Common Gnoseological Meaning of Goedel and Caratheodory Theorems

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Common Gnoseological Meaning of Goedel and Caratheodory Theorems

1. Introduction

Vienna Circle

logical positivism - physicalism

Rudolph Carnap Otto von Neurath :

1931 - 1935

"Any scientifically meaningful statement

is expressible in physical terms

- about a movement in the observable space and time -

or, if *the statement* ***is not expressible this way***

it is meaningful scientifically

when it is convertible to a statement about a language,

otherwise *it is of* ***no scientific meaning."***

$\Rightarrow$

mutual **relation** among **structures/languages** of:

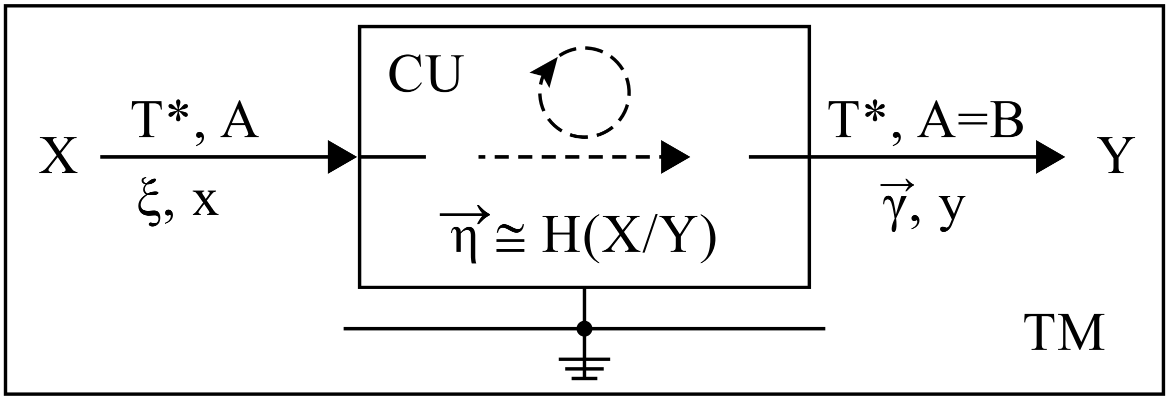
Thermodynamics – Energy Transformation

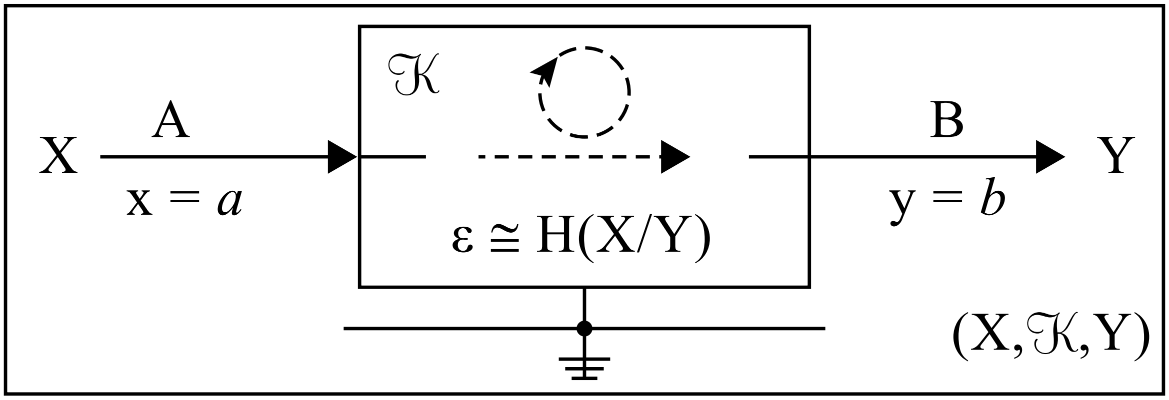
Information Theory – Message/Information Transfer

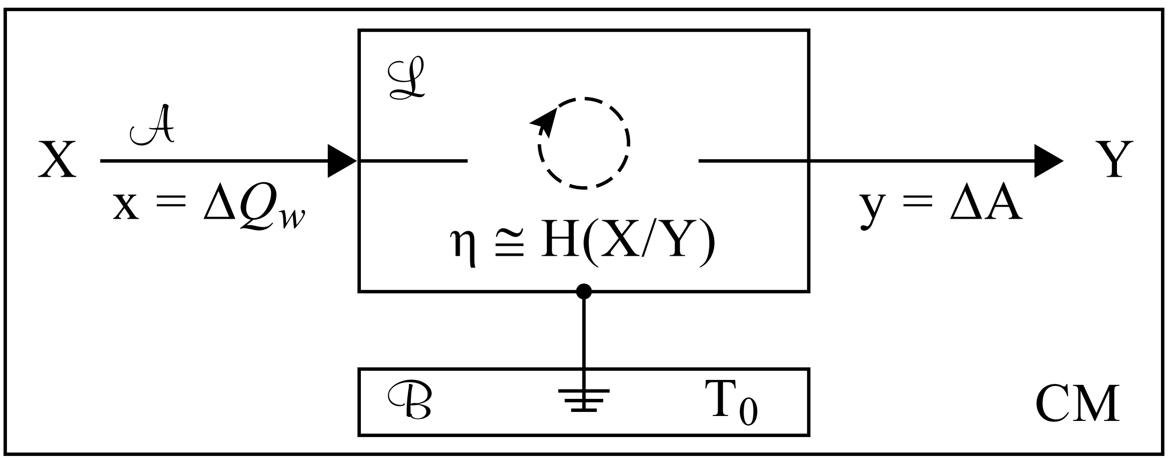
Computing Theory – Computing/Inference

Adiabatic Theory

COMPUTING THEORY	TURING MACHINE TM FINITE-STATE CONTROL UNIT	COMPUTING/INFERENCE PROCESS STATE/CONFIGURATION TRANSFORMATIONS
INFORMATION THEORY	SHANNON TRANSFER CHAIN $(\mathbf{X}, \mathcal{K}, \mathbf{Y})$ TRANSFER CHANNEL	MESSAGE/INFORMATION TRANSFER PROCESS
THERMODYNAMICS	CARNOT MACHINE CM CARNOT CYCLE	HEAT ENERGY TRANSFORMATION PROCESS
THERMODYNAMIC ADIABATIC THEORY	THERMODYNAMIC ADIABATIC SYSTEM $\mathfrak{L}$	STATES' DEVELOPING PROCESS







Processes in all these structures
run in the finite physical world
and **follow its laws.**

We model them by the states' $\theta^{\mathfrak{L}}_{[\cdot]}$ trajectories $l_{\mathfrak{Q}_{\mathfrak{L}}}$
within the heat isolated
- $[\mathrm{d}]\mathbf{Q}_{\mathbf{Ext}}=0$ **-**
adiabatic system $\mathfrak{L}/\mathfrak{Q}_{\mathfrak{L}}$ where is valid:

Caratheodory common formulation of the II. P.T. :

> ***In the arbitrary vicinity of every state***
> ***of the state space $\mathfrak{Q}_{\mathfrak{L}}$ of the adiabatic system $\mathfrak{L}$***
> ***exist states not reachable***
> ***from the starting state adiabatically*** ([d]$\mathbf{Q}_{\mathbf{Ext}}$=**0**)
> *(or the states not reachable by the system at all).* [1]

For the consistency of
the **Peano arithmetic theory** $\mathcal{T}_{\mathcal{PA}}$
the analog is expressed by:

Gödel incompleteness theorems:[2]

> ***For the theory $\mathcal{T}_{\mathcal{PA}}$ exists the true ("1") CLAIM***
> ***that either this CLAIM and its NEGATION***
> ***is NOT PROVABLE within the system $\mathcal{P}/\mathcal{T}_{\mathcal{PA}}$.*** [3]

- ***CLAIM*** **about the $\mathcal{T}_{\mathcal{PA}}$ consistency especially** -

> ***The CLAIM saying that theory $\mathcal{T}_{\mathcal{PA}}$ is consistent***
> ***is not PROVABLE by its means*** ($\mathcal{P}$) **- by itself.** [4]

---

[1] **Along the given the trajectory $l_{\mathfrak{Q}_{\mathfrak{L}}}$ with the given starting point, reversibly or irreversibly. Or such states which are the part of the $\mathfrak{L}/\mathfrak{Q}_{\mathfrak{L}}$'s** ***outer construction*** **and thus of the whole $l_{\mathfrak{Q}_{\mathfrak{L}}}$'s** ***definition.***

[2] **Rosser-Gödel theorem.**

[3] **Far from (!) "In...." Attempts to prove/*TO PROVE/INFER*** it **within the system $\mathcal{P}/\mathcal{T}_{\mathcal{PA}}$ leads to the inconsistency of the consistent (!) system $\mathcal{P}_{\kappa}$** (in fact we are entering into the **inconsistent metasystem $\mathcal{P}^{\star}$** - the real sense of the Proposition V).

