" pravoúhlého potenciálového schodu



$$\psi_I(x) = e^{ikx} + Re^{-ikx} \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi_{II}(x) = Te^{ik'x} \qquad k' = \frac{\sqrt{2m(E-V_0)}}{\hbar}$$



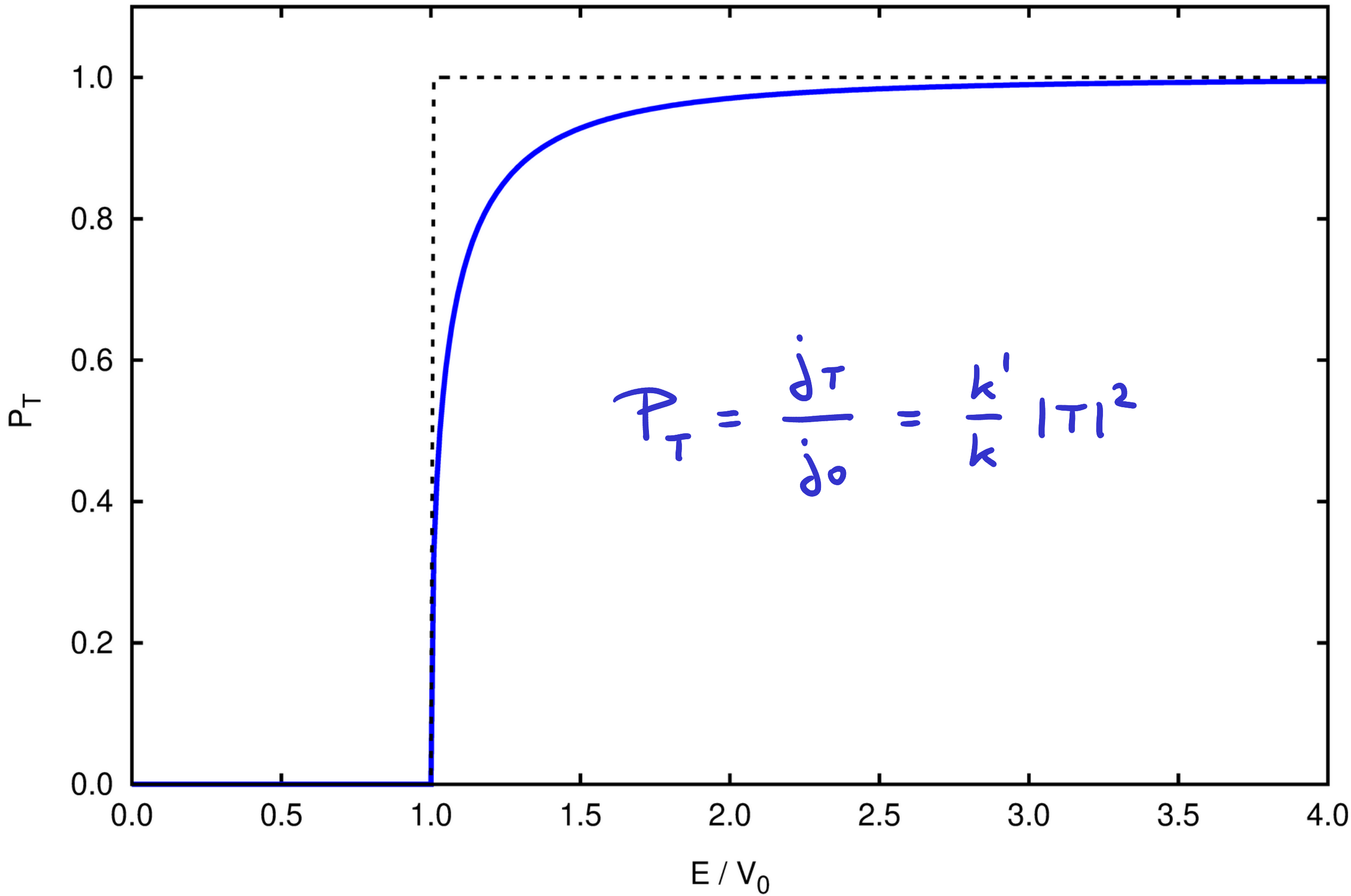
$$\psi_I(x) = e^{ikx} + Re^{-ikx} \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi_{II}(x) = Ae^{-\varkappa x} \qquad \varkappa = \frac{\sqrt{2m(V_0-E)}}{\hbar}$$

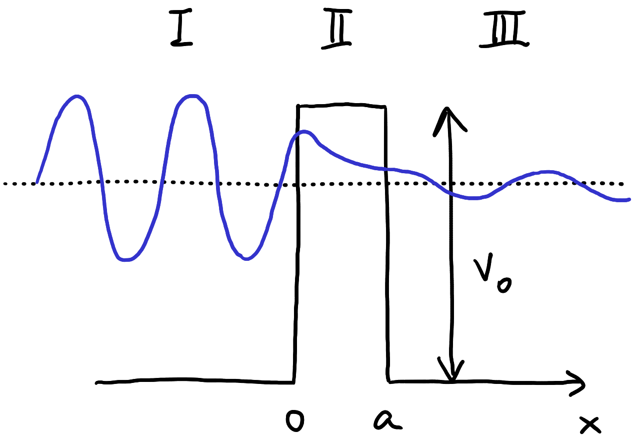
v obou případech „sešívací" podmínky : $\psi_I(0) = \psi_{II}(0)$ spojitost

