Kurt Gödel Day 2021
&
Czech Gathering of Logicians 2021

25–26 June 2021
Brno, the Czech Republic

Book of abstracts

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Locations

Conference

– Fri 25/6/2021 – Sat 26/6/2021

– Brno Observatory and Planetarium, Kraví hora 522/2, Brno

– main building

Conference lunch

– Fri 25/6/2021, 14:00

– Brno Observatory and Planetarium, Kraví hora 522/2, Brno

– main building

Conference banquet

– Fri 25/6/2021, 19:00

– Brno Observatory and Planetarium, Kraví hora 522/2, Brno

– main building

Website

`https://www.physics.muni.cz/~godel/kgd2021/`

Sponsors

Brno Observatory and Planetarium

Czech Society for Cybernetics and Informatics

Kurt Gödel Society in Brno

Masaryk University

Call for papers

This community event aims at bringing together researchers in logic and related areas. The event is open to all researchers interested in logic, while contributions related to Gödel's work are especially welcome. Promoting the heritage of Kurt Gödel, the Kurt Gödel Prize will be awarded during the meeting by the Kurt Gödel Society in Brno and the recipient will deliver a lecture.

We cordially invite researchers working in a field relevant to the conference to submit a short plain text abstract of approximately 200 words, and an extended abstract of at most 1.000 words (references included) through EasyChair at https://easychair.org/conferences/?conf=kgd2021cgl2021.

Accepted papers (contributing talks) will be presented in 30 minute slots including discussion. Abstracts must be written in Czech/English; uploaded extended abstract must be in pdf format, using the EasyChair LaTeX style of submissions, https://easychair.org/publications/for_authors.

Deadline for paper submission: Monday 24 May 2021 (extended). Notification of acceptance: Monday 7 June 2021 (extended). Conference fee (includes booklet of abstracts, coffee breaks and conference dinner): CZK 1.000 (for students: CZK 200) (please contact Helena Durnová (hdurnova [at] ped.muni.cz). Registration of attendees is required because of Covid restrictions, please contact Helena Durnová (hdurnova [at] ped.muni.cz).

Programme and organising committees

Programme committee

Marta Bílková (Czech Academy of Sciences, Prague) (chair)
Carles Noguera (Czech Academy of Sciences, Prague)
Jan Novotný (Masaryk University, Brno)
Jiří Raclavský (Masaryk University, Brno)

Organising committee

Helena Durnová
Jiří Dušek
Zuzana Haniková
Jan Paseka
Jiří Raclavský
Blažena Švandová

Organisation

Brno Observatory and Planetarium
Kurt Gödel Society in Brno
Masaryk University
Institute of Computer Science, Academy of Sciences
Union of Czech Mathematicians and Physicists, Brno branch

Programme

Friday 25 June 2021

10:00	*registration*	
10:30	**opening**	
	Prof Jan Paseka	(Kurt Gödel Society in Brno)
	Dr Jiří Dušek	(Brno Observatory and Planetarium)
	Prof Jan Novotný	(Kurt Gödel Society in Brno)
10:45	**KG prize ceremony**	
	invited talk	*chair*: Jan Paseka
11:00	Matthias Baaz	Kurt Gödel and Alfred Tarski: The Extremes of Logic
12:00	*coffee break*	
	contributed talks	*chair*: Zuzana Haniková
12:30	Marie Duží	From Gödel to Henkin; Completeness vs Incompleteness
13:00	Kentarô Yamamoto	The Automorphism Group of the Fraïssé Limit of Finite Heyting Algebras
13:30	Adam Přenosil	Universal Horn Properties of Upsets of Distributive Lattices
14:00	**lunch**	

	invited talk	*chair*: Jiří Raclavský
15:00	Petr Cintula	Completeness and Incompleteness in Logics with Non-classical Proposition Core
	contributed talks	
16:00	Igor Sedlár	Reasoning About Graded While Programs
16:30	*coffee break*	
16:45	Helena Durnová	Popularising Modern Logic in 1950s Czechoslovakia
17:15	Azza Gaysin	H-coloring Dichotomy in Proof Complexity
17:45	*coffee break*	
		chair: Helena Durnová
18:00	Marta Bílková, Sabine Frittella, Ondrej Majer, Sajad Nazari	Probabilistic Reasoning Based on Incomplete and Inconsistent Information
18:30	Marta Bílková, Sabine Frittella, Daniil Kozhemiachenko	Two-dimensional Logics of Comparative Uncertainty
19:00	**banquet**	

Saturday 26 June 2021

	invited talk	*chair*: Zdeněk Pospíšil
09:00	Vítězslav Švejdar	From Arithmetization to Interpretability Principles
	contributed talk	
10:00	Jamie Wannenburg	Beth Definability in Relevance Logics with the Gödel-Dummett axiom
10:30	*coffee break*	
	invited talk	*chair*: Jan Pavlík
11:00	Pavol Zlatoš	Hilbert's Program and Gödel's Incompleteness Theorems
	contributed talks	
12:00	Stéphane Le Roux, Érik Martin-Dorel, Jan-Georg Šmaus	Existence of Nash Equilibria in Preference Priority Games Proven in Isabelle
12:30	*coffee break*	*chair*: Jiří Raclavský
13:00	Libor Běhounek	Do These Degrees Really Go to Eleven?
13:30	Kadir Emir, David Kruml, Jan Paseka, Thomas Vetterlein	Why are Models of Quantum Logic Infinite

14:00 end of the conference

I. Invited speakers

Matthias Baaz

Kurt Gödel and Alfred Tarski: The Extremes of Logic

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In this lecture we will compare the eminent founders of modern logic connected by a strange coincidence by the date of January, 14: Kurt Gödel and Alfred Tarski.

Kurt Gödel has been driven by the the possibility that the individual thinking might transgress its own limits. The solutions of mathematical problems are the models, not the aims of his thinking. He strongly believed in the simplicity of all solutions maybe beyond language. Therefore he chose very carefully the next generation scientists with whom he communicated (basically Georg Kreisel, Gaisi Takeuti and Hao Wang).

Alfred Tarski on the other hand grew up in the logical traditions of Poland. He considered logic as a mathematical subject based on a mathematical language, which is very able to contribute to mathematics as algebra, topology etc. do. He emphasized the formal semantical relations as entailment, satisfaction and truth. He educated many students and influenced not only logic and mathematics but also formal linguistics by his thorough mathematical rigor.

In a more general sense, this lecture tries to discuss (but not to answer) the question: "In which sense does Mathematics matter to Logics?" where Kurt Gödel and Alfred Tarski had complementary views.

Petr Cintula

Completeness and Incompleteness in Logics with Non-classical Proposition Core

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Kurt Gödel is celebrated for many famous results. In this talk we focus on two fundamental logical results he proved first: the completeness theorem [2] and the first incompleteness theorem [3] and explore how we can generalize them to predicate logics whose propositional core is other than the classical logic.

In the first part of the talk (based on paper [4]) we survey the history of generalizing the completeness theorem and present a very general framework which allows us to prove it for a rather extensive family of non-classical logics which included the most of the prominent logics studied in the literature. Thus we demonstrate that a very little of propositional logic is needed to obtain a reasonable first-order logic with a meaningful syntax–semantics connection.

In the second part (based on paper [1]) we do analogous analysis of the first incompleteness theorem in its guise of essential undecidability of (Peano's) arithmetics [5, 6]. Here we show non-only that a very little of propositional logic is needed to obtain the result; we also show that a very weak arithmetics suffices (a variant of Robinson's R with non-total relations instead of addition and multiplication).