[4] **It is the *META-CLAIM*** not writable within the $\mathcal{T}_{\mathcal{PA}}$ *language.*

The adiabatic trajectories $l_{\mathfrak{Q}_{\mathfrak{L}}}$ - within the $\mathfrak{L}/\mathfrak{Q}_{\mathfrak{L}}$[5]

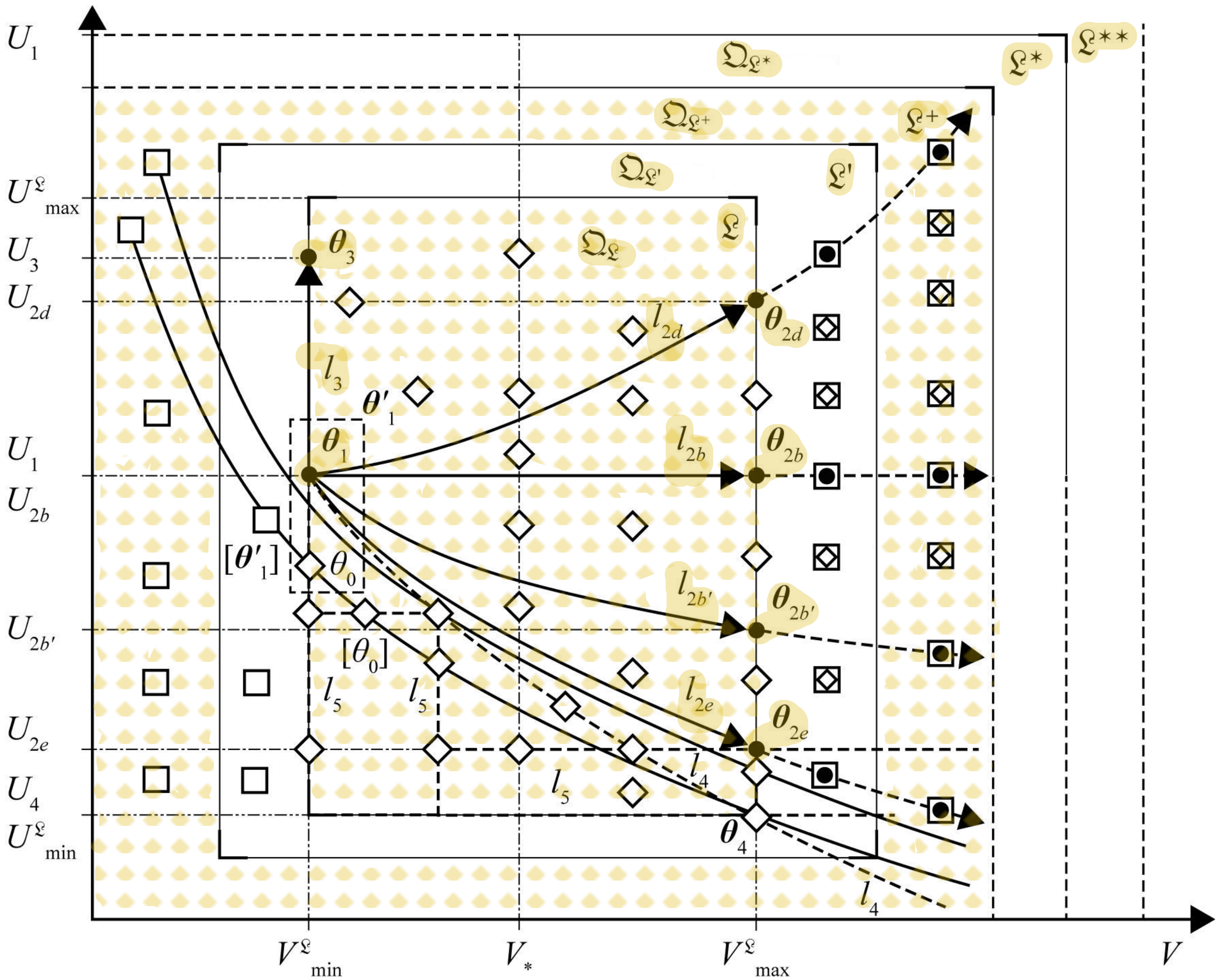
l_{2b} ***isothermic*** *irreversible expansion*

$l_{2b'}$ ***adiabatic*** *irreversible expansion*

l_{2d} ***izobaric*** *irreversible expansion*

l_{2e} ***izentropic*** *reversible expansion*

l_{3} ***izochoric*** *irreversible change*



l_4 ***not possible***

[5]**Which is the part of the wider meta-language with the vocabulary $\{p, V, T\}$ in which we construct/define this adiabatic state space $\mathfrak{Q}_{\mathfrak{L}}$/system $\mathfrak{L}$ by its complement $\{p, V, T\} - \mathfrak{Q}_{\mathfrak{L}}$ and the trajectory $l_{\mathfrak{Q}_{\mathfrak{L}}}$ by its complement $\mathfrak{Q}_{\mathfrak{L}} - l_{\mathfrak{Q}_{\mathfrak{L}}}$ $[\{p, V, T\} - (\mathfrak{Q}_{\mathfrak{L}} - l_{\mathfrak{Q}_{\mathfrak{L}}})]$ but, including its initial state from $\mathfrak{Q}_{\mathfrak{L}}$ - from outside.**

Peano Axioms/Inference system $\mathcal{P}$/Theory $\mathcal{T}_{\mathcal{PA}}$

1/$\mathcal{P}$ $\mathbb{N}_\mathbf{0} = \mathbb{N} \cup \{\mathbf{0}\}$;

2 $\forall_{\mathbf{x}\in\mathbb{N}_\mathbf{0}}|[\exists_{\mathbf{y}\in\mathbb{N}}|[\mathbf{y} = \mathbf{f}(\mathbf{x})]]$;

3/$\mathcal{P}$ $\forall_{\mathbf{x}\in\mathbb{N}_\mathbf{0}}|[\mathbf{0} \neq \mathbf{f}(\mathbf{x})]$;

4 $\forall_{\mathbf{x}\in\mathbb{N}_\mathbf{0}}|[[\mathbf{f}(\mathbf{x}) \neq \mathbf{f}(\mathbf{y})] \Rightarrow (\mathbf{x} \neq \mathbf{y})]$;

5/$\mathcal{P}$ *axiom/axiomatic schema of the* ***mathematical induction***:

$$[[\varphi(\mathbf{0}) \wedge \forall_{\mathbf{x}\in\mathbb{N}_\mathbf{0}}|\varphi(\mathbf{x}) \Rightarrow \varphi[\mathbf{f}(\mathbf{x})]] \Rightarrow \forall_{\mathbf{x}\in\mathbb{N}_\mathbf{0}}|\varphi(\mathbf{x})]$$

Inference rule ***Modus Ponens***[a]

$$\frac{\vdash \mathbf{b},\ \vdash (\mathbf{b} \Rightarrow \mathbf{c})}{\vdash \mathbf{c}},\quad \mathbf{c}\text{ - } \textit{immediate consequence of } \mathrm{b}$$

[a]**Besides the Generalization.** The **Substitution function** from the ***Principia Mathematica*** is used for the evaluation of the variables be their values or their quantification.

"1" - arithmeticity of the $\mathcal{P}$

$\cong$

adiabaticity of the $\mathfrak{L}/\mathfrak{Q}_\mathfrak{L}$.

Consistent $\mathcal{T}_{\mathcal{PA}}$ **inference** within $\mathcal{P}$

$\cong$

moving along trajectories $\mathrm{l}_{\mathfrak{Q}_\mathfrak{L}}$ within the $\mathfrak{Q}_\mathfrak{L}/\mathfrak{L}$.