$\psi_I'(0) = \psi_{II}'(0)$ hladkost

"Propustnost" pravoúhlého potenciálového schodu



"Propustnost" pravoúhlé bariéry

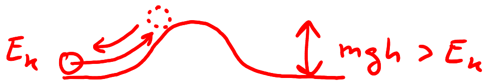


$$\psi_I(x) = e^{ikx} + Re^{-ikx}$$

$$\psi_{II}(x) = Ae^{\varkappa x} + Be^{-\varkappa x}$$

$$\psi_{III}(x) = Te^{ik(x-a)}$$

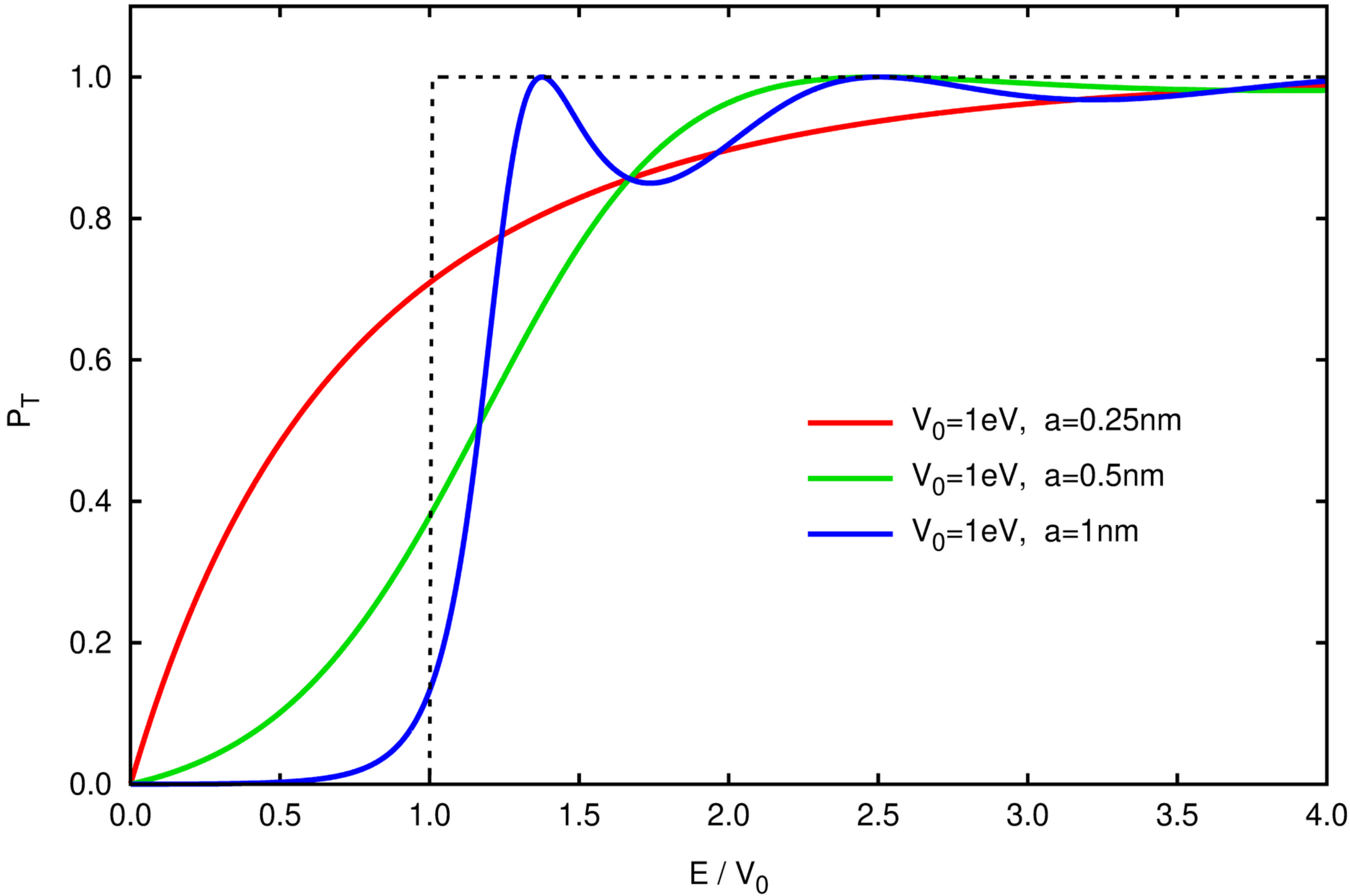
$$k = \frac{\sqrt{2mE}}{\hbar} \qquad \varkappa = \frac{\sqrt{2m(V_0-E)}}{\hbar}$$



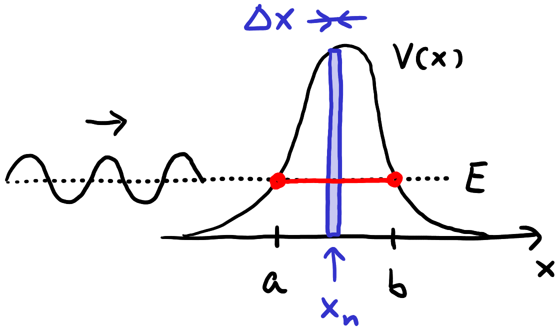
pravděpodobnost tunelování

$$P_T = |T|^2 = \frac{1}{1+\frac{1}{4}\left(\frac{\varkappa}{k}+\frac{k}{\varkappa}\right)^2 \sinh^2 \varkappa a} \sim e^{-2\varkappa a}$$