References

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Vítězslav Švejdar

From Arithmetization to Interpretability Principles

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The notion of interpretability, defined in [4], can be used to prove the relative consistency of axiomatic theories. One of many examples is the following. If the Zermelo-Fraenkel set theory ZF is extended by adding the continuum hypothesis CH as an additional axiom, then the resulting theory $(\mathsf{ZF} + \mathrm{CH})$ is interpretable in ZF. In symbols, $\mathsf{ZF} \rhd (\mathsf{ZF} + \mathrm{CH})$. This fact shows that $(\mathsf{ZF} + \mathrm{CH})$ is consistent provided that ZF is consistent. Interpretability of axiomatic theories can also serve as a tool for comparing the strength of axiomatic theories: if T is interpretable in S but not vice versa, then one can conclude that S is considerably stronger than T. In this sense, $(\mathsf{ZF} + \mathrm{CH})$ is stronger but not considerably stronger than ZF.

Feferman in [1] linked interpretability to Gödel's second incompleteness theorem. For example, if a theory T is sufficiently strong, a formula $\tau(z)$ describes its axioms and $\mathsf{Con}(\tau)$ is the formalized consistency statement created from τ, then $(T + \neg\mathsf{Con}(\tau))$ is interpretable in T, but $(T + \mathsf{Con}(\tau))$ is not. Feferman also listed some general properties of the interpretability relation like the transitivity. This way, interpretability became not only a tool, but also a field of study. Further relevant results were obtained by Petr Hájek who added some principles to those listed by Feferman. He also noticed that the two popular set theories, the Zermelo-Fraenkel set theory ZF and the Gödel-Bernays set theory GB, while being identical as to the provability of set sentences, differ in interpretability. In particular, the conditions $\mathsf{GB} \rhd (\mathsf{GB} + \varphi)$ and $\mathsf{ZF} \rhd (\mathsf{ZF} + \varphi)$, for a set sentence φ, are not equivalent. Hájek's work directly inspired other researchers (Solovay, Lindström, Pudlák, Visser, ...) and gave rise to interpretability logic, a modal propositional logic with two modalities, a unary symbol $\Box$ for provability and a binary symbol $\rhd$ for interpretability.

The talk is meant as a contribution to studying the early history of interpretability and its principles.

References

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Pavol Zlatoš

Hilbert's Program and Gödel's Incompleteness Theorems

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Gödel's Incompleteness Theorems belong to the most remarkable achievements of the 20^{th} century mathematics, shedding light on the limitations of formal methods in mathematics and still raising philosophical questions about the nature of human thought, its relations to our brains and to computers, etc.

> *Kurt Gödel's achievement in modern logic is singular and monumental — indeed, it is more than a monument, it is a landmark which will remain visible far in space and time. [...] The subject of logic has certainly completely changed its nature and possibilities with Gödel's achievement.*
>
> (John von Neumann)

We will explain the meaning, impact and importance of Gödel's Incompleteness Theorems viewed through the prism of Hilbert's Program. At the same time we will present some myths and oversimplified interpretations of both Hilbert's Program and Gödel's Incompleteness Theorems, and try to set measures to them. To this end we will briefly survey the development of mathematics and the circumstances which finally led to its crisis at the turn of the 19^{th} and the 20^{th} century, as well as the reactions to and proposed ways out of it. From this point of view Hilbert's Program appears not just as one of the proposals how to overcome the crisis but also as a serious attempt to face the challenge raised by Brouwer's intuitionistic revolt, the most radical one from among of these reactions. Then Gödel's Theorems — albeit they show the impossibility to carry out Hilbert's Program in its original form — fall fully within the guidelines set up by Hilbert.

Hilbert's Program was an ambitious and wide-ranging project in the philosophy and foundations of mathematics. In order to "dispose of the foundational questions in mathematics once and for all", Hilbert proposed a two-pronged approach in 1921: first, classical mathematics should be formalized in axiomatic systems; second, using only restricted, "finitary" means, one should give proofs of the consistency of these axiomatic systems. Although Gödel's Incompleteness Theorems show that the program, as originally conceived, cannot be carried out, it had many partial successes, and generated important advances in logical theory and meta-theory, both at the time and since.

(Richard Zach,
Hilbert's Program Then and Now, arXiv:math/0508572)

II. Contributed talks

Do These Degrees Really Go to Eleven?

Libor Běhounek

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Abstract

The contribution describes and compares two recently proposed models of determinate truth under the degree-theoretic semantics of vagueness. Although both models admit the graded nature of determinate truth and represent it by an upper set of truth degrees, they differ significantly regarding the logical properties of the proposed semantics. Besides comparing the advantages and drawbacks of both models, their combination is discussed and dismissed as a remedy to their respective weaknesses.

Two different ways of representing determinate truth in the degree-theoretic semantics of vagueness have recently been proposed [2, 4]. Both proposals subscribe to fuzzy plurivaluationism, which answers the most common objections to degree theories of vagueness by considering a class of admissible fuzzy models for vague predicates rather than a single fuzzy model [8]. Both proposals employ (a suitable) fuzzy logic [5] to govern vague propositions, as a means to resolve the sorites paradox [6] and to logically ground the fuzzy plurivaluationistic semantics [1]. Moreover, both proposals represent the notion of determinate truth by an upper set of truth values, conforming to the intuition that the intensity of a vague property F can be different even among determinately-F individuals (as, e.g., determinately tall people can still be ordered by tallness). However, the two proposals differ in several important respects regarding the inferential behaviour of determinately true vague propositions. In this contribution (titled in reference to [4]), I point out the differences between the two approaches and compare their relative advantages and drawbacks. I will also discuss the possibility of combining both approaches and the drawbacks that cannot be eliminated by the combination.

The approach of [4] represents determinate truth by a crisp closed upper set of designated truth degrees. The truth degrees can either be taken from the interval $[0, 1]$, in which case the designated set is $[e, 1]$ for some $e \in (0, 1)$; or isomorphically, from the whole extended real line $[-\infty, +\infty]$, where the designated set is $[1, +\infty]$. In the former case, the propositional connectives are those of (a suitable) *uninorm* fuzzy logic [7], while for the latter case, an extension of the Łukasiewicz $[0, 1]$-valued connectives to $[-\infty, +\infty]$ is proposed. Uninorm fuzzy logics define the consequence relation in the standard way as the preservation of designated truth values.

In [2], on the other hand, determinate truth is represented by a *fuzzy* upper subset of $[0, 1]$; in particular, by a fuzzy set of degrees so close to 1 as to be indistinguishable from 1 by any *small number* of inferential steps in a suitable *t-norm* fuzzy logic (e.g., the well known Łukasiewicz logic). Consequently, in all reasonably short arguments, such degrees consistently behave as if they actually equalled the full truth 1, and only artificially long arguments such as the sorites paradox reveal, by way of arriving at a contradiction, their actual deficiency in truth. The fuzzy semantics of the vague predicate *small number* (of inferential steps; cf. [6]) determines the degree to which $\alpha \leq 1$ belongs to the fuzzy set of degrees representing determinate truth in a given model.