The states on the adiabatic trajectories

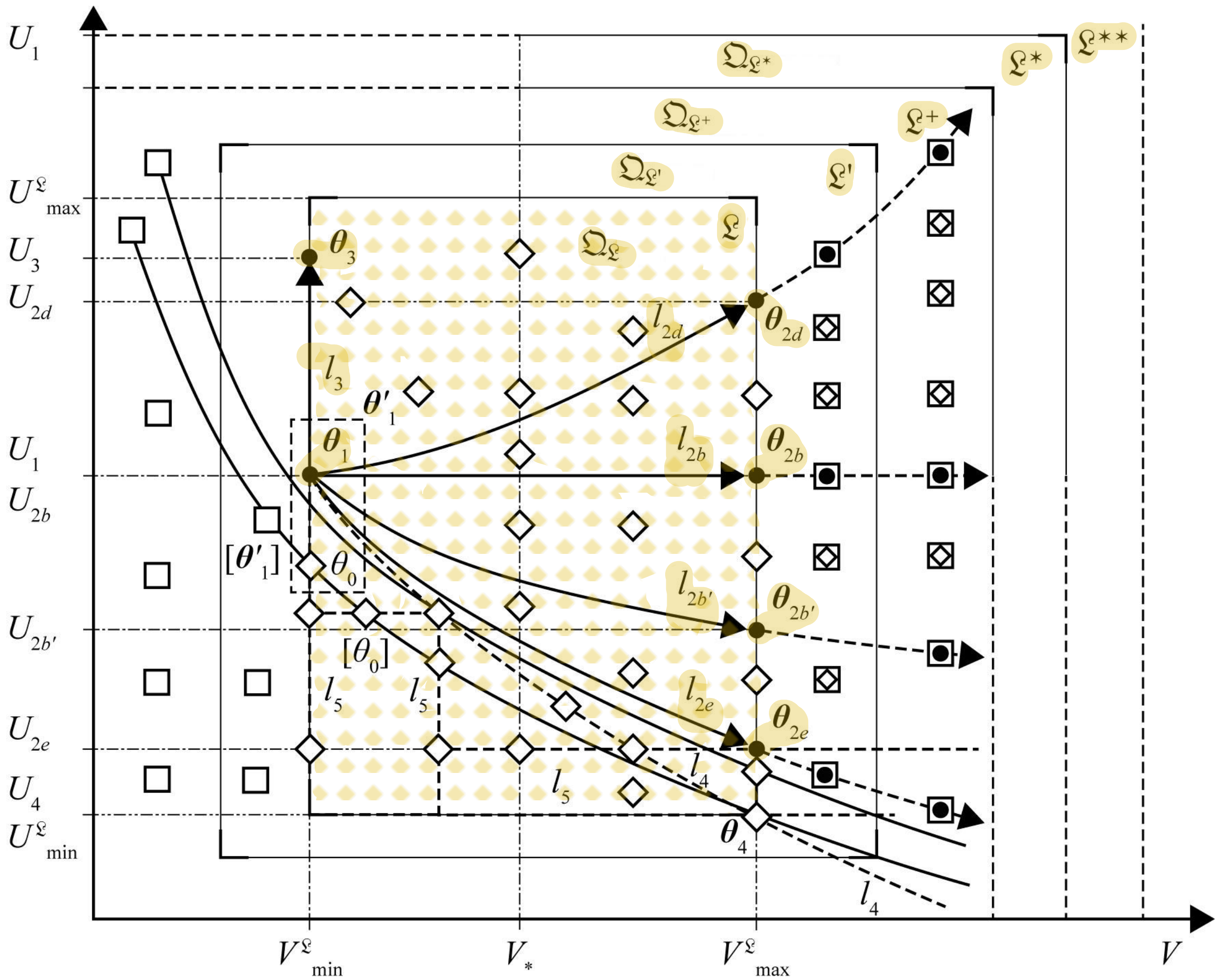
(also irreversible)

then **model the consistently**

inferred/inferrable *PA-FORMULAS*.

2. Autoreference and Caratheodory

Any adiabatic trajectory $\mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}$ is defined
by its complement $\mathfrak{Q}_{\mathfrak{L}} - \mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}$,
within its definition space $\mathfrak{Q}_{\mathfrak{L}}$/system $\mathfrak{L}$ **and**,
the $\mathfrak{Q}_{\mathfrak{L}} - \mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}$ is not reachable within $\mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}$ itself.



For the trajectory $\mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}$ could 'prove' - by itself -
its own adiabatic property $[\mathrm{d}]\mathbf{Q}_{\mathbf{Ext}}\mathbf{=0}$
it should have to contain its own definition
as its own status!

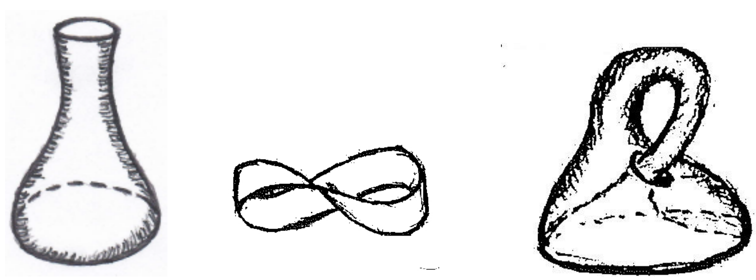
The $\mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}$ would be autoreferential/autoconstructive:
as the adiabatic one to construct, not adiabatically, $\mathrm{d}\mathbf{Q}_{\mathbf{Ext}}\neq 0$, the adiabatic, $[\mathrm{d}]\mathbf{Q}_{\mathbf{Ext}}\mathbf{=0}$, spaces $\mathfrak{Q}_{\mathfrak{L}}/\mathfrak{Q}_{\mathfrak{L}} - \mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}$
and define, by this way, itself - as its own status.

It is the **Klein bottle** building from the **original's** inside.

The original's outer surface defines its inner surface, which is now the model of the trajectory $\mathfrak{l}_{\mathfrak{Q}_{\mathfrak{L}}}$ and its outer surface is the model of the $\mathfrak{Q}_{\mathfrak{L}} - \mathfrak{l}_{\mathfrak{Q}_{\mathfrak{L}}}$.

Within the **inner surface** we want to **prove its internality** by **reaching its outer surface with an inner? curve**.

Within the **Klein bottle, constructible only by the outer manipulation with the original one, one curve is possible but, it is crossing, contradictorily against the original bottle, its inner and outer space simultaneously.**[6]



Only we, as the outer constructers of the original bottle know its all properties.

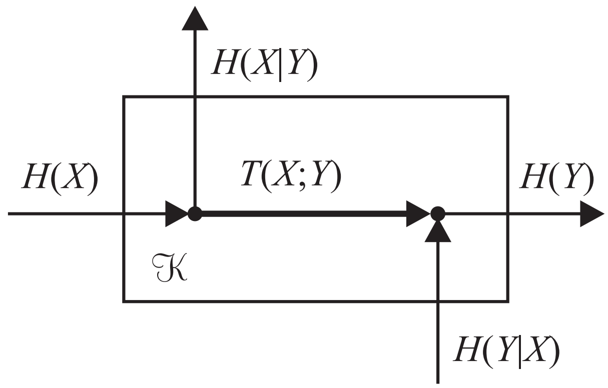
The adiabatic trajectory $\mathfrak{l}_{\mathfrak{Q}_{\mathfrak{L}}}$ does not conatin itself as the object of its own - it does not know its own properties - and it itself, by its means, does not reveal them.

Only we, as the outer constructers of the $\mathfrak{Q}_{\mathfrak{L}}/\mathfrak{L}//\mathcal{P}$ know their adiabaticity//consistency.

[6]**The original bottle should be the autoconstructive/autoreferential. The bottle should construct itself by stepping out from itself and form itself from its material as the two surfaces again and, by not with their own means.**

3. Information Transfer Channel

$$\mathcal{K} \stackrel{\text{Def}}{=} [\mathbf{X},\ \varepsilon,\ \mathbf{Y}]$$



$\mathbf{X} \stackrel{\text{Def}}{=} [\mathbf{A},\ \mathbf{p_X}(\cdot)]$ - the ***transmitter*** of *input* messages $\mathbf{x} \in \mathbf{A}^+$

$\mathbf{Y} \stackrel{\text{Def}}{=} [\mathbf{B},\ \mathbf{p_Y}(\cdot)]$ - the ***receiver*** of *output* messages $\mathbf{y} \in \mathbf{B}^+$,[7]

ε - the *maximal* ***probability*** of $\mathbf{y} = \mathbf{b}$ **erroneous** for $\mathbf{x} = \mathbf{a}$,

$\mathbf{p_X}(\cdot),\ \mathbf{p_Y}(\cdot)$ - the ***probability distribution*** **on A and B**,

$$\mathbf{A} = \mathbf{B} = \mathbf{T}$$

$\mathbf{H}(\mathbf{X}),\ \mathbf{H}(\mathbf{Y})$ - the ***input/output information entropies***[8]

$$\mathbf{H}(\mathbf{X}) \stackrel{\text{Def}}{=} -\sum_{\mathbf{A}} \mathbf{p_X}(\cdot) \ln \mathbf{p_X}(\cdot)$$

$$\mathbf{H}(\mathbf{Y}) \stackrel{\text{Def}}{=} -\sum_{B} p_Y(\cdot) \ln p_Y(\cdot)$$

$$i(\cdot) = -\ln(\cdot), \quad i(\cdot|\cdot) = -\ln(\cdot|\cdot)$$

[7] **A, B - a finite *alphabets* of elements x of X and y of Y.**

[8] ***Shannon entropies - average amounts of information in*** *any* $\mathbf{x} \in \mathbf{A}$ *and* $\mathbf{y} \in \mathbf{B}$.