"Propustnost" pravoúhlé bariéry



Gamowova formule



bariéru v klasicky nedostupné oblasti $(E < V(x))$ nahradíme sadou N tenkých pravoúhlých bariér s výškami kopírujícími $V(x)$

pravděpodobnost tunelování bariérkou n: $\quad P_n \sim e^{-2\varkappa_n \Delta x}$

$$\varkappa_n = \frac{1}{\hbar}\sqrt{2m\left[V(x_n) - E\right]}$$

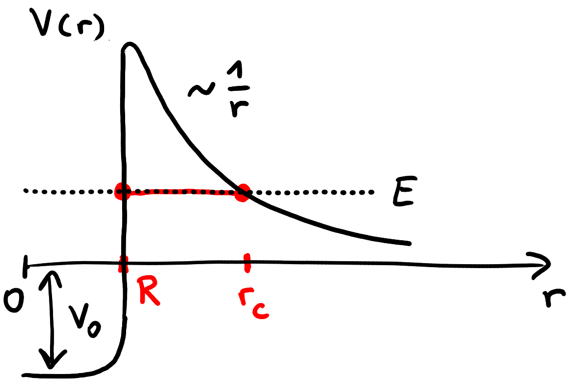
celková pravděpodobnost

$$P_T = P_1 \cdot P_2 \cdot P_3 \cdot \ldots \cdot P_n \cdot \ldots \cdot P_N$$

$$\ln P_T = \ln P_1 + \ln P_2 + \ln P_3 + \ldots + \underbrace{\ln P_n}_{-2\varkappa_n \Delta x} + \ldots + \ln P_N$$

$$\ln P_T = \sum -2\varkappa_n \Delta x = \int_a^b -2\varkappa(x)\,dx \longrightarrow P_T \approx \exp\left\{\overbrace{-\frac{2}{\hbar}\int_a^b \sqrt{2m\left[V(x) - E\right]}\,dx}^{\text{Gamowův faktor } G}\right\}$$

α-rozpad jader



pravděpodobnost tunelování α-částice ven z jádra podle Gamowa

$$P_T = e^{-G} \qquad G \approx \beta_1 \frac{Z}{\sqrt{E}} - \beta_2 Z^{\frac{2}{3}}$$

rozpadový zákon

$$\frac{dN}{dt} = -\frac{1}{\tau} N \rightarrow N(t) = N_0 e^{-t/\tau}$$

$\sim P_T \times$ Frekvence nárazů f_0

Geiger - Nutall

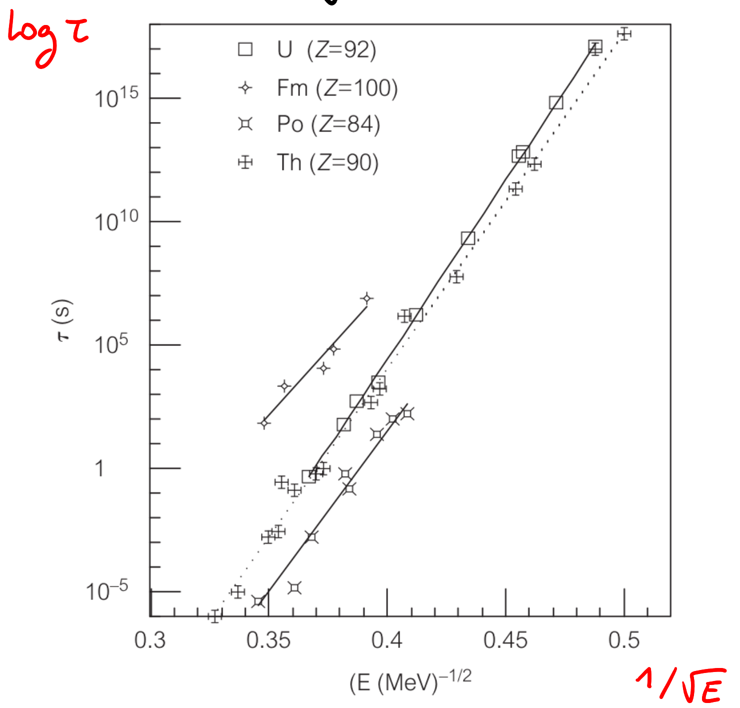
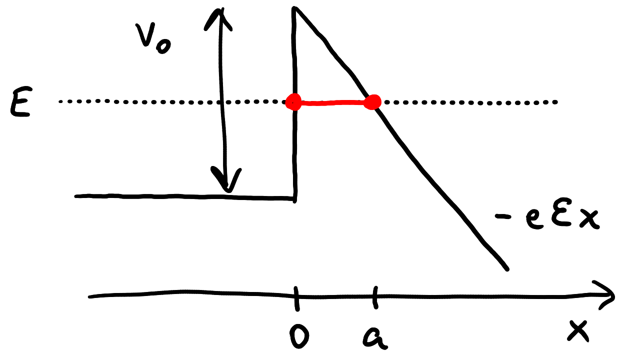


Figure 11.17. Semilog plot of α-decay lifetime (τ in seconds) versus $1/\sqrt{E_\alpha}$ (in MeV) for four different radioactive decay series, the so-called Geiger–Nutall plot. The data are taken from a recent edition of the Chart of the Nuclides (Walker (1983).)

$$\ln \tau = \beta_1 Z \frac{1}{\sqrt{E}} - \beta_2 Z^{\frac{2}{3}} - \ln f_0$$

Studená emise (autoemise)



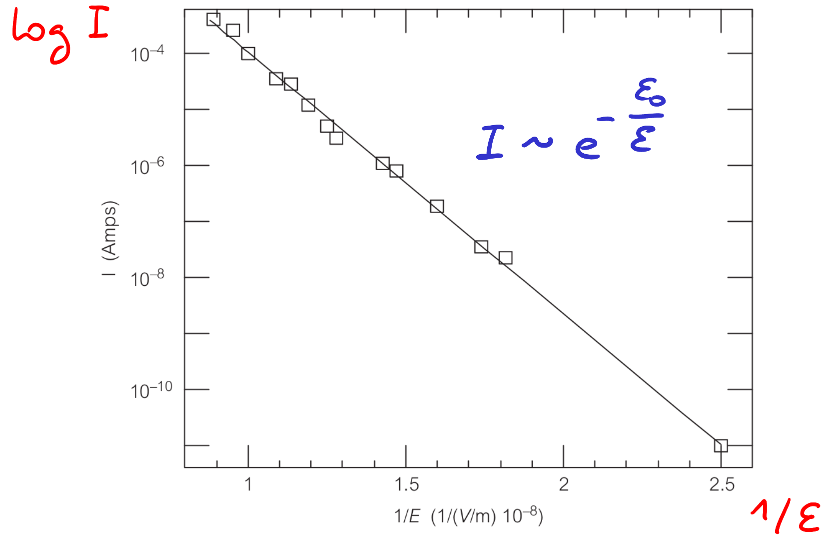


Figure 11.11. Semilog plot of tunneling current, I, versus $1/\mathcal{E}$ where $\mathcal{E}$ is the applied electric field, illustrating field emission. The data are taken from Millikan and Eyring (1926).