In this brief abstract, let me just hint at some of the advantages and drawbacks of the respective approaches; more details will be given in the talk. Based on the underlying fuzzy logics, let us call the approach of [4] the *uninorm semantics* and that of [2] the *t-norm semantics.*

An advantage of the t-norm semantics is that by regarding determinate truth as a graded indistinguishability-based notion, it naturally accommodates second-order vagueness, while the representation of determinate truth by a crisp set of degrees makes the uninorm semantics susceptible to the sorites paradox for the (crisp) predicate *determinately-F*. Although the restored paradox can be avoided by excluding the operator △ (indicating the designated values) from the propositional language, the uninorm semantics does not seem to offer an explanation why exactly this connective should be banned (whereas in the t-norm semantics, △ is clearly illegitimate since it disregards the indistinguishability of degrees). Additionally, uninorm connectives exhibit a less classical behaviour than the t-norm ones (mainly by invalidating the inference rule of weakening); thus, for example, the conjunctive meaning of the fusion connective (which in the t-norm based semantics of vagueness helps explain the natural-language ambiguity between lattice conjunction and fusion) is partly lost in the uninorm semantics, as the fusion of a determinately true and a determinately false proposition can still be determinately true. Some language constructions (such as relativized quantification) can also be more straightforwardly formalized by means of t-norm connectives than by the uninorm ones.

On the other hand, certain arguments are more straightforwardly formalizable by means of the uninorm semantics—e.g., the 'positive' inductive step in the sorites reasoning (i.e., that *adding* a grain to a determinate heap increases its heapness). Moreover, the preservation of determinate truth in the sense of [4] is directly represented by the standard Tarski consequence relation of the uninorm fuzzy logic, while representing it in the t-norm semantics requires using a non-Tarskian consequence relation of the kind that has only recently been introduced in [3].

The fact that both semantics have some advantages over one another suggests that neither is a completely satisfactory degree-theoretic model of determinate truth. A tempting option might be to combine both approaches, i.e., to fuzzify the uninorm semantics by the graded relation of inferential indistinguishability. Nevertheless, while this would indeed combine some strengths of both approaches, it would also inevitably combine their drawbacks (resulting in the non-Tarskian consequence relation being further complicated by the failure of weakening). Thus, in spite of being a possibly interesting area for future research, the combination can hardly solve the imperfections of both semantics. Therefore, until a better model is found, we have to regard both variants just as two competing imperfect approximations to a fully satisfactory degree-theoretic account of determinate truth.

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Two-dimensional logics of comparative uncertainty

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Abstract

We introduce two-dimensional logics based Gödel logics to formalize paraconsitent fuzzy reasoning. The logics are interpreted on matrices, where the common underlying structure is the bi-lattice (twisted) product of the $[0,1]$ interval. The first (resp. second) coordinate encodes the positive (resp. negative) information one has about a statement. We propose constraint tableaux that provide a modular framework to address their completeness and complexity. We also discuss the compactness of entailment and duality between algebraic and frame semantics.

General project. This work is a part of the project introduced in [2]. We are developing a modular logical framework for reasoning based on uncertain, incomplete or inconsistent information. In this framework, an agent is constructing their belief using probabilistic incomplete and/or conflicting information aggregated from multiple sources. We formalize such probabilistic reasoning using the framework of two-layer modal logics first introduced in [5, 7] and then developed by [4] and [1]. Two-layer modal logics to formalise such probabilistic reasoning in a potentially paraconsistent context were proposed in [2]. These logics work roughly as follows. First, the information given by the agent's sources is given on the lower layer. It is then lifted up to the upper layer by belief modalities. Finally the reasoning with the agent's belief is encoded there.

Two-dimensional treatment of uncertainty. For the purpose of our talk, we consider agents who although not being always able to give an exact level of their certainty in some proposition, can compare their certainty in one proposition to the certainty in the other. Thus, we are interested in the expansions of Gödel logic which can be treated as the logic of comparative truth (or comparative certainty).

Two-dimensionality comes from the definition of the logics using expansions of the product bilattice $[0,1]\odot[0,1]$. While $\wedge$ and $\vee$ are defined in a standard way, there are several ways to define implication. We consider two possibilities: $\rightarrow$ dualizes implication by co-implication, and $\rightarrowtriangle$ understands negative support of an implication as a conjunction of the positive support of the antecedent with the negative support of the consequent. Furthemore, in each of these interpretations, we consider different possible entailments. Thus we have two families of logics. The first of them which we call $\mathsf{G}^2(\rightarrow)$ connects to one of Wansing's logic of [10], namely $\mathsf{I_4C_4}$, and goes back to bi-intuitionistic logic [6, 9], the second option $\mathsf{G}^2(\rightarrowtriangle)$ connects to Nelson's logic $\mathsf{N4}$ [8].

Definition 1 (G^2 logics)**.** For all $a,b\in[0,1]$, we set $a\wedge b:=\min(a,b)$, $a\vee b:=\max(a,b)$ as well as

$$a\rightarrow_G b:=\begin{cases}1, & \text{if} \quad a\leq b\\ b & \text{else}\end{cases}\qquad\qquad b\prec_G a:=\begin{cases}0, & \text{if}\quad b\leq a\\ b & \text{else}\end{cases}$$

Negation and 1 are defined as $\sim_{\mathsf{G}}a:=a\rightarrow_{\mathsf{G}}0$, and $1:=\sim_{\mathsf{G}}0$, respectively.

Now fix a countable set Prop of propositional letters and consider the following language:

$$\phi:=\mathbf{0}\mid\mathbf{1}\mid p\mid\neg\phi\mid(\phi\wedge\phi)\mid(\phi\vee\phi)\mid(\phi\rightarrow\phi)\mid(\phi\prec\phi)\mid(\phi\rightarrowtriangle\phi)$$

where $p \in \mathsf{Prop}$. We define $\sim\phi := \phi \to \mathbf{0}$, and $\sim_w\phi := \phi \multimapdotbothB \mathbf{0}$.

Let $v : \mathsf{Prop} \to [0,1] \times [0,1]$, and denote v_1 and v_2 its left and right coordinates, respectively. We extend v as follows.

$$\begin{array}{rclrcl} v(\mathbf{0}) & = & (0,1) & v(\phi_1 \wedge \phi_2) & = & (v_1(\phi_1) \wedge v_1(\phi_2), v_2(\phi_1) \vee v_2(\phi_2)) \\ v(\mathbf{1}) & = & (1,0) & v(\phi_1 \vee \phi_2) & = & (v_1(\phi_1) \vee v_1(\phi_2), v_2(\phi_1) \wedge v_2(\phi_2)) \\ v(\neg\phi) & = & (v_2(\phi), v_1(\phi)) & v(\phi_1 \to \phi_2) & = & (v_1(\phi_1) \to_{\mathsf{G}} v_1(\phi_2), v_2(\phi_2) \prec_{\mathsf{G}} v_2(\phi_1)) \\ & & & v(\phi_1 \multimapdotbothB \phi_2) & = & (v_1(\phi_1) \to_{\mathsf{G}} v_1(\phi_2), v_1(\phi_1) \wedge v_2(\phi_2)) \end{array}$$

Definition 2 ($(x,y)^{\uparrow}$-validity and entailment)**.** Let $(x,y)^{\uparrow} = \{(z,z') : z,z' \in [0,1], z \geq x, z' \leq y\}$. ϕ is $(x,y)^{\uparrow}$-*valid* iff $v(\phi) \in (x,y)^{\uparrow}$ for any v. Γ $(x,y)^{\uparrow}$-*entails* ψ ($\Gamma \vDash_{\mathsf{G}^2_{(x,y)}} \psi$) iff for any v s.t. $v(\phi) \in (x,y)^{\uparrow}$ for all $\phi \in \Gamma$, we have $v(\psi) \in (x,y)^{\uparrow}$.