$\mathbf{H}(\mathbf{X}|\mathbf{Y})$, $H(Y|X)$ - the ***loss/noise entropy***

$$\mathbf{H}(\mathbf{X}|\mathbf{Y}) \overset{\text{Def}}{=} -\sum_{\mathbf{A}}\sum_{\mathbf{B}} \mathbf{p}_{\mathbf{X},\mathbf{Y}}(\cdot,\cdot)\ln \mathbf{p}_{\mathbf{X}|\mathbf{Y}}(\cdot|\cdot)$$
$$H(Y|X) \overset{\text{Def}}{=} -\sum_{A}\sum_{B} p_{X,Y}(\cdot,\cdot)\ln p_{Y|X}(\cdot|\cdot)$$

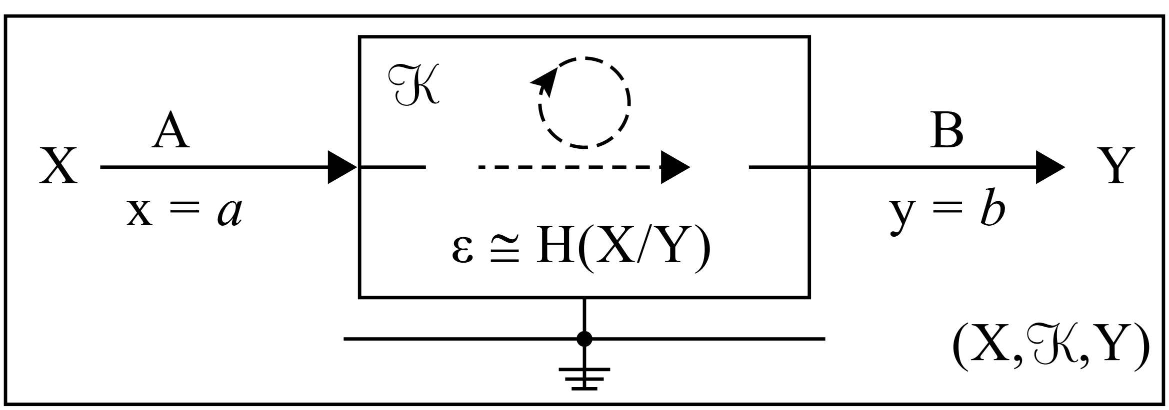
For the ***transinformation*** $\mathbf{T}(\mathbf{X};\mathbf{Y})$, $T(Y;X)$[9]

$$\mathbf{T}(\mathbf{X};\mathbf{Y}) \overset{\text{Def}}{=} \mathbf{H}(\mathbf{X}) - \mathbf{H}(\mathbf{X}|\mathbf{Y})$$
$$T(Y;X) \overset{\text{Def}}{=} H(Y) - H(Y|X)$$

the **channel equation** is valid

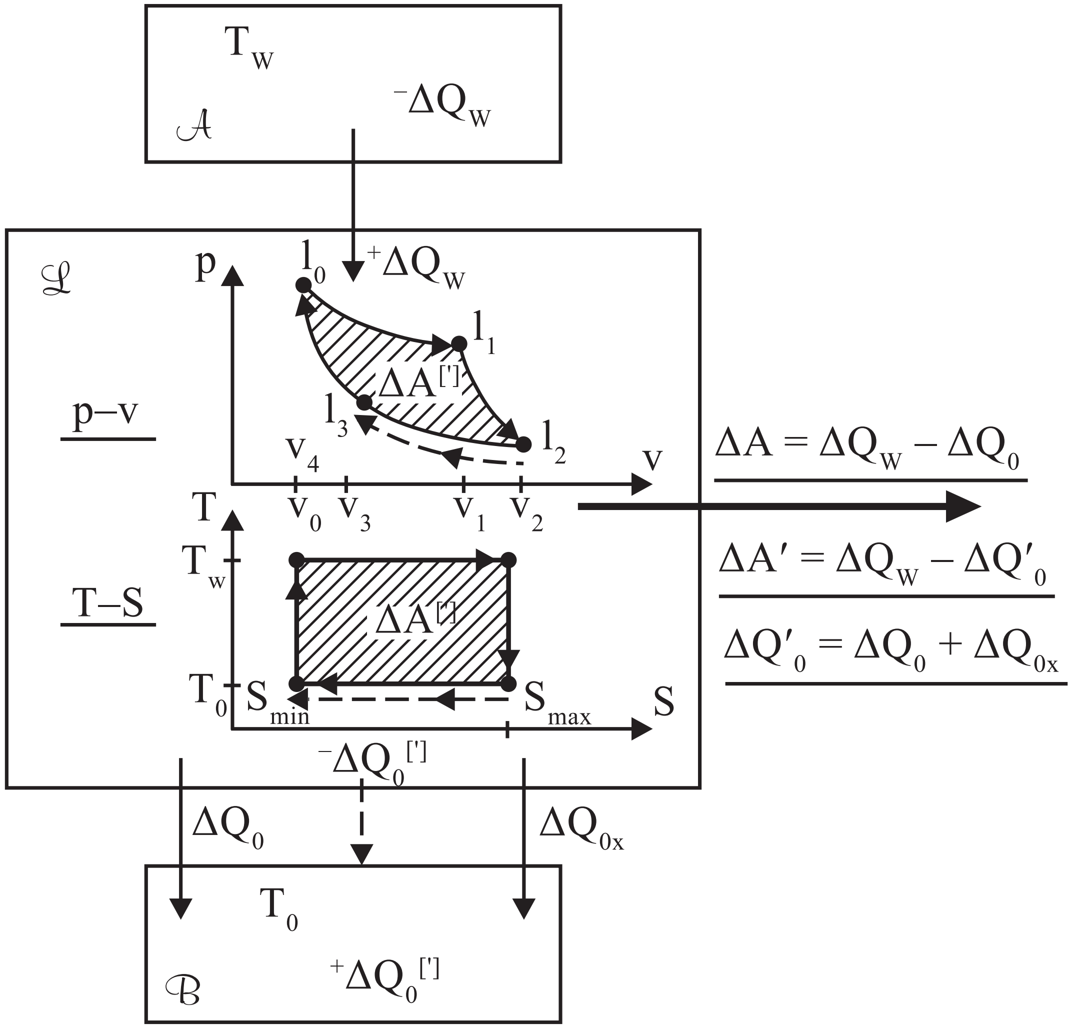
$$\mathbf{H}(\mathbf{X}) - \mathbf{H}(\mathbf{X}|\mathbf{Y}) = H(Y) - H(Y|X)$$

$(\mathbf{X},\ \mathcal{K},\ \mathbf{Y})$ - ***Shannon Transfer Chain***



[9] **Also it is valid for the information i with the probability** $\mathbf{p}_{\cdot}(\cdot)$, $\mathbf{i}_{\mathbf{X}} + \mathbf{i}_{\mathbf{Y}|\mathbf{X}} = \mathbf{i}_{\mathbf{Y}} + \mathbf{i}_{\mathbf{X}|\mathbf{Y}}$, $\mathbf{i}_{\cdot} = -\ln \mathbf{p}_{\cdot}(\cdot)$.

4. Carnot Machine



l_0-l_1: ***izothermal exp.*** transfers the heat $\mathbf{\Delta Q_W}$ from $\mathcal{A}$ to $\mathcal{L}$, the work $\mathbf{\Delta A_{0,1} = \Delta Q_W}$ is **given** at $\mathbf{T_W}$

l_1-l_2: ***adiabatic exp.*** cools $\mathcal{L}$ from $\mathbf{T_W}$ to $\mathbf{T_0}$, the work $\mathbf{\Delta A_{1,2} = -\Delta U}$ is **given** from the internal energy $\mathbf{U}$ of $\mathcal{L}$

l_2-l_3: ***izothermal comp.*** transfers the heat $\mathbf{\Delta Q_0 < \Delta Q_W}$[10] from $\mathcal{L}$ to $\mathcal{B}$ at $\mathbf{T_0}$, **consumes** the work $\mathbf{-\Delta A_{2,3} < \Delta A_{0,1}}$

l_3-l_0: ***adiabatic comp.*** heats $\mathcal{L}$ from $\mathbf{T_0}$ to $\mathbf{T_W}$, $\mathbf{\Delta U > 0}$, and **consumes** the work $\mathbf{-\Delta A_{3,4} = \Delta A_{1,2}}$

[10] **In the reversible Carnot Cycle is $\mathbf{\Delta Q_{0x} = 0}$, no *production* of (positive) *noise* heat, $\mathbf{\Delta Q_{0x} > 0}$, arises.**

The **resulting output work** for a **reversible** Carnot Cycle $\mathcal{O}$ is

$$\begin{aligned} \mathbf{\Delta A} &= \mathbf{\Delta Q_W} - |\mathbf{\Delta Q_0}| \\ \mathbf{\Delta A} &= \Delta A_{l_0-l_1} + \Delta A_{l_1-l_2} + \Delta A_{l_2-l_3} + \Delta A_{l_3-l_0} \end{aligned}$$