V_o – výstupní práce

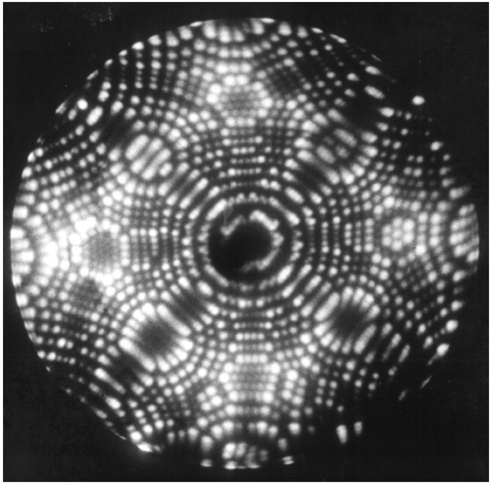
$\mathcal{E}$ – intenzita elektrického pole

klasicky zakázaná oblast $x \in [0, a = \frac{V_o - E}{e\mathcal{E}}]$

pravděpodobnost tunelování $P_T \approx e^{-G} = e^{-\frac{\mathcal{E}_0}{\mathcal{E}}}$

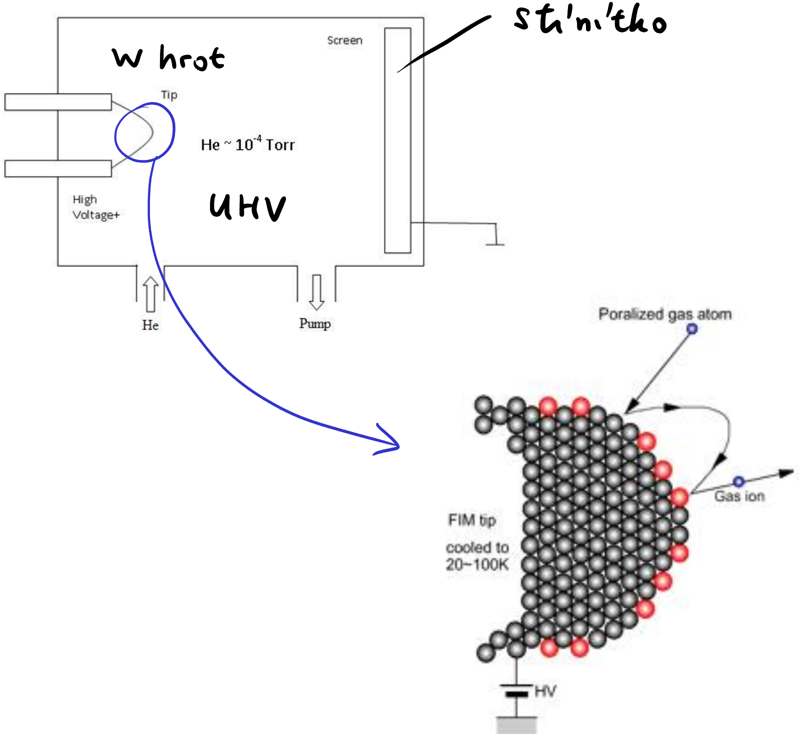
→ tunelový proud $I = I_o e^{-\frac{\mathcal{E}_0}{\mathcal{E}}}$ → $\ln I = -\frac{\mathcal{E}_0}{\mathcal{E}} + \ln I_o$ (Fowler & Nordheim, 1928)

Iontová autoemisní mikroskopie



FIM obrázek Pt hrotu

Tatsuo Iwata, z wikipedie



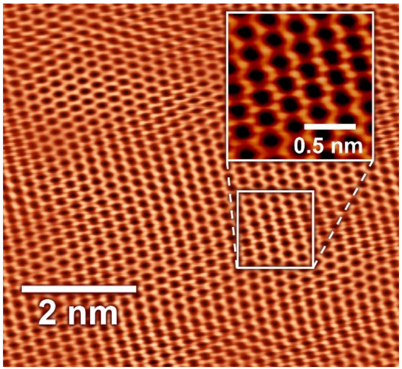
- do UHV vpuštěny ionty pracovního plynu (He, Ne)
- usedají na chlazený hrot (20-100K), ionizují se elektrickým polem u hrotu ($\sim 10^{11}\ \mathrm{V m^{-1}}$) a odlétají zanechat stopu na stínítku

Erwin Wilhelm Müller

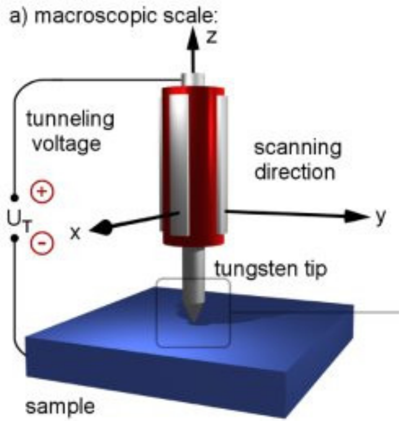
1911: Born June 13.
1936: Engineering Degree with Gustav Hertz.
1937: Invented the Field Emission Microscope.
1941: Discovered Field Desorption.
1951: Invented the Field Ion Microscope.
1952: Joined the Penn State Faculty.
1956: First observation of individual atoms.
1967: Invented the Atom-Probe.
1975: Elected, National Academy of Science and National Academy of Engineering.
1976: Under consideration (with Ernst Ruska) for the 1986 Nobel Prize in Physics.
1977: Died May 17 of a heart attack.
1977: Awarded the National Medal of Science (Posthumously from President Carter).