Results. We establish connections between $\mathsf{G}^2_{(x,1)}(\multimapdotbothB)$'s and $\mathsf{N4}^{\perp}$ by [8] as well as between $\mathsf{G}^2_{(x,y)}(\to)$'s and $\mathsf{I_4C_4}$ from [10]. In particular, we show that the set of valid formulas of any $\mathsf{G}^2_{(x,y)}(\to)$ coincides with that of $\mathsf{I_4C_4}^{\perp} + (\phi \to \psi) \vee (\psi \to \phi)$ while the set of valid formulas of any $\mathsf{G}^2_{(x,1)}(\multimapdotbothB)$ coincides with that of $\mathsf{N4}^{\perp} + (\phi \to \psi) \vee (\psi \to \phi)$. Thus, just as G is a prelinear extension of the intuitionistic logic, G^2's are prelinear extensions of its expansions with strong negation.

Furthermore, we present a unified sound and complete tableau system for all G^2's introduced in [3]. We also show the expected duality between algebraic semantics on the one hand, and prelinear frames for $\mathsf{N4}^{\perp}$ and $\mathsf{I_4C_4}^{\perp}$ on the other hand. We use this duality to prove that any $\vDash_{G^2_{(x,y)}(\to)}$ is compact as long as $(x,y)^{\uparrow}$ extends $(x,1)^{\uparrow}$ or $(0,y)^{\uparrow}$ or does not contain any (z,z), and that $\vDash_{G^2_{(x,1)}(\multimapdotbothB)}$'s are compact as well.

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Probabilistic reasoning based on incomplete and inconsistent information

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Abstract

This work is part of a wider project aiming at developing a modular logical framework to formalize probabilistic reasoning based on incomplete and/or inconsistent information [3]. *Non-standard probabilities* were introduced in [5] to generalize the notion of probabilities to paraconsistent reasoning based on *First Degree Entailment* (FDE) [1]. Here we study the meaning of probabilities, conditional update, belief functions and their aggregation in the framework of FDE.

Motivation. We often have access to a multitude of information coming from different sources. This information is often incomplete, conflicting and uncertain. Here we aim at formalizing how a rational agent forms beliefs based on such information.

Scenarios we have in mind can be illustrated by the following toy example. An investigator needs to know if one of the suspects was present at the crime scene. She collects information from various sources (CCTV camera recordings, ATM logs, witnesses' statements etc.). The sources of evidence confirming investigator's hypothesis (the suspect was present at the place of crime) are different from, and in general independent of, those rejecting it (there is a CCTV camera closed to the crime scene vs. ATM in a supermarket in a different city). A lack of evidence supporting the hypothesis is not a reason to reject it. In the end the investigator has to aggregate the available information and form some beliefs about what likely happened.

Probabilistic reasoning based on incomplete and inconsistent information. We take *First Degree Entailment* FDE [1] as our base logic and study the meaning of probabilities, conditional update, belief functions and their aggregation in that framework. We base our work on *non-standard probabilities* [5] to account for potentially contradictory information about events. A *probabilistic model* is a tuple $\mathscr{M} = \langle \Sigma, \mu, v^+, v^- \rangle$ where Σ is a finite set of states, $v^+, v^- : \Sigma \times \mathrm{Prop} \to \{0,1\}$ are valuations representing respectively the positive and negative information and μ is a probability measure on the powerset algebra $\mathscr{P}(\Sigma)$. Let $|\varphi|^+_{\mathscr{M}} = \{s \in \Sigma : v^+(\varphi) = 1\}$ and $|\varphi|^-_{\mathscr{M}} = \{s \in \Sigma : v^-(\varphi) = 1\}$. The *non-standard probability function* based on $\mathscr{M}$ is the couple of maps (p^+_μ, p^-_μ) where $p^+_\mu(\varphi) := \mu(|\varphi|^+_{\mathscr{M}})$ (resp. $p^-_\mu(\varphi) = \mu(|\varphi|^-_{\mathscr{M}})$) represents the positive (resp. negative) probabilistic evidence for φ. Non-standard probabilities satisfy the following axioms: (i) if $A \vdash_{\mathsf{FDE}} B$ then $p^+(A) \leq p^+(B)$, (ii) $p^+(A \wedge B) + p^+(A \vee B) = p^+(A) + p^+(B)$, and (iii) $p^+(\neg A) = p^-(A)$. Notice that one can no longer prove that $p^+(\varphi) + p^+(\neg\varphi) = 1$. Indeed the two values are independent here.

Belief functions interpreted over FDE. To handle cases where available information does not allow to define the probability of some formulas (i.e. some subsets of Σ are non-measurable), we define *partial probabilistic models* $\mathscr{M} = \langle \Sigma, \mathscr{X}, \mu, v^+, v^- \rangle$, where $\mathscr{X}$ is a σ-algebra of measurable subsets of Σ. Notice that p^+_μ and p^-_μ are partial functions on the measurable formulas. We use inner and outer measures to reason about non-measurable elements as follows

$$(p^+_\mu)_*(\varphi) = \sup\{p^+_\mu(\psi) : |\psi|^+ \subseteq |\varphi|^+ \text{ and } |\psi|^+ \in \mathscr{X}\}.$$

Another way of reasoning with non-standard probabilities covering the non-measurable elements is to use belief functions instead of probability measures. The above method is a special case of the latter case, based on the fact that inner (resp. outer) measures are belief (resp. plausibility) functions.

Conditional update for non-standard probabilities. [5] already propose several kinds of conditioning over non-standard probabilities. We propose two new definitions in order to be able to talk about conditioning on partial probabilistic models.

The classical standard conditioning is not able to talk about non-measurable elements. We generalise the standard conditioning to non-Boolean structures containing non-measurable elements as follows. The positive and negative probability structures associated to a partial probabilistic model are $\langle \mathscr{L}, L, \eta^+\rangle$ and $\langle \mathscr{L}^{op}, L^{op}, \eta^-\rangle$ where $\mathscr{L}$ is the Lindenbaum algebra, L and L^{op} are respectively the sublattices $\{[\varphi] : [\varphi] \in \mathscr{L}, |\varphi|^+_{\mathscr{M}} \in \mathscr{X}\}$, $\{[\varphi] : [\varphi] \in \mathscr{L}, |\varphi|^-_{\mathscr{M}} \in \mathscr{X}\}$ and $\eta^+([\varphi]) = p^+_\mu(\varphi)$ and $\eta^-([\varphi]) = p^-_\mu(\varphi)$. Let $M = \langle \mathscr{L}, L, \eta\rangle$ be a probability structure, $a \in L$ and $\eta(a) \neq 0$. Conditioning on a gives rise to the probability structure $M_a = \langle \mathscr{L}_a, L_a, \eta_a\rangle$ where $\mathscr{L}_a$ is the congruence lattice based on a, $L_a = \{[c] : c \in \mathscr{L}$ and $c \wedge a \in L\}$ is a sublattice of $\mathscr{L}_a$, and η_a is defined over L_a as follows: $\eta_a([c]) = \frac{\eta(c\wedge a)}{\eta(a)}$. For every $c \in \mathscr{L}$:

$$(\eta_a)_*([c]) = \frac{\eta_*(a \wedge c)}{\eta(a)} \quad \text{and} \quad (\eta_a)^*([c]) = \frac{\eta^*(a \wedge c)}{\eta(a)}.$$

We also propose a second way of conditioning following [4] which is based on the fact that belief functions are the lower envelopes of the the probability measures consistent with the given belief function.

Future directions and context. We develop the duality between probabilistic models and non-standard probabilities over de Morgan algebras and to adapt standard definitions and tools of (imprecise) probability theory to non-classical reasoning. In addition we still need to fully understand the philosophical meaning of non-standard probabilities, their associated belief functions and conditional updating.