Kelvin's form of the $II.$ **P. T.** for a **reversible** case is

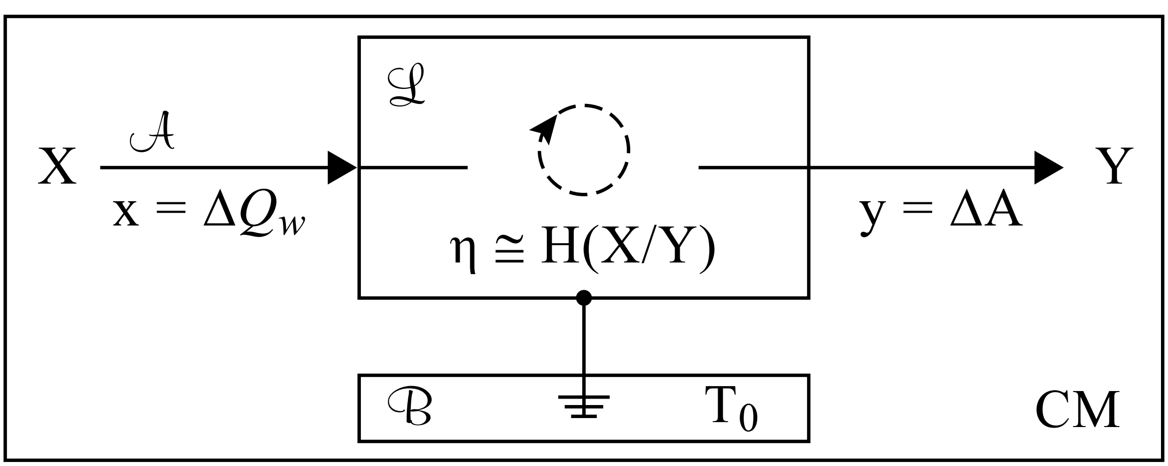
$$\sum_{\mathbf{i}\in[\mathbf{W},\mathbf{0}]} \frac{\mathbf{\Delta Q_i}}{\mathbf{T_i}} \stackrel{\triangle}{=} \oint_{\mathcal{O}} \frac{\delta \mathbf{Q}(\mathbf{\Theta})}{\mathbf{\Theta}} = \mathbf{0}$$

Thomson-Planck's formulation of the $II.$ **P. T.** says:

It is impossible to construct a heat cycle transforming all heat delivered to the medium $\mathcal{L}$ (going through this cycle) **into the *equivalent amount* of the mechanical work** $\mathbf{\Delta A}$.

The $I.$ **P.T.** is valid $\Leftrightarrow$ the ***tansformation efficiency*** $\eta_{\mathbf{max}}$:

$$\eta_{\mathbf{max}} \stackrel{\text{Def}}{=} \frac{\mathbf{\Delta A}}{\mathbf{\Delta Q_W}} = \frac{\mathbf{\Delta Q_W} - |\mathbf{\Delta Q_0}|}{\mathbf{\Delta Q_W}} = \frac{\mathbf{T_W} - \mathbf{T_0}}{\mathbf{T_W}} < \mathbf{1}$$



Recording/transmitting/computing an information $\mathbf{\Delta I}$
at the temperature $\mathbf{\Theta}$ requires the energy $\mathbf{\Delta W}$

$$\mathbf{\Delta W \geq k \cdot \Theta \cdot \Delta I}, \quad \text{now} \quad \Delta W \stackrel{\triangle}{=} \mathbf{\Delta Q_W}$$

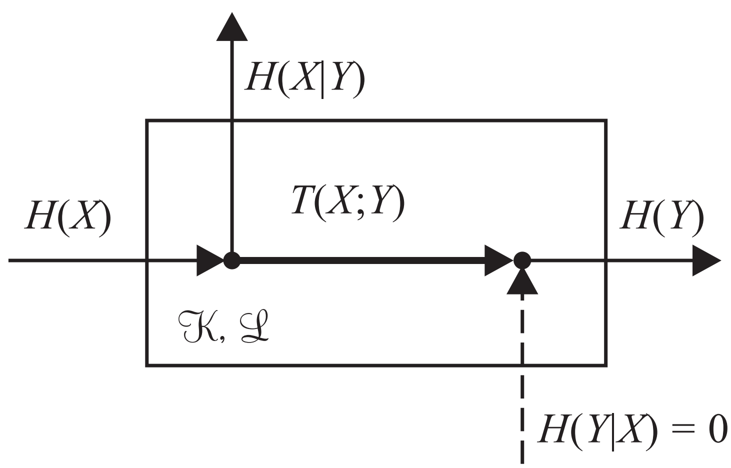
The changes of the entropies of the medium $\mathcal{L}$ *with* $\mathcal{O}$
are now considered informationally on a $\mathcal{K}$

$$\mathbf{H(X)} \stackrel{\text{Def}}{=} \frac{\mathbf{\Delta Q_W}}{\mathbf{kT_W}}, \qquad \mathbf{H(Y|X)} \stackrel{\text{Def}}{=} \mathbf{0}$$

$$\mathbf{H(Y)} \stackrel{\text{Def}}{=} \frac{\mathbf{\Delta A}}{\mathbf{kT_W}} = \frac{\mathbf{\Delta Q_W - \Delta Q_0}}{\mathbf{kT_W}}$$ [11]

$$= \frac{\mathbf{\Delta Q_W}}{\mathbf{kT_W}} \cdot \eta_{\mathbf{max}} = \mathbf{H(X)} \cdot \eta_{\mathbf{max}} \stackrel{\triangle}{=} \mathbf{\Delta I}$$

$$\mathbf{H(Y) - H(Y|X) = H(X) - H(X|Y)}$$

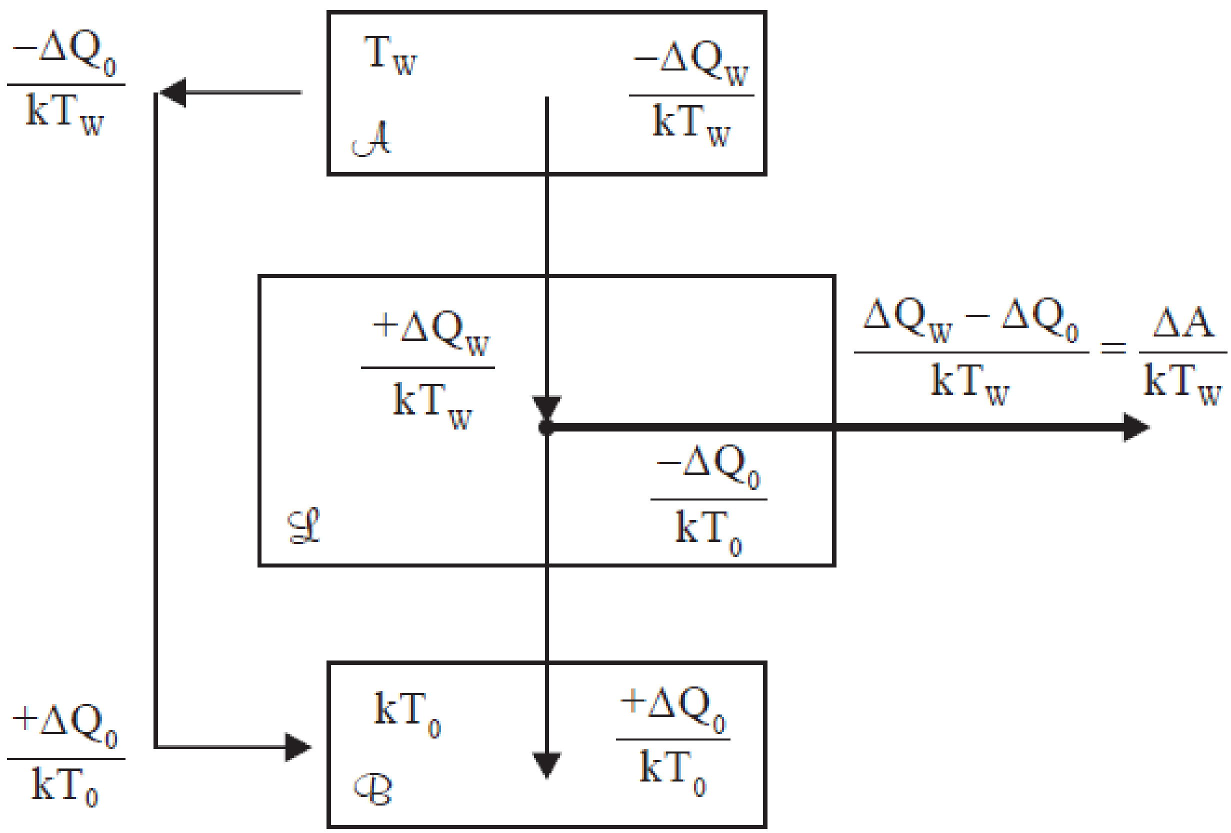


With our thermodynamic substitutions we gain:

[11] $H(X) \geq H(Y) = T(X;Y) = \Delta I \geq 0$; **the *information* form of the II. P.T. is implied for the reversible case; Brillouin, Landauer, Gershenfeld, Bennet.**

$$\frac{\Delta Q_W}{kT_W} \cdot \eta_{max} - 0 = \frac{\Delta Q_W}{kT_W} - H(X|Y)$$

$$\boxed{H(X|Y) = \frac{\Delta Q_W}{kT_W} \cdot (1 - \eta_{max}) = \frac{\Delta Q_0}{kT_W}}$$



The change $\Delta S_{\mathcal{AB}}$ within the change $\Delta S_{\mathcal{C}}$

of the global ***CM***'s heat entropy $S_{\mathcal{C}}$

within its subsystem $\mathcal{AB}$ is

$$\boxed{\Delta S_{\mathcal{AB}} = -\frac{\Delta Q_0}{T_W} + \frac{\Delta Q_0}{T_0} = \frac{\Delta Q_0}{T_0} \cdot \eta_{max} = \frac{\Delta Q_W}{T_W} \cdot \eta_{max}}$$