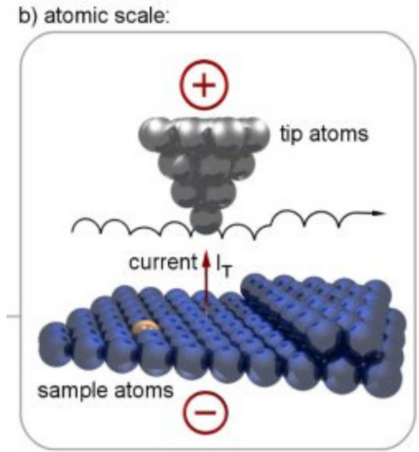
STM - Scanning Tunneling Microscopy

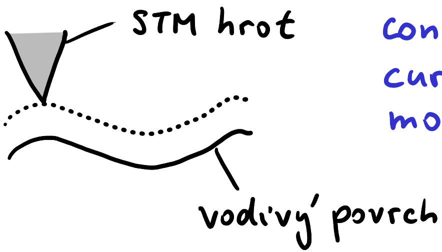
1981 Gerd Binnig & Heinrich Rohrer @ IBM   NC 1986



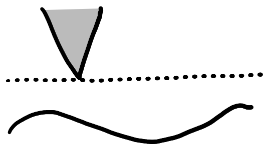
STM snímek povrchu grafitu







constant
current
mode



constant
height
mode

Tunelový proud v STM mikroskopu

M. Komai et al. / Applied Surface Science 146 (1999) 158–161

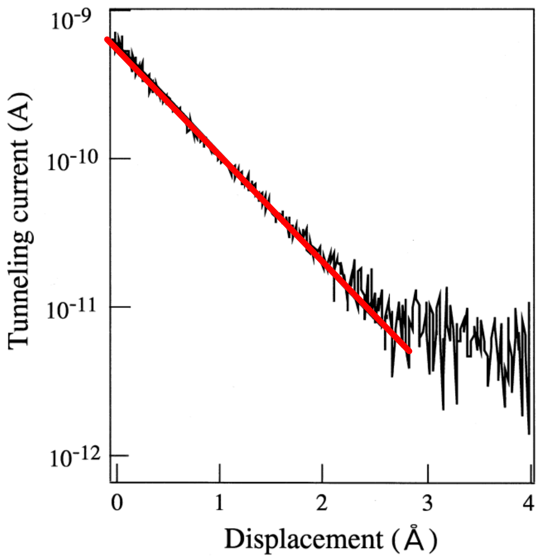
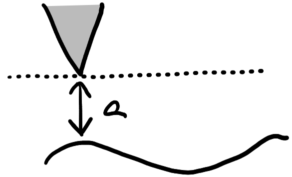


Fig. 4. Tunneling current obtained from the static measurement for Ba-deposited Si(111)-(3×1) surfaces as a function of the tip–sample distance. The tunneling current exponentially decays with the tip–sample distance over two orders of magnitude. The gradient of the ln $I_T - s$ plot is 1.77 (Å^{-1}).

$$P_T \approx e^{-2\varkappa a}$$



$$I_T \approx I_0\, e^{-2\varkappa a}$$

$$\rightarrow \ln I_T = \ln I_0 - 2\varkappa a$$

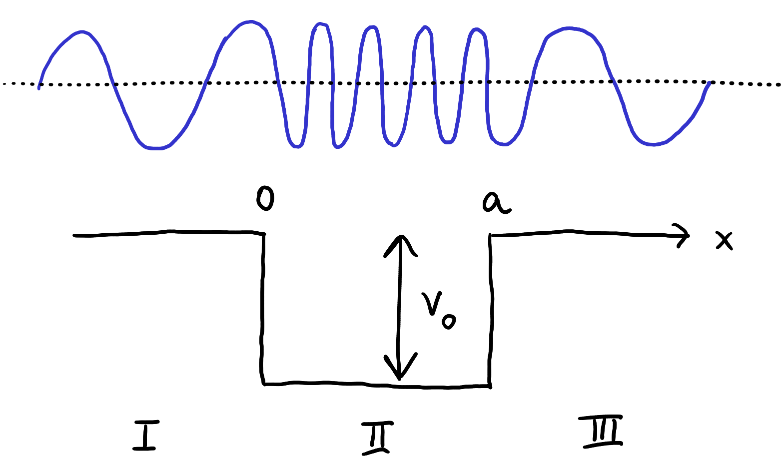
$2\varkappa$ je $1.77\ \text{Å}^{-1}$

$$2\varkappa = \frac{2\sqrt{2m}}{\hbar}\sqrt{V_0 - E}$$

výstupní práce

$$V_0 - E = \left(\frac{\hbar\, 2\varkappa}{2\sqrt{2m}}\right)^2 \approx 3.0\ \text{eV}$$