This work is part of a wider project aiming at developing a modular logical framework to formalize probabilistic reasoning based on incomplete and/or inconsistent information [3]. We propose a *two-layer modal logical framework* [2]. The bottom layer is to be that of events or facts, represented by probabilistic information provided by sources available to an agent with a certain degree of reliability. The modalities connecting bottom layer to the top layer, are that of belief of the agent (e.g. about an event taking place) based on the information from the sources in terms of (various kinds of) aggregation. The top layer is to be the logic of thus formed beliefs. This work focuses on the semantics of the modalities that connect the bottom and the upper layer.

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Popularising modern logic in 1950s Czechoslovakia

Abstract

Otakar Zich jr. (1908-1984), a leading logician in postwar Czechoslovakia, was the main figure also behind the publication of a book popularising modern logic in Czechoslovakia. This was not his first popular book on logic, nor was it the only publication on logic by a mathematician. Zich himself published "An Introduction to the Philosophy of Mathematics" a decade earlier and another eminent mathematician, Miroslav Katětov (1918-1995), published a book on the foundations of mathematics in logic. From today's perspective, these books may well be early steps in convincing a majority of mathematicians and especially mathematics teachers that logic is only a part of mathematics. In my talk, I will try to show how the actual wording of the popular books on logic from the early postwar decade could have contributed to the view (held by mathematicians) that logic is a subdiscipline of mathematics, rather than philosophy.

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In 1958, the Czechoslovak Society for the Spreading of Political and Scientific Knowledge[1] published the content of popular lectures on logic held in the previous year in the book *Modern* Logic (Zich et al. 1958). The group of Czech logicians was led by Otakar Zich jr. (1908-1984),[2] the head of the department of logic in Prague under its various institutional incarnations.

Already a decade before that, Zich wrote a popular book *Introduction to the philosophy of mathematics* (Zich 1947), most of which was (content-wise) also devoted to logic. Similarly, the slim volume by Katětov (1946), *What is the logical construction of mathematics?* was devoted to explaining basic notions of logic and their use in mathematics. All three books have something in common, although they have different audience: Katětov's (1946) book was written for young mathematicians, and so was Zich's (1947) *Introduction to Philosophy of Mathematics* (they were both published within the series directed towards young mathematicians), while the book (Zich et al. 1958) was explicitly *not* written for mathematicians, nor people with solid mathematical background. Furthermore, even if they start from philosophy of mathematics (Zich 1947), all three books soon embark upon the explication of the technicalities of (modern) logic and advocate its usefulness for mathematics. In this respect, the three books are reminiscent of one of three major strands of philosophy of mathematics, namely logicism (the other two being formalism and intuitionism), and especially Bertrand Russell's *Introduction to Mathematical Philosophy* (1919).

The three books invite us to pose questions about the relationship between mathematics and logic. How was this relationship shaped by popular books on mathematics and logic? Was it more convenient (within the context of postwar Czechoslovakia) to present logic in an apolitical way through giving it a

1 Cf. Olšáková (2014).

2 The other authors of the booklet were Karel Berka, Miroslav Jauris, Pavel Materna, Miroslav Mleziva and Ota Weinberger.

mathematical appearance, or was it a matter of taste, or even fashion? Finally, could (or can) such popularising efforts have impact on how a disciplines (in this case mathematics and logic) are perceived not just by the general public, but also by the practitioners of the respective fields of research?

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From Gödel to Henkin;

Completeness vs Incompleteness

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Church's typed lambda calculus with its standard semantics is sufficiently rich to encode arithmetic. Hence, by Gödel's incompleteness theorems, it is *incomplete*. Yet, Leon Henkin is best known for his proof of *completeness* in the theory of types. How is it possible, isn't it a contradiction? No, it is not, of course. What Henkin proved is completeness with respect to *general models*. If he tried to enrich the calculus, it would be of no way since Gödel's incompleteness theorems ensure that no deductive calculus can achieve completeness with respect to the *standard models*. Hence, Henkin voted for changing the semantics. Instead of having in each level of the typed hierarchy the set of *all* the functions belonging to the corresponding type, he admitted only *some subsets* of them, namely those that are *lambda definable*.

As a starting point, I first discuss the difference between *completeness of a logic*, *completeness of a calculus* and *completeness of a theory*. Though all the three notions have much in common, namely, they express some sufficiency, there are also significant differences between them. For instance, the first-order theory of Peano arithmetic is an incomplete theory defined within a complete logic. In contrast, the second-order Peano arithmetic is a complete theory defined within an incomplete logic.

A *logic* can be semantically defined as the *set of logical truth* of the language L. A *logic is complete* iff there exists an algorithm that recursively enumerates the truths (validities) of that logic.

Gödel was probably the first to conceive the 'completeness of a logic' as a way to analyse computational complexity of the class of logically valid formulas. He also realised that there is a big difference between semantic notions and proof-theoretic ones, as the following citation illustrates:

> '[…] "valid" refers to the nondenumerable totality of functions, while "provable" presupposes only the denumerable totality of formal proofs.' (Goedel 1930, p. 117)

In principle, it is *not necessary* to define a *deductive calculus for the logic*; *any recursive procedure* able to generate logical truths would do. From a computational point of view, *decidability implies completeness* (but not vice versa). Gödel addressed the issue of FOL completeness in the absence of a previous result on decidability for logical truths; a negative solution to the decision problem (Entscheidungsproblem) was given six years later by Church in 1935-6 and independently by Turing in 1936-7. Thus, Gödel faced a dilemma:

- Either semantics is decidable, in which case the completeness of the logic is trivial or,
- completeness is a critical property that cannot be obtained as a corollary of a previous decidability result.

Gödel rightly concentrated on the second issue. He had solved the problem for first-order logic positively, and negatively for any ω-consistent recursively axiomatised logical theory in which arithmetic could be embedded. Thus, the first-order predicate logic (FOL) is complete; the set of its logical truths is *recursively enumerable*. Yet, it is *not recursive*, as FOL is not decidable.

Henkin's primary goal was to prove that Church's typed *lambda calculus* (1940) is applicable, as it is complete with respect to some well-defined structures. Church made a clear distinction between the value of a function at a given argument, *F*(*x*), and the function itself, $\lambda x\ F(x)$. The latter makes the *naming* of functions in the language possible. *Definability* was then a hot topic, but recall that Church's thesis, identifying effectively computable functions with λ-*definable* ones had already been formulated, and Henkin was one of Church's students. He showed that if the terms of the calculus were interpreted in a less rigid way, accepting hierarchies of types that did not necessarily have to contain *all* the functions but did contain the *named* functions, i.e. λ-*definable* ones, one could easily show that *all consequences of a set of hypotheses were provable in the calculus*. By relaxing the conditions on the structures in which the language is interpreted, there are more *general* models in which the formulas must be true, and therefore the set of valid formulas is reduced to the recursive set. In other words, the formulas true in all general models coincide with the formulas generated by the rules of calculus.

When Henkin was developing the hierarchy of objects with a proper name, he realised that by utilising the λ-conversion rules, equivalence classes of provably equivalent terms could be defined; these classes form a model isomorphic to the new hierarchy of types formed by the named elements. It remained to ensure that the objects named by the propositions were just the truth values. To this end, Henkin expanded the set of axioms to form a *maximally consistent* set. Then he proved that every consistent set of formulas *T* has a general model that satisfies exactly the formulas of *T*; the elements of such model are the *equivalence classes of the terms themselves*. In this way, he would have managed to give proof of the completeness of the deductive calculus.