The change $\mathbf{\Delta S_{\mathcal{L}}}$ of the heat entropy $\mathbf{S_{\mathcal{L}}}$ within

the change $\mathbf{\Delta S_{\mathcal{C}}}$ of the whole heat entropy **$\mathbf{S_{\mathcal{C}}}$ of the *CM***[12]

is

$$\boxed{\mathbf{\Delta S_{\mathcal{L}}} = \oint_{\mathcal{O}} \frac{\delta \mathbf{Q}}{\mathbf{T}} = \frac{\mathbf{\Delta Q_W}}{\mathbf{T_W}} - \frac{\mathbf{\Delta Q_0}}{\mathbf{T_0}} = \mathbf{0}}$$

The *resultant* change $\mathbf{\Delta S_{\mathcal{C}}}$ of *CM* and the **output $\mathbf{\Delta I}$** is

$$\boxed{\mathbf{\Delta S_{\mathcal{C}}} = \mathbf{\Delta S_{\mathcal{L}}} + \mathbf{\Delta S_{\mathcal{AB}}} = \frac{\mathbf{\Delta Q_W}}{\mathbf{T_W}} \cdot \eta_{\mathbf{max}} = \mathbf{k \cdot \Delta I} = \mathbf{k \cdot H(Y)}}$$

The **Brillouin's *extended* form of the II. P.T.**

is valid[13]

$$\boxed{\mathrm{d}(\mathbf{S_{\mathcal{C}}} - \mathbf{k \cdot I}) \;\geq\; \mathbf{0}}$$

$$\Delta S_{\mathcal{C}} - k \cdot T(X;Y) \;=\; k \cdot H(X) \cdot (\eta_{max} - \eta_{max})$$
$$\Delta S_{\mathcal{C}} - k \cdot \Delta I = 0 \qquad \Delta(S_{\mathcal{C}} - k \cdot I) = 0$$

[12] Or of the whole system in which the ***CM*** is running.

[13] **The *information* member I does not exist in the traditional (differential) formulation of this theorem; it is $\mathrm{d}S \geq 0$ only.**

★ The ***reversible*** **Carnot Cycle** $\mathcal{O}$

■ the **medium** $\mathcal{L}$ going through the $\mathcal{O}$

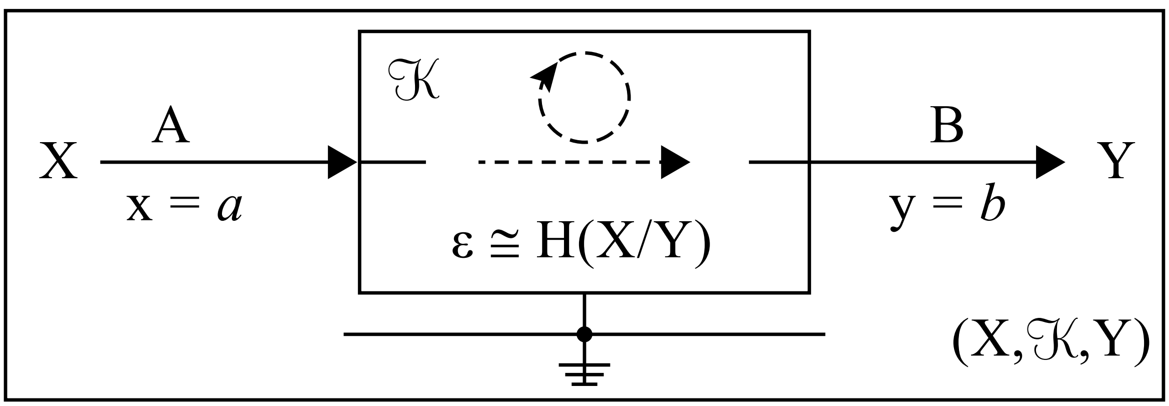
• the **whole** ***CM***

work as ***thermodynamic*** **models** of

★ the **information transfer process** $\mathcal{T}$ ***without*** **noise**,[14]

■ the **channel** $\mathcal{K}$ with its transfer process $\mathcal{T}$

• the ***Shannon Transfer Chain*** ($\mathbf{X}$, $\mathcal{K}$, $\mathbf{Y}$)



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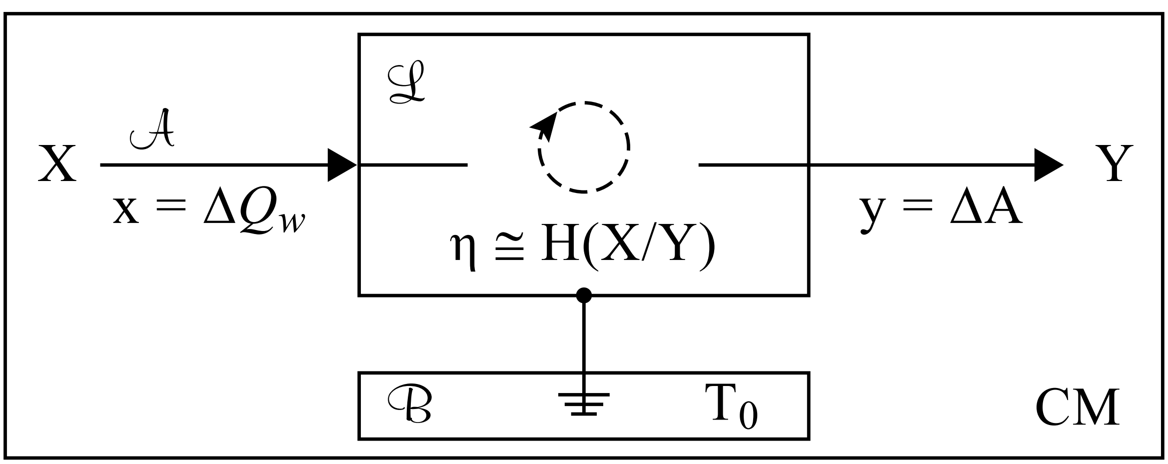


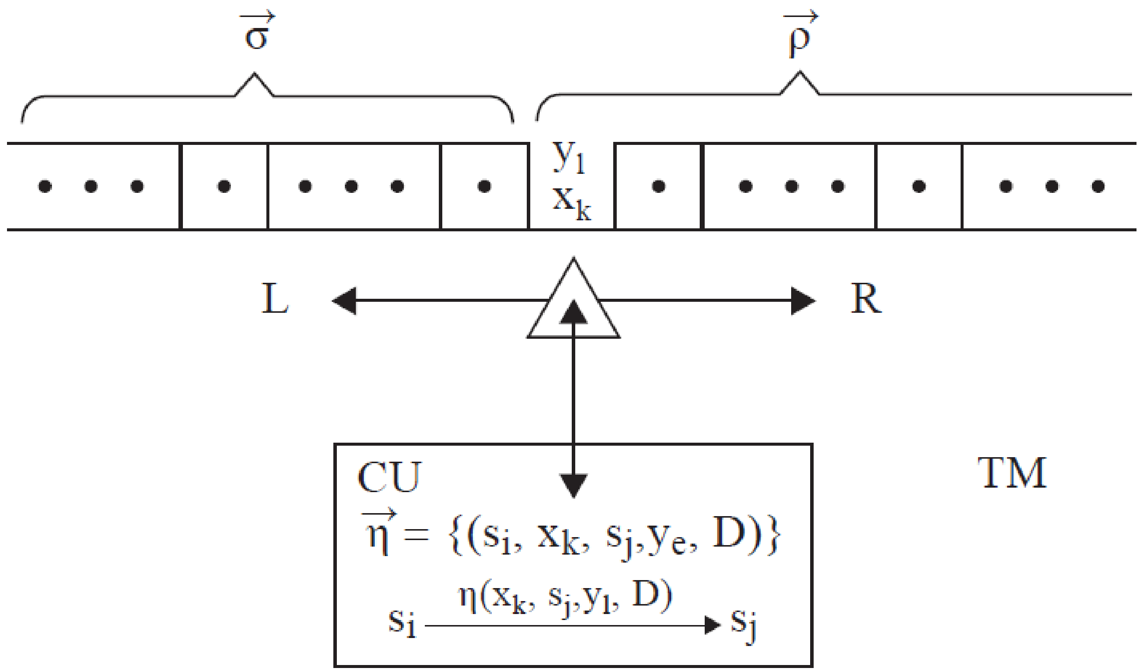
[14] $\mathbf{H(Y|X) = 0}$

6. Turing Machine

Turing Machine TM **- driven by the *program*** $\overrightarrow{\eta}$

$$\overrightarrow{\eta} = (\eta_p)_{p=1}^{p\in\mathbb{N}} = [(\mathbf{s_i},\ \mathbf{x_k},\ \mathbf{s_j},\ \mathbf{y_l},\ \mathbf{D})_\mathbf{p}]_{\mathbf{p=1}}^{\mathbf{p}\in\mathbb{N}},\quad |\overrightarrow{\eta}| \in \mathbb{N}$$

$$\eta_{[\cdot]} = (\mathbf{s_{i[\cdot]}},\ \mathbf{x_{k[\cdot]}},\ \mathbf{s_{j\cdot[+1]}},\ \mathbf{y_{l[\cdot]}},\ \mathbf{D})$$