Rezonanční průchod nad kvantovou jámou



$$\psi_I(x) = e^{ikx} + R e^{-ikx}$$

$$\psi_{II}(x) = A e^{ik'x} + B e^{-ik'x}$$

$$\psi_{III}(x) = T e^{ik(x-a)}$$

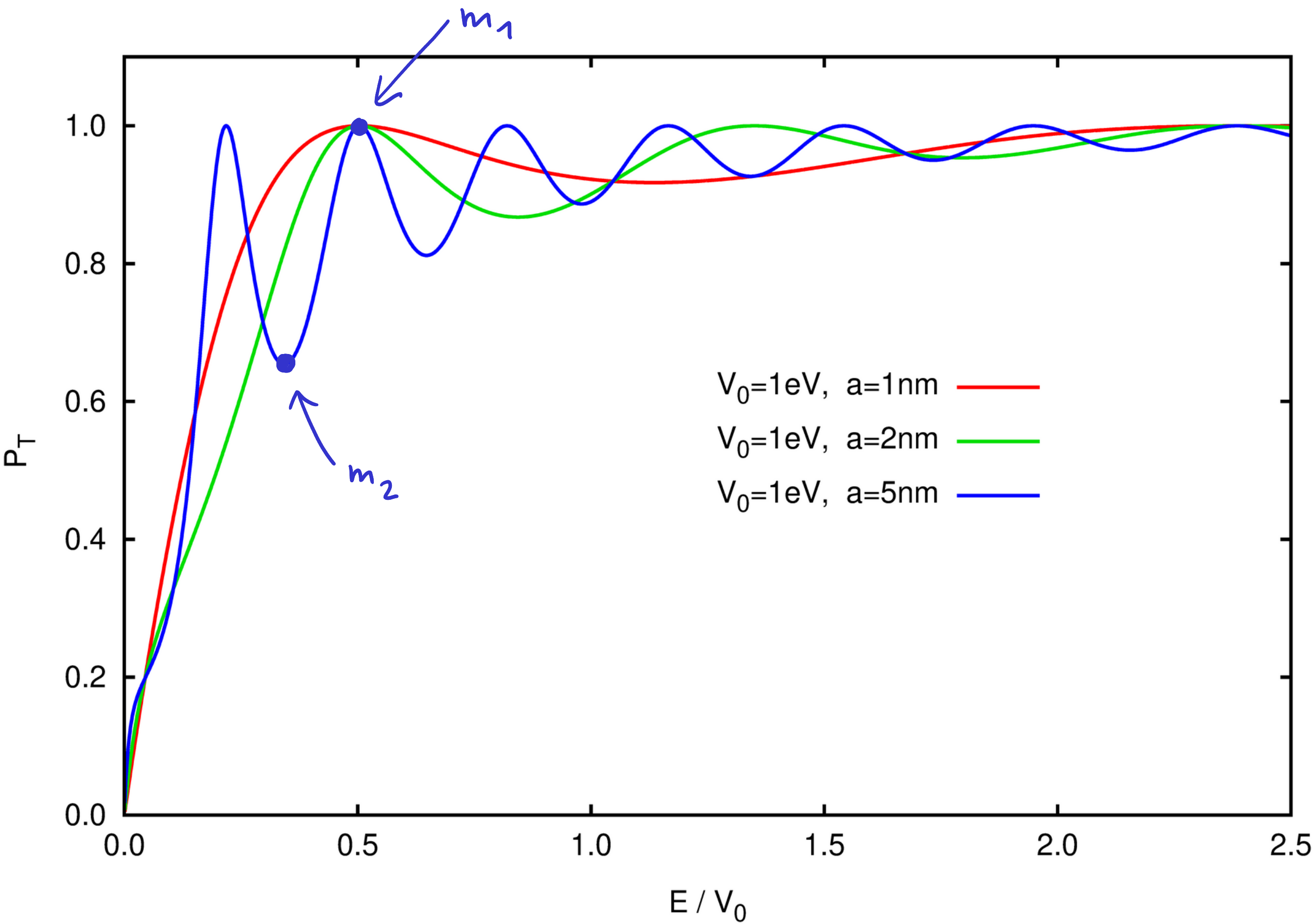
$$k = \frac{\sqrt{2mE}}{\hbar} \qquad k' = \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

pravděpodobnost průchodu $P_T = |T|^2 = \dfrac{1}{1 + \frac{1}{4}\left(\frac{k'}{k} - \frac{k}{k'}\right)^2 \sin^2 k'a}$

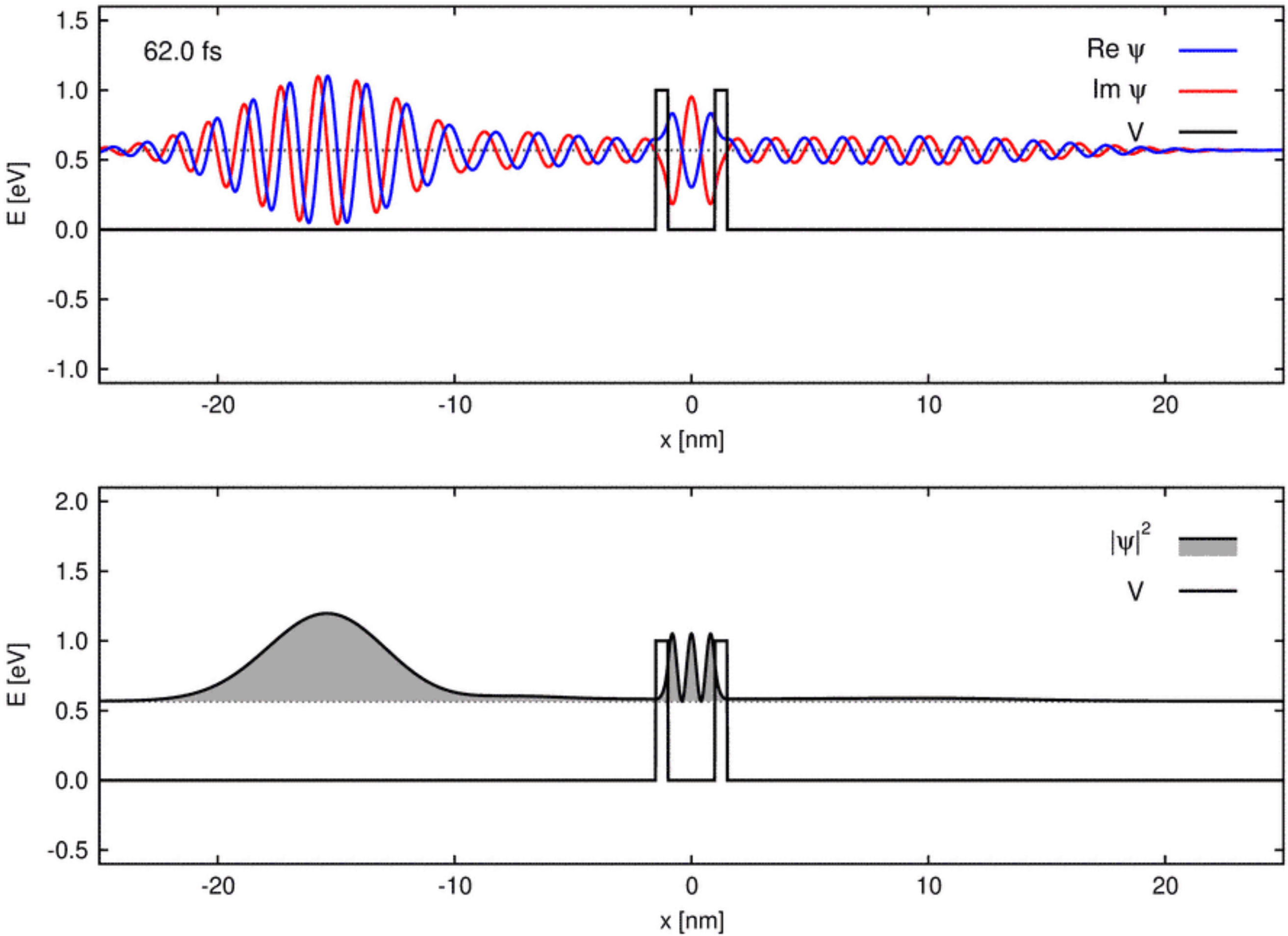
maxima $P_T = 1$ pro $k'a = n\pi \;\rightarrow\; E_0 + V_0 = \dfrac{\hbar^2 \pi^2}{2ma^2} n^2$