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H-coloring dichotomy in proof complexity

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Abstract

The $\mathcal{H}$-coloring problem for undirected simple graphs is a computational problem from a huge class of the constraint satisfaction problems (CSP): an $\mathcal{H}$-coloring of a graph $\mathcal{G}$ is a homomorphism from $\mathcal{G}$ to $\mathcal{H}$ and the problem is to decide for fixed $\mathcal{H}$, given $\mathcal{G}$, if a homomorphism exists or not.

The dichotomy theorem for the $\mathcal{H}$-coloring problem was proved by Hell and Nešetřil (1990, J. Comb. Theory Ser. B, 48, 92–110) and it says that for each $\mathcal{H}$ the problem is either p-time decidable or NP-complete. Since negations of unsatisfiable instances of CSP can be expressed as propositional tautologies, it seems to be natural to investigate the proof complexity of CSP.

We show that the decision algorithm in the p-time case of the $\mathcal{H}$-coloring problem can be formalized in a relatively weak theory and that the tautologies expressing the negative instances for such $\mathcal{H}$ have polynomial proofs in propositional proof system $R^*(log)$. To establish this, we use a well-known connection between theories of bounded arithmetic and propositional proof systems.

We complement this result by a lower bound that holds for many weak proof systems for a special example of NP-complete case of the $\mathcal{H}$-coloring problem.

Existence of Nash Equilibria in Preference Priority Games Proven in Isabelle

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1 Introduction

This work is about formalising game-theoretic results using proof assistants [EJ91, LeR09, Nip09]. In previous work [LMS17], we have formalised [LeR14, Lemma 2.4] both in Coq and Isabelle. The result is as follows: starting from a two-player game with finitely many outcomes, one may derive a game by rewriting each of these outcomes with either the basic outcome "Player 1 wins" or "Player 2 wins". If all ways of deriving such a *win/lose (w/l) game* lead to a game where one player has a winning strategy, then the original game has a Nash equilibrium (NE).

Here, we present an application of this work to parity games and priority games using Isabelle. Dittmann has proven in Isabelle that parity games are positionally determined. First, we generalise this result to priority games, where parity is replaced by an arbitrary winning set W. Secondly, we consider *preference priority games*, i.e., sequential games where players have preferences over outcomes. We show that such games have an NE.

2 Positional Determinacy of Priority Games

We consider *sequential* games: there is a graph partitioned so that each vertex is owned by one of the two players, and a play is a path through this graph. The path starts in the initial vertex, and in each vertex, the owner decides where to go next according to some strategy.

Definition 1. An *arena* is a tuple (V_1, V_2, v_0, E) where $V_1 \cap V_2 = \emptyset$, and $v_0 \in V := V_1 \cup V_2$, and $E \subseteq V^2$ is such that for all $v \in V$, the set $vE := \{u \in V \mid (v, u) \in E\}$ is non-empty.

A *positional strategy* of Player 1 in an arena (V_1, V_2, v_0, E) is a function $s : V_1 \to V$ such that $(v, s(v)) \in E$ for all $v \in V_1$ ("positional" because the history is ignored; in the sequel, we only consider positional strategies).

In a straightforward way, a strategy pair induces a unique infinite path denoted by $\langle s_1, s_2 \rangle$.

Definition 2. A *priority game form* is an arena together with a priority function $\pi : V \to \mathbb{N}$.

For an infinite path with bounded π, the least priority occurring infinitely often as a label of a visited vertex is called *induced priority*.

Definition 3. A *w/l priority game* consists of a priority game form and a subset $W \subseteq \mathbb{N}$. A run ρ is winning for Player 1 iff the induced priority of ρ is in W.

If $W := 2\mathbb{N}$, the w/l priority game is called a *parity game*.

Definition 4. Given a w/l priority game $(V_1, V_2, v_0, E, \pi, W)$, a Player 1 *winning strategy* s_1 is such that for all Player 2 strategy s_2, the induced priority of $\langle s_1, s_2 \rangle$ is in W. A w/l priority game such that one player has a winning strategy is said to be *weakly positionally determined*.

The term "weakly" indicates that a winning (positional) strategy runs against a strategy of the opponent that is itself also positional. Usually, positional determinacy means that a positional strategy wins even against more general (non-positional) strategies.

Dittmann [Dit16] has shown in Isabelle that parity games are (not just weakly!) positionally determined [EJ91]. Based on a transformation from priority games to parity games, we can extend the statement to the priority games:

Lemma 5. W/l priority games with bounded π are positionally determined.

We have also shown the result in Isabelle but only *weak* determinacy. Note that Lemma 5 could be proved by applying [GZ05, Thm. 2, Cor. 7], but the proof that we formalize is more direct when assuming positional determinacy of parity games.

The Isabelle formalisation of this lemma with all the preliminaries comprises approximately 1350 lines of proof script, about as much as our entire Isabelle development of [LMS17]. The difficulty is that infinite paths are defined coinductively, and thus statements relating different priority and parity games must be proven by coinduction.

3 Nash Equilibria for Preference Priority Games

We now consider *simultaneous* or *one-shot* games, as opposed to sequential games.

Definition 6. A *game form* is a tuple $\langle S_1, S_2, O, v\rangle$ where S_1 and S_2 are the strategies of Players 1 and 2, resp.; O is a nonempty set (of possible outcomes); $v : S_1 \times S_2 \to O$ is the outcome function. A game form endowed with two binary relations $\prec_1, \prec_2$ over O for each player (modeling her preference) is called a *game*.

A *w/l game* is a game where $O = \{\mathit{True}, \mathit{False}\}$ and the preferences are $\mathit{False} \prec_1 \mathit{True}$ and $\mathit{True} \prec_2 \mathit{False}$. If one player has a winning strategy the game is said to be *determined*.

Definition 7. Let $\langle S_1, S_2, O, v, \prec_1, \prec_2\rangle$ be a game. A strategy profile (s_1, s_2) in $S_1 \times S_2$ is a *Nash equilibrium* (NE) if it makes both players stable:

$$(\forall s_1' \in S_1,\ v(s_1, s_2) \not\prec_1 v(s_1', s_2)) \wedge (\forall s_2' \in S_2,\ v(s_1, s_2) \not\prec_2 v(s_1, s_2'))$$

Given a game form and a set $W \subseteq O$, one can *derive* a w/l game in a straightforward way: all outcomes in W are mapped to *True* (Player 1 wins). If a game form is such that for *every* W, the derived w/l game is determined, we call the game form itself *determined*. Fig. 1 shows an example with $S_1 = \{1_t, 1_b\}$, $S_2 = \{2_l, 2_r\}$, and $O = \{\heartsuit, \clubsuit, \diamondsuit\}$.

In [LMS17], we have formalised in Isabelle and Coq a theorem [LeR14] stating that a game g whose game form is determined has an NE. Fig. 1 shows the main theorem in Isabelle code.

We now link sequential games to simultaneous games by putting a black box around the process of constructing an infinite sequence using strategies and then extracting a number from it. The black box is a (simultaneous) game form: it takes two strategies and returns a number.

At the same time, we can define *preference-priority games*, which are sequential games where rather than having a winning set W, we have preferences of the players on the outcomes in $\mathbb{N}$. By linking preference-priority games to the simultaneous setting, we can apply the main theorem above to show that preference-priority games also have an NE:

```
theorem equilibrium_transfer_finite :
  assumes finiteO : "finite (range (form g))"
      and trans1 : "trans (pref1 g)"
      and irref1 : "irrefl (pref1 g)"
      and trans2 : "trans (pref2 g)"
      and irref2 : "irrefl (pref2 g)"
      and det : "determinedForm (form g) R1 R2"
  shows "∃ s1∈R1. ∃ s2∈R2. isNash g s1 s2"
```

	2_l	2_r
1_t	$\heartsuit$	$\clubsuit$
1_b	$\diamondsuit$	$\diamondsuit$

- $\diamondsuit \mapsto \mathit{True} : 1_b$ wins
- $\diamondsuit, \heartsuit \mapsto \mathit{False} : 2_l$ wins
- ...