$\mathbf{s_i}$ - the ***status*** of the $\mathbf{CU}_{TM}$ in the actual **step** $\mathbf{p} \in \mathbb{N}$

$\mathbf{x_k}$ - the ***input symbol*** on the ***input-output tape*** in the **p**

$\mathbf{y_l}$ - the ***output symbol*** **overwriting** $\mathbf{x_k}$ in the step **p**

$\mathbf{s_j}$ - the ***defined*** $\mathbf{CU}_{TM}$**'s status** for the step $\mathbf{p+1}$

D - the $\mathbf{CU}_{TM}$ ***read-write head moving*** *Left/Right*
after $\mathbf{y_l}$ **has been written instead of** $\mathbf{x_k}$ **in the step p**

$$[\mathbf{y_l},\ \mathbf{x_k} \in \mathbf{T} = \mathbf{A} = \mathbf{B}]$$

$(\overrightarrow{\sigma}, \mathbf{s_i}, \overleftarrow{\rho})/(\mathbf{x_k}, \mathbf{s_i}, \mathbf{y_l})$ **-** the ***TM***'s/$\mathbf{CU}_{TM}$**'s configurations**

With the I/O transformations $\mathbf{x_k} \longrightarrow \mathbf{y_l}$,

the $\mathbf{CU}_{\boldsymbol{TM}}$'s states' ***transitions*** are performed,

$$\mathbf{s_{i_p}} \overset{(\mathbf{x_{k_p}},\ \mathbf{y_{l_p}},\ \mathbf{D_p})}{\longrightarrow} \mathbf{s_{j_{p[+1]}}}$$

defining the regular grammar and the language $\mathbf{L}_{\mathbf{CU}_{\boldsymbol{TM}}}$

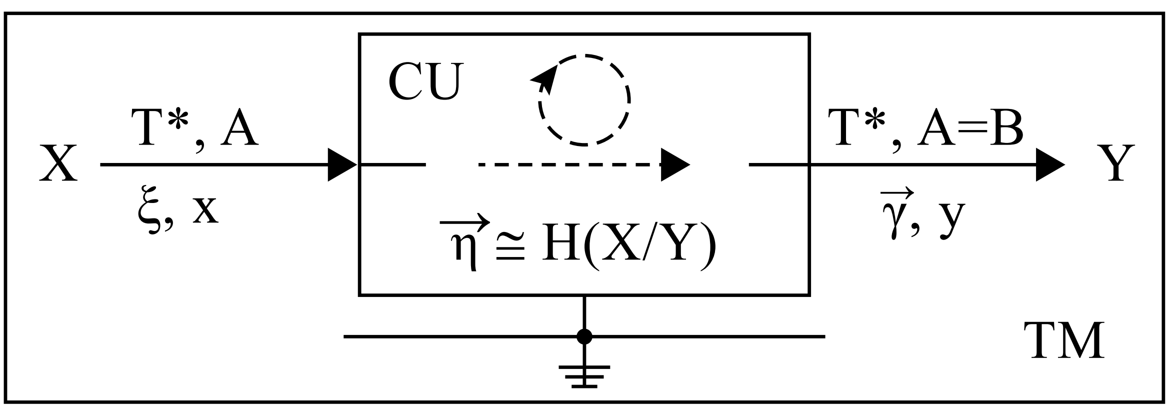
$$\mathbf{s_{i_p}} \longrightarrow (\mathbf{x_{k_p}}, \mathbf{y_{l_p}}, \mathbf{D_p})\mathbf{s_{j_{p[+1]}}}$$

$$\mathbf{L}_{\mathbf{CU}_{\boldsymbol{TM}}} = \{(\mathbf{x_{k_p}}, \mathbf{y_{l_p}}, \mathbf{D_p})\}_{\mathbf{p=1}}^{\mathbf{p=last}}$$

and the regular language $\mathbf{L}_{\boldsymbol{TM}}$ of the configurations which the ***TM*** has gone through so far[15]

$$\mathbf{S_{i_p}} \longrightarrow (\overrightarrow{\sigma}_{\mathbf{p}}, \mathbf{s_{ip}}, \overrightarrow{\rho}_{\mathbf{p}})\mathbf{S_{j_{p[+1]}}}$$

$$\mathbf{L}_{\boldsymbol{TM}} = \{(\overrightarrow{\sigma}_{\mathbf{p}}, \mathbf{s_{ip}}, \overrightarrow{\rho}_{\mathbf{p}})\}_{\mathbf{p=1}}^{\mathbf{p=p_{last}}}$$



[15] Terminal symbols $\mathbf{T} = \{I,\ B\}$ • the *instruction* (s_i, x_k, s_j, y_l, D) • the *configuration* $(\overrightarrow{\sigma}, s_{[\cdot]}, \overrightarrow{\rho})$ • the *configuration type* $(\varepsilon\,[\boldsymbol{\sigma}, s_{[\cdot]}, \boldsymbol{\rho}]\,\varepsilon)$ • $\boldsymbol{X} = (\overrightarrow{\boldsymbol{\sigma}}, s_{[\cdot]},\ \overrightarrow{\boldsymbol{\rho}}) \triangleq (\mathbf{B}\boldsymbol{\sigma}, s_{[\cdot]}, \boldsymbol{\rho}\mathbf{B})$ the *general configuration type*, e.g. $\mathbf{BI}\overrightarrow{\mathbf{IB}}s_{[\cdot]}\overrightarrow{\mathbf{BI}}\mathbf{B}$. Also $\mathbf{S_{i_p}} \longrightarrow (\mathbf{s_{i_p}}, \mathbf{x_{k_p}}, \mathbf{s_{j_{p[+1]}}}, \mathbf{y_{l_p}}, \mathbf{D_p})\mathbf{S_{j_{p[+1]}}}$, $\mathbf{L'}_{\boldsymbol{TM}} = \{(\mathbf{s_{i_p}}, \mathbf{x_{k_p}}, \mathbf{s_{j_{p[+1]}}}, \mathbf{y_{l_p}}, \mathbf{D_{[\cdot]}})\}_{\mathbf{p=1}}^{\mathbf{p=p_{last}}}$, instructions have been performed yet, [14, 19, 18].

7. Inference as Informatiom Transfer

The right TM's program $\overrightarrow{\eta}$[16]
generates the resultative TM's configuration sequence
and
similar is valid for the information transfer acts
and the $(\mathbf{X},\ \mathcal{K},\ \mathbf{Y})$'s configurations.

Now, the inferred $CLAIM$ $\mathbf{a_i}$ is the last member
of the input $FORMULAE$ chain $\overrightarrow{\mathbf{x}}$[17]
and the $\mathbf{a_i}$'s $\mathcal{P}/\mathcal{T}_{\mathcal{P}\mathcal{A}}$ inference by ***Modus Ponens***
is realized as the $\mathbf{y}$'s information transfer $\mathcal{T}$ in $\mathcal{K}$.

$$[\overrightarrow{\mathbf{x}|\mathbf{y}}] = [\mathbf{a_0},\ \mathbf{a_1},\ \mathbf{a_2},\ \ldots\ \mathbf{a_{i-1}}] \sqsubset [\overrightarrow{\mathbf{x}}]$$

$$[\overrightarrow{\mathbf{x}}] = [\mathbf{a_0},\ \mathbf{a_1},\ \mathbf{a_2},\ \ldots\ \mathbf{a_{i-1}};\ \mathbf{a_i}]$$