(rezonance při $E + V_0$ rovné energiím v ∞ hluboké jámě)

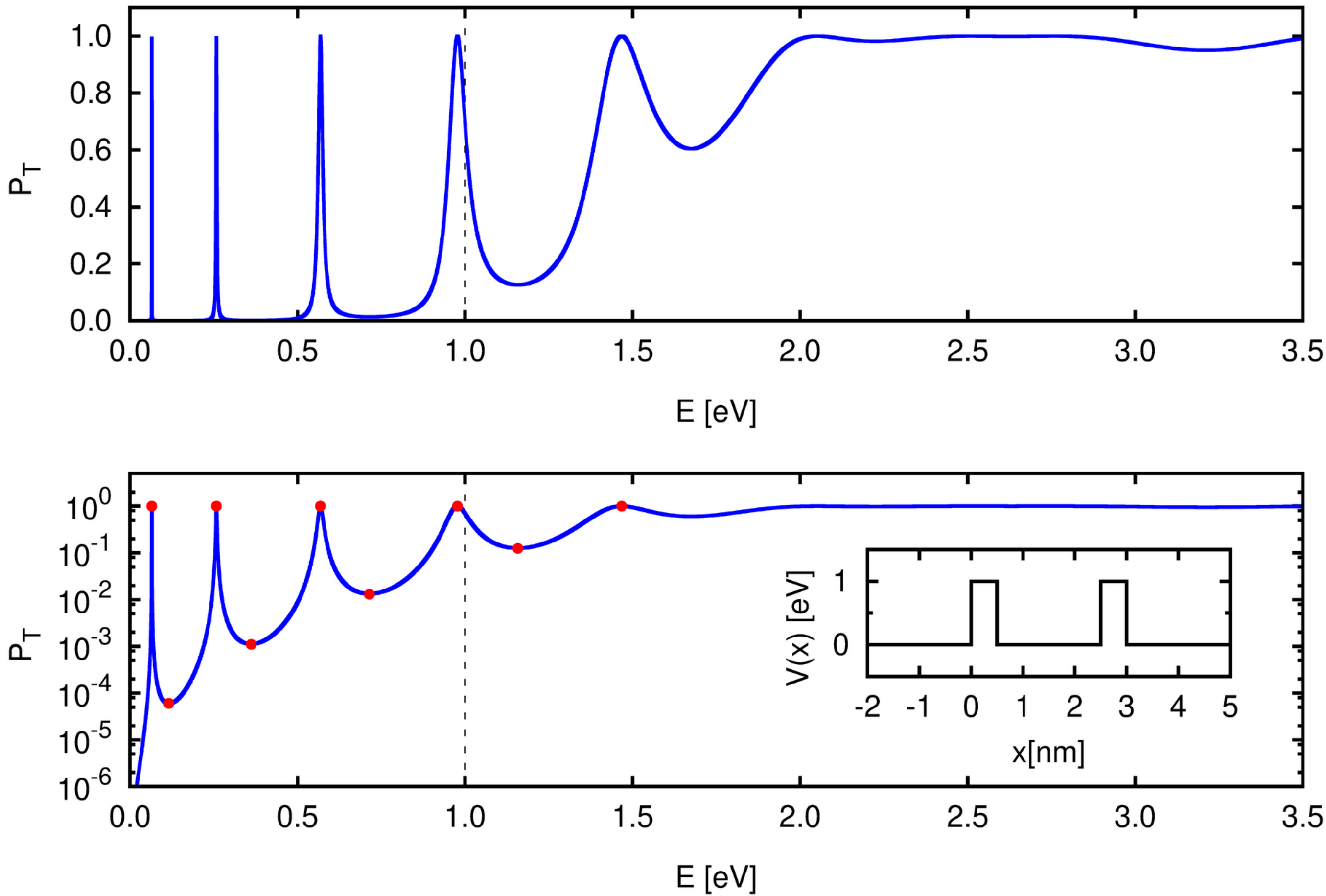
Rezonanční průchod nad kvantovou jámou



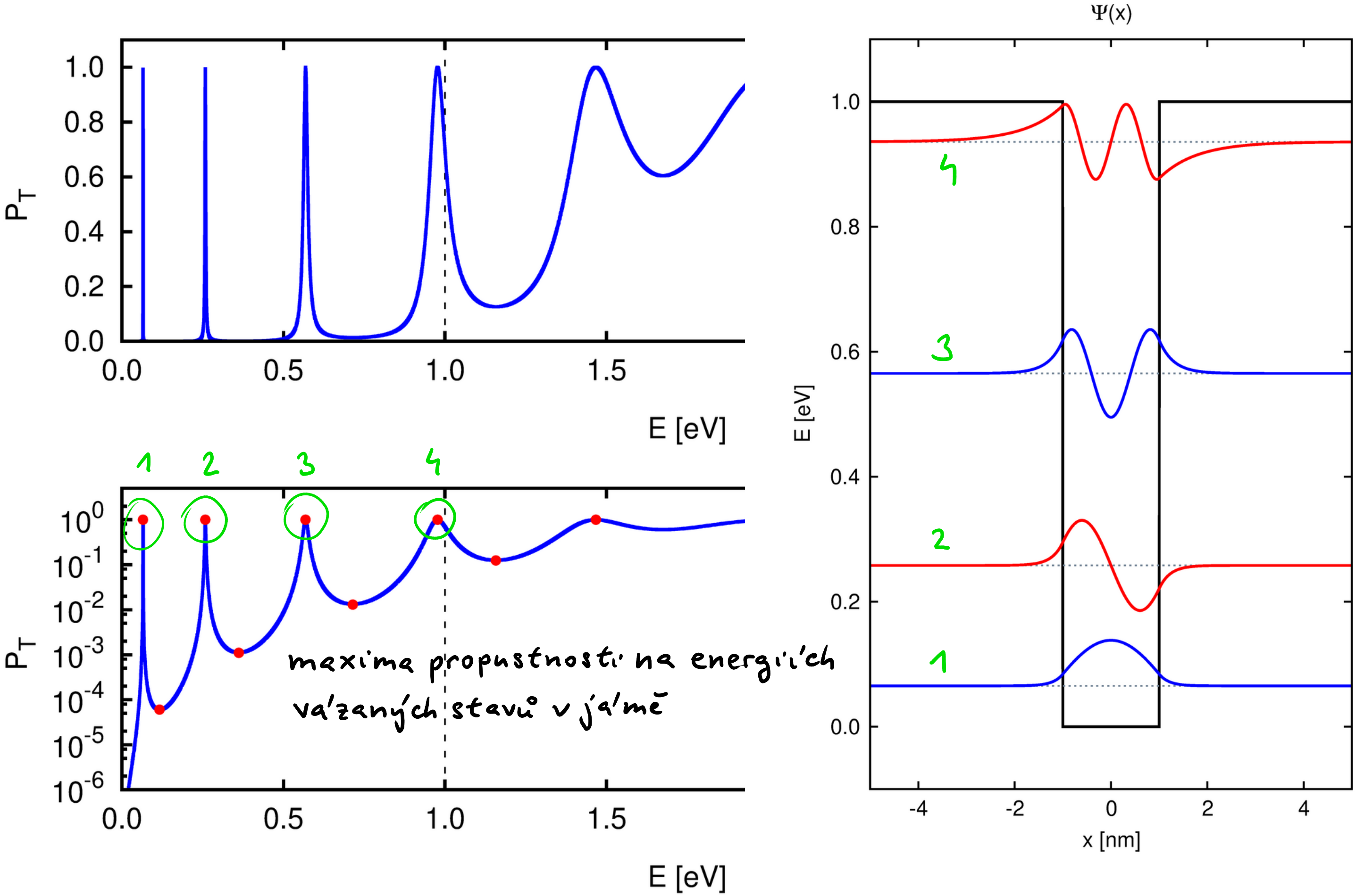
Metastabilní stavy



"Propustnost" pravoúhlé dvojbariéry

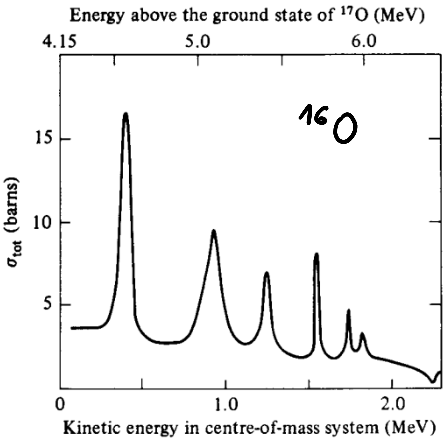


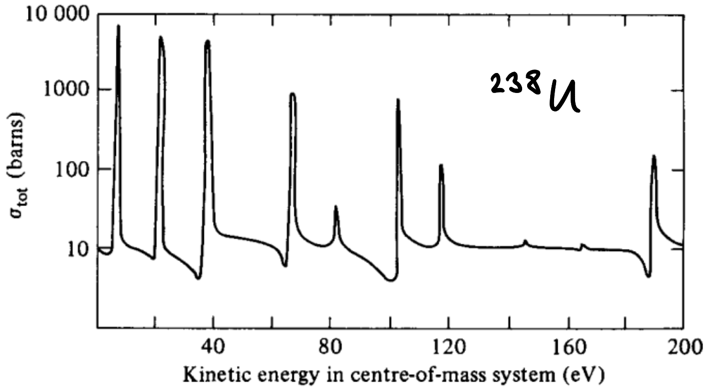
"Propustnost" pravoúhlé dvojbariéry



Rezonanční záchyt neutronů (volná analogie)

- účinný průřez záchytu neutronů v jádrech ^{16}O a ^{238}U





- úzké rezonance odpovídají utváření excitovaných stavů jader ^{17}O a ^{239}U