Figure 1: The main theorem of [LMS17], and a game whose game form is determined

```
lemma equilibriumTransferPreferencePriority :
  assumes "snd (fst PPG) ∈ V_fst (fst PPG)"
  and "ParityGame (fst (fst PPG))"
  and "deadendFree (fst (fst PPG))"
  and acyclic1 : "acyclic ((fst ∘ snd) PPG)"
  and acyclic2 : "acyclic ((snd ∘ snd) PPG)"
  shows "∃ s1. ∃ s2. isNashPPG PPG s1 s2"
```

We have proven this result on paper and using Isabelle. The Isabelle formalisation of this lemma with all the preliminaries comprises approximately 450 lines of proof script.

As a future work, we would like to lift the restriction of *weak* positional determinacy.

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Why are models of quantum logic infinite

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To grasp the essential properties of the basic model used in quantum physics, David Foulis and his collaborators coined in the 1970s the notion of an orthogonality space [1, 7]. The idea was to reduce the involved structure of a complex Hilbert space to the minimum of what is really needed. An orthogonality space is a set endowed with a binary relation about which not more than symmetry and irreflexivity is assumed. The canonical example is the projective Hilbert space together with the orthogonality relation. The approach can be seen as an attempt to increase the level of abstraction in quantum logic to its limits: From the quantum-physical perspective, solely the aspect of distinguishability of measurement results is taken into account; from the logical perspective, solely the aspect of mutual exclusiveness is exploited.

Orthogonality spaces were recently rediscovered and they have proven as a basis of quantum logic in an amazingly effective way [3, 4, 5, 6]. In fact, each orthogonality space gives rise to a test space; see [7]. Test spaces can in turn be understood as an abstract way to model quantum-mechanical propositions. The latter are, in the standard approach, modelled by subspaces of Hilbert spaces. It has turned out that the transition from orthogonality spaces to inner-product spaces is possible on the basis of a remarkably simple condition, to which we refer to as linearity. The rank of an orthogonality space is, loosely speaking, the maximal number of mutually orthogonal elements. In case that the rank is at least 4, linearity is sufficient to lead from the simple relational structure of an orthogonality space to a Hermitian space.

Accordingly, the focus of investigations of orthogonality spaces has up to now mostly been on the case that the rank is 4 or higher. In contrast, this lecture is focussed on the case that the orthogonality space is finite and of rank 2 or 3, where there is no general representation theory. Often adopting the point of view of graph theory, we establish for such spaces a number of interesting combinatorial properties.

We can summarize our results which are related to the results of Eckmann and Zabey [2] as follows.

Theorem 1. *Let $(X, \perp)$ be a linear orthogonality space of finite rank m. Then*

(i) *If $m = 2$ then X is either finite with even cardinality or infinite.*

(ii) *If $m \geq 3$ then X is infinite.*

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Universal Horn properties of upsets of distributive lattices

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Given a class of partially ordered structures, such as distributive lattices or Boolean algebras, we may ask: which kinds of upward closed sets are there in this class of algebras? More precisely, which properties of upsets can we express using some limited syntactic means?

Such properties may for example be expressed by first-order formulas in a signature whose algebraic part contains all the connectives of distributive lattices and whose relational part only contains a single unary predicate, to be interpreted by our upset. We shall in fact be interested in properties expressible more specifically by means of so-called universal Horn formulas. These are universally quantified implications between a finite set of atomic premises and a single atomic conclusion. An atomic formula in this context is a unary predicate applied to a term in the relevant algebraic signature. For example, an upset U of a distributive lattice $\mathbf{L}$ is a lattice filter if and only if the universal Horn sentence $x \in U \;\&\; y \in U \implies x \wedge y \in U$ is satisfied in $\mathbf{A}$.

We show that the only properties of upsets of distributive lattices and unital meet semi-lattices expressible by a universal Horn sentence are the properties of being a lattice n-filter:

$$y_1 \in U \;\&\; \dots \;\&\; y_{n+1} \in U \implies x_1 \wedge \cdots \wedge x_{n+1} \in U, \text{ where } y_i := \bigwedge_{j \neq i} x_j.$$

For example, 2-filters are defined by the following universal Horn property:

$$x_1 \wedge x_2 \in U \;\&\; x_2 \wedge x_3 \in U \;\&\; x_3 \wedge x_1 \in U \implies x_1 \wedge x_2 \wedge x_3 \in U.$$

The proof hinges on two facts. Firstly, the n-filters on a distributive lattice are precisely the intersections of prime n-filters (prime in the sense that $x \vee y \in U$ implies $x \in U$ or $y \in U$). Secondly, the prime n-filters on a distributive lattice are precisely the homomorphic preimages of the n-filter of non-zero elements of the finite Boolean algebra with n atoms. The most important properties of filters on distributive lattices therefore extend to n-filters.

In Boolean algebras, the situation is more complicated, since we can express more properties using the richer algebraic signature of Boolean algebras. Nevertheless, the above properties still play a privileged role among all the universal Horn properties of upsets of Boolean algebras.

Reasoning About Graded While Programs

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A *while program* is a program containing instructions of the form **while** φ **do** α (while loops), where φ, the condition, is a statement and α is an instruction that is executed repeatedly while the condition is true. Interesting properties of while programs, related to program verification, include *termination* (does the program halt?) and *correctness* (does the program halt in a state satisfying some specific postcondition whenever it is run in a state satisfying a specific precondition?). Various formalisms were proposed to formalize (and automate) reasoning about these properties and their variants. One such formalism *dynamic logic* [3, 4], a version of modal logic with modal operators indexed by instructions, that interprets instructions semantically as binary input-output relations on a set of states.

Following the work of Zadeh on fuzzy algorithms [6], *graded while programs* can be defined as abstractions of while programs where the condition φ in while loops is a statement admitting *degrees of truth* strictly between the Boolean truth values 0 (falsity) and 1 (truth). Such abstractions may be useful when it is impossible or impractical to give a precise statement of the condition. (Graded while programs are related to systems of if-then rules in fuzzy controllers; the latter, however, do not contain explicit loops.)

In this talk we present a version of propositional dynamic logic based on finite Łukasiewicz chains that allows to reason about termination and (partial) correctness of graded while programs. Unlike previously published versions of dynamic logic based on finite Łukasiewicz chains [2, 5], our version allows to interpret instructions, not only statements, as graded. We will focus on motivations and expressivity, but we will also outline our technical results which include a sound and weakly complete axiomatization and a proof that the validity problem in finitely-valued Łukasiewicz dynamic logics is EXPTIME-complete. (Technical results were not established in previous "exploratory" papers such as [1].)

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Beth definability in relevance logics with the Gödel-Dummett axiom*

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We provide some sufficient conditions for a relevance logic $\mathbf{L}$ to posses the *(infinite) Beth (definability) property*, i.e., whenever a set of variables is defined *implicitly* in terms of other variables by means of some formulas over $\mathbf{L}$, then it can also be defined *explicitly*. The relevance logics in question are the axiomatic extensions of Anderson and Belnap's principle relevance logic $\mathbf{R^t}$ (see [1]) with the *Gödel-Dummett* axiom

$$(p \to q) \vee (q \to p), \tag{1}$$

as well as the axiomatic extensions of $\mathbf{R^t_+M}$ with (1), where $\mathbf{R^t_+M}$ is the negation-less fragment of $\mathbf{R^t}$ with *mingle* $p \to (p \to p)$. One can obtain the negation-free (*positive*) fragment of Gödel logic by extending $\mathbf{R^t_+M}$ with (1) and *weakening* $p \to (q \to p)$.

Famously, Urquhart showed that $\mathbf{R^t}$ does not have the Beth property [10]. We use the tools of abstract algebraic logic to investigate which of its extensions do. On the other hand, Maksimova proved that a strong version of the Beth property holds for Gödel logic [5], as well as for its positive fragment [6]. It was later established in [2] that all the axiomatic extensions of the afore mentioned logics have the Beth property. Our results concerning the negation-less fragment of $\mathbf{R^t}$ generalizes these results.

A Dunn monoid $\mathbf{A} = \langle A; \wedge, \vee, \cdot, \to, e\rangle$ comprises a distributive lattice $\langle A; \wedge, \vee\rangle$, a commutative monoid $\langle A; \cdot, e\rangle$ that is square-increasing ($x \leqslant x \cdot x$), and a binary operation $\to$ satisfying the *law of residuation*

$$x \cdot y \leqslant z \text{ iff } y \leqslant x \to z.$$

We may enrich the language of Dunn monoids with an *involution* $\neg$ that satisfies $x = \neg\neg x$ and $x \to \neg y = y \to \neg x$, thus obtaining De Morgan monoids.

A Dunn monoid is *idempotent* if it satisfies $x \cdot x = x$ and a Dunn (or De Morgan) monoid is said to be *semilinear* if it embeds into a direct product of totally ordered algebras.

Our interest in these algebras stem from the fact that the subvarieties of De Morgan monoids [resp. idempotent Dunn monoids] algebraize the extensions of $\mathbf{R^t}$ [resp. $\mathbf{R^t_+M}$]. In particular, extending these logics with (1) amounts to imposing semilinearity on the respective varieties. If, in addition, one imposes *integrality* (i.e., $x \leq e$) on Dunn monoids, one obtains *relative Stone algebras*—the algebraic counterpart of positive Gödel logic.

Let us then introduce the algebraic property that corresponds to the Beth property. A homomorphism $f\colon \mathbf{A} \to \mathbf{B}$ between algebras in a variety K is an *epimorphism* provided it is right-cancellative, i.e., for every pair of homomorphisms $g, h\colon \mathbf{B} \to \mathbf{C}$ with $\mathbf{C} \in \mathsf{K}$,

$$\text{if } g \circ f = h \circ f, \text{ then } g = h.$$

Clearly any surjective homomorphism is an epimorphism. The converse need not hold. Indeed, rings and distributive lattices each form varieties in which non-surjective epimorphisms arise.

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As it happens, this reflects the absence of unary terms defining multiplicative inverses in rings, and complements in distributive lattices, despite the uniqueness of those entities when they exist. Such constructs are said to be *implicitly* (and not *explicitly*) *definable*. We say that a variety K has the *epimorphism surjectivity (ES) property*, if it precludes phenomena of this kind, i.e., if epimorphisms in K are surjective.

When a variety K algebraizes a logic $\vdash$, then K has the ES property if and only if $\vdash$ has the Beth property [3]. We are therefore looking for varieties of Dunn and De Morgan monoids that have the ES property, and we can state the first result to this effect.

Theorem 1. *The variety of semilinear idempotent Dunn monoids has the ES property.*

This result uses a characterization of homomorphisms between totally ordered idempotent Dunn monoids, that is stated by means of a representation of these algebras from [4].

An element a of a Dunn/De Morgan monoid $\mathbf{A}$ is said to be *negative* if $a \leqslant e$ in $\mathbf{A}$, and $\mathbf{A}$ is said to be *negatively generated* if $\mathbf{A}$ is generated by its negative elements. It turns out that negatively generated semilinear Dunn monoids are idempotent, and they are called *generalized Sugihara monoids* (cf. [9]). We show that the classes of negatively generated semilinear Dunn/De Morgan monoids are varieties (and thus correspond to axiomatic extensions of the logics above). Note that every relative Stone algebra is obviously negatively generated, because e is its greatest element, so the following theorem applies to all varieties of relative Stone algebras (a result that was established in [2]).

Theorem 2. *Every variety of negatively generated semilinear Dunn/De Morgan monoids has surjective epimorphisms.*

Apart from the representations mentioned after Theorem 1, this result also uses a representation theorem for De Morgan monoids in [7, 8]. Note that in general the ES property is not inherited by subvarieties, so Theorem 2 is labour saving.

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The automorphism group of the Fraïssé limit of finite Heyting algebras

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Consider a countable class of isomorphism types of finitely generated structures with the amalgamation property, the joint embedding property, and the hereditary property. By the *Fraïssé limit* of such a class $\mathcal{K}$, we mean the unique countable structure M whose age, i.e., the class of finitely generated substructures of M, is $\mathcal{K}$ up to isomorphism.

One pervasively studied aspect of ultrahomogenous structures—the Fraïssé limit of some classes of structures—is their automorphism groups (see, e.g., Macpherson [3]). Many studies on the automorphism groups of concrete ultrahomogeneous structures involved uniformly locally finite ones, which are necessarily ω-categorical. (For instance, the normality of the automorphism group of the countable atomless Boolean algebra, which is ultrahomogeneous and uniformly locally finite, was established by Anderson [1].) In the present work, we offer a case study on the automorphism group of a natural non-uniformly locally ultrahomogeneous structure: the Fraïssé limit L of finite Heyting algebras, whose existence follows from Maksimova's result [4] on the Craig interpolation theorem for intuitionistic logic.

First of all, we would like to know if $\mathrm{Aut}(L)$ is indeed different from that of, say, the countable atomless Boolean algebra. Recall that a permutation group G on a countable set X is *oligormorphic* if the induced action of G on X^n has only finitely many orbits for all $n < \omega$. We take a theorem of Tsankov [6] as the definition of Roelcke precompactness: a topological group G is *Roelcke precompact* if G is the inverse limit of some inverse system of oligomorphic permutation groups.

Theorem. The topological group $\mathrm{Aut}(L)$ is not Roelcke precompact. *A fortiori*, it is not realized as the automorphism group of of any countable ω-categorical structure.

Every uniformly locally finite ultrahomogeneous structure has a non-locally compact automorphism group [3]. This is true of our structure as well:

Proposition. The topological group $\mathrm{Aut}(L)$ is not locally compact.

The *extreme* amenability of the automorphism group of an ultrahomogeneous structure characterizes the Ramsey property of its age [2]. We are unable to prove or disprove the existence of an ultrahomogeneous expansion of L whose automorphism group is extremely amenable at this point. However:

Theorem. $\mathrm{Aut}(L)$ is not amenable.

Finally, we focus on the abstract group structure of $\mathrm{Aut}(L)$. Our methodology is applicable to other Fraïssé classes of lattices with the superamalgamation property, which is related to interpolation theorems of nonclassical logics (see, e.g., [4]).

Theorem. Let M be a countable ultrahomogeneous structure with the age of M having the superamalgamation property. Moreover, assume that the amalgamation property of the age is witnessed canonically and functorially in the sense of Tent and Ziegler [5, Example 2.2.1]. Then, the abstract group $\mathrm{Aut}(M)$ is simple.

Corollary. $\mathrm{Aut}(L)$ is simple.

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