$$[\mathbf{y}] = \mathbf{a_i} \sqsubset [\overrightarrow{\mathbf{x}}]$$

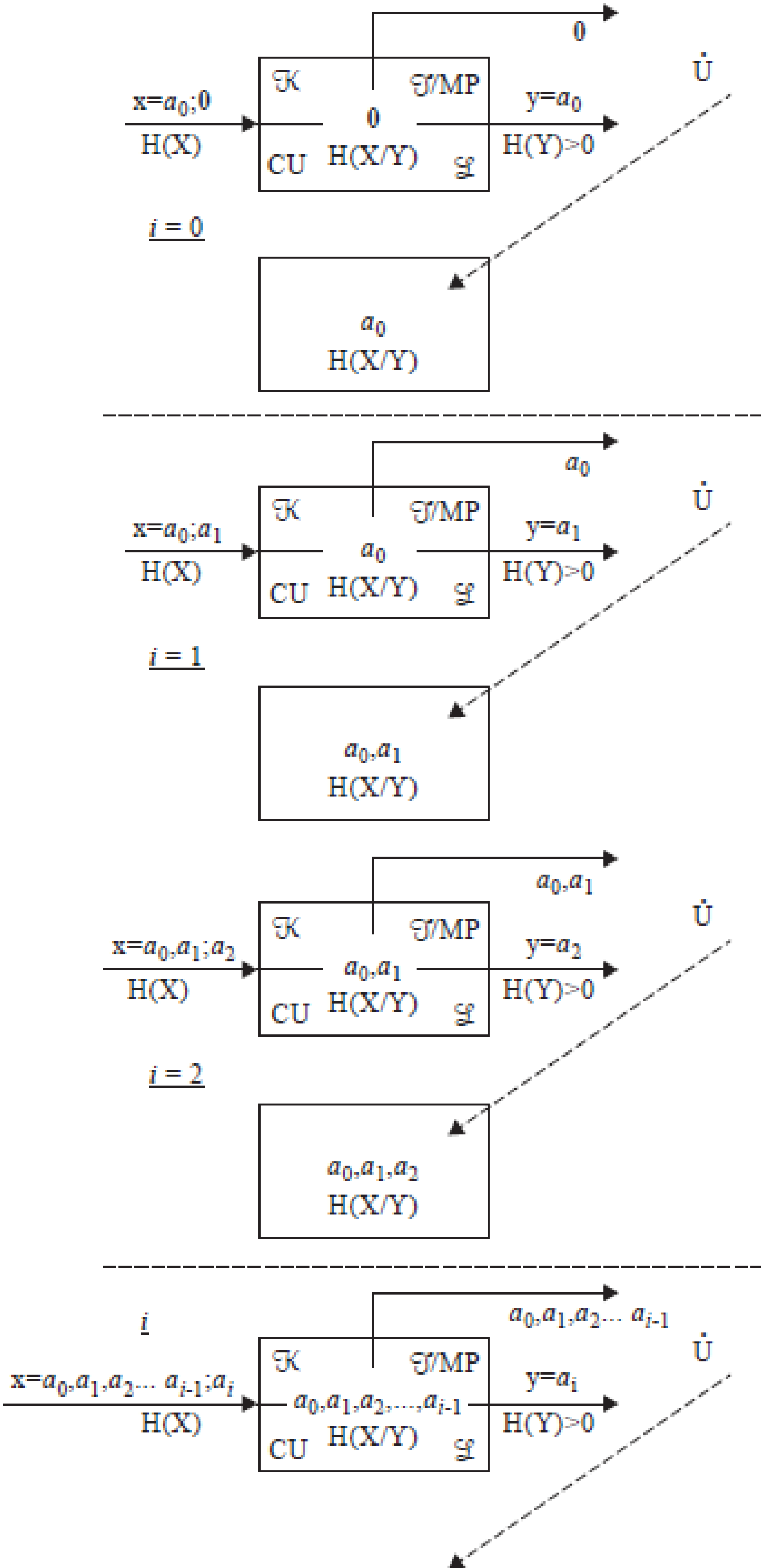
Entropies for this $\mathbf{a_i}$'s $\mathcal{P}/\mathcal{T}_{\mathcal{P}\mathcal{A}}$ inference
from $\overrightarrow{\mathbf{x}}$ realized by the $\mathbf{y} = \mathbf{a_i}$ transfer from $\overrightarrow{\mathbf{x}}$
through a $\mathcal{K}$ in its status $[\overrightarrow{\mathbf{x}|\mathbf{y}}]$ are:

$$\mathbf{H}(\mathbf{X}|\mathbf{Y}) \cong \mathbf{H}(\mathbf{a_0},\mathbf{a_1},\ldots,\mathbf{a_{i-1}}) = \mathbf{H}(\overrightarrow{\mathbf{x}|\mathbf{y}}),\ \mathbf{H}(\mathbf{Y}) = \mathbf{H}(\mathbf{y})$$

$$\mathbf{H}(\mathbf{X}) \cong \mathbf{H}(\mathbf{a_0},\mathbf{a_1},\ldots,\mathbf{a_{i-1}};\mathbf{a_i}) = \mathbf{H}(\overrightarrow{\mathbf{x}})$$

$$\mathbf{H}(\mathbf{Y}) \cong \mathbf{H}(\mathbf{a_0},\mathbf{a_1},\ldots,\mathbf{a_{i-1}};\mathbf{a_i}) - \mathbf{H}(\mathbf{a_0},\mathbf{a_1},\ldots,\mathbf{a_{i-1}})$$

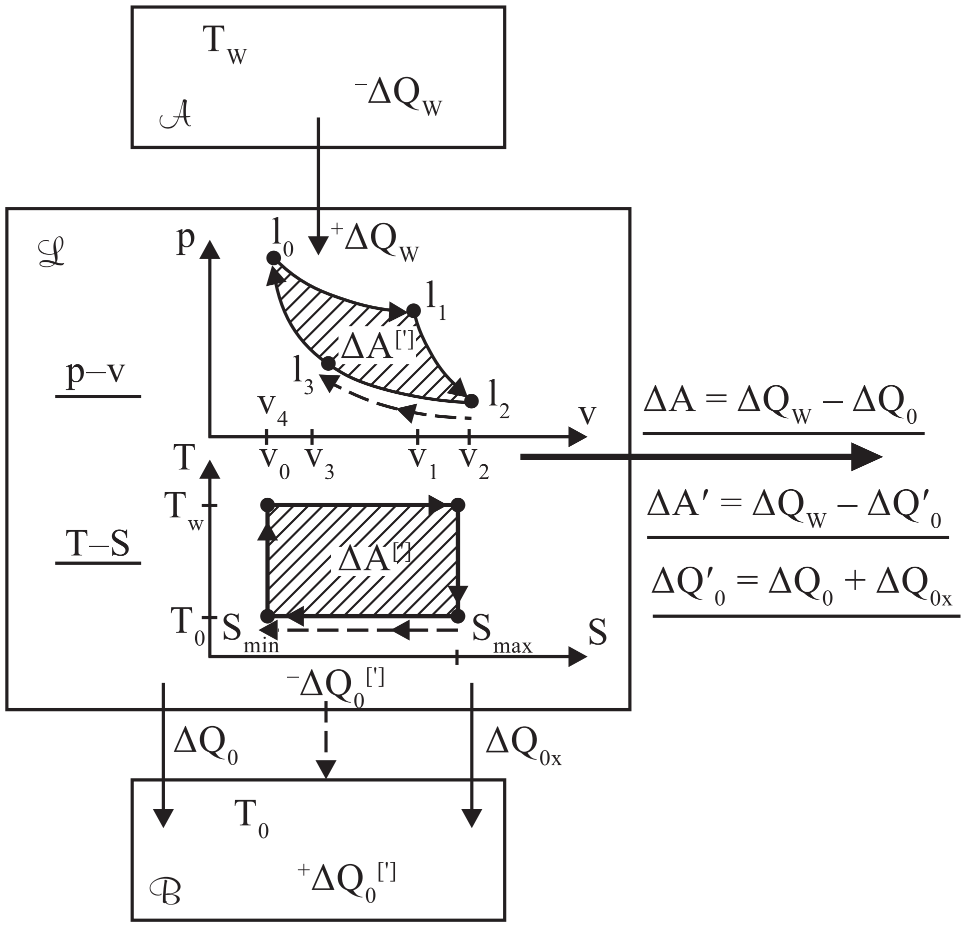
[16] **In our thermodynamic analogy following the Caratheodory** *II*. **P.T.**
[17] With the Gödel number **x**,



0
𝔎
𝔗/MP
x=a_0;0
H(X)
0
H(X/Y)
CU
𝔏
y=a_0
H(Y)>0
U̇
i = 0
a_0
H(X/Y)
a_0
𝔎
𝔗/MP
x=a_0;a_1
H(X)
a_0
H(X/Y)
CU
𝔏
y=a_1
H(Y)>0
U̇
i = 1
a_0,a_1
H(X/Y)
a_0,a_1
𝔎
𝔗/MP
x=a_0,a_1;a_2
H(X)
a_0,a_1
H(X/Y)
CU
𝔏
y=a_2
H(Y)>0
U̇
i = 2
a_0,a_1,a_2
H(X/Y)
i
a_0,a_1,a_2… a_{i-1}
𝔎
𝔗/MP
x=a_0,a_1,a_2… a_{i-1};a_i
H(X)
a_0,a_1,a_2,…,a_{i-1}
H(X/Y)
CU
𝔏
y=a_i
H(Y)>0
U̇

8. Carnot Cycle, Automata and Information Transfer

Any $\mathcal{CM}$, now with $\mathcal{O}$, is describable as an automaton[18]



with the resulting regular grammar and language L,

$$\{\theta^{i}_{l_0} \rightarrow (\Delta A^{i})\theta^{i}_{l_0}\}, \qquad L = \{\Delta A^{i}\}$$

and, as

for $\mathcal{K}$

$$H^{i}(X) = \frac{\Delta A^{i}_{l_0-l_1}}{kT^{i}_{W}}, \qquad H^{i}(X|Y) = \frac{|\Delta A^{i}_{l_2-l_3}|}{kT^{i}_{W}}$$

$$H^{i}(Y) = H^{i}(X) - H^{i}(X|Y) = \frac{\Delta A^{i}}{kT^{i}_{W}} > 0$$

$$\Delta A^{i} = \Delta A^{i}_{l_0-l_1} + \Delta A^{i}_{l_1-l_2} + \Delta A^{i}_{l_2-l_3} + \Delta A^{i}_{l_3-l_0}$$

[18]Now the Moore's but not only. Information transmission (not cyclical or cyclical) or the heat energy transformation (not cyclical or cyclical) is also describable by the terminology of *regular grammars* and *finite automata*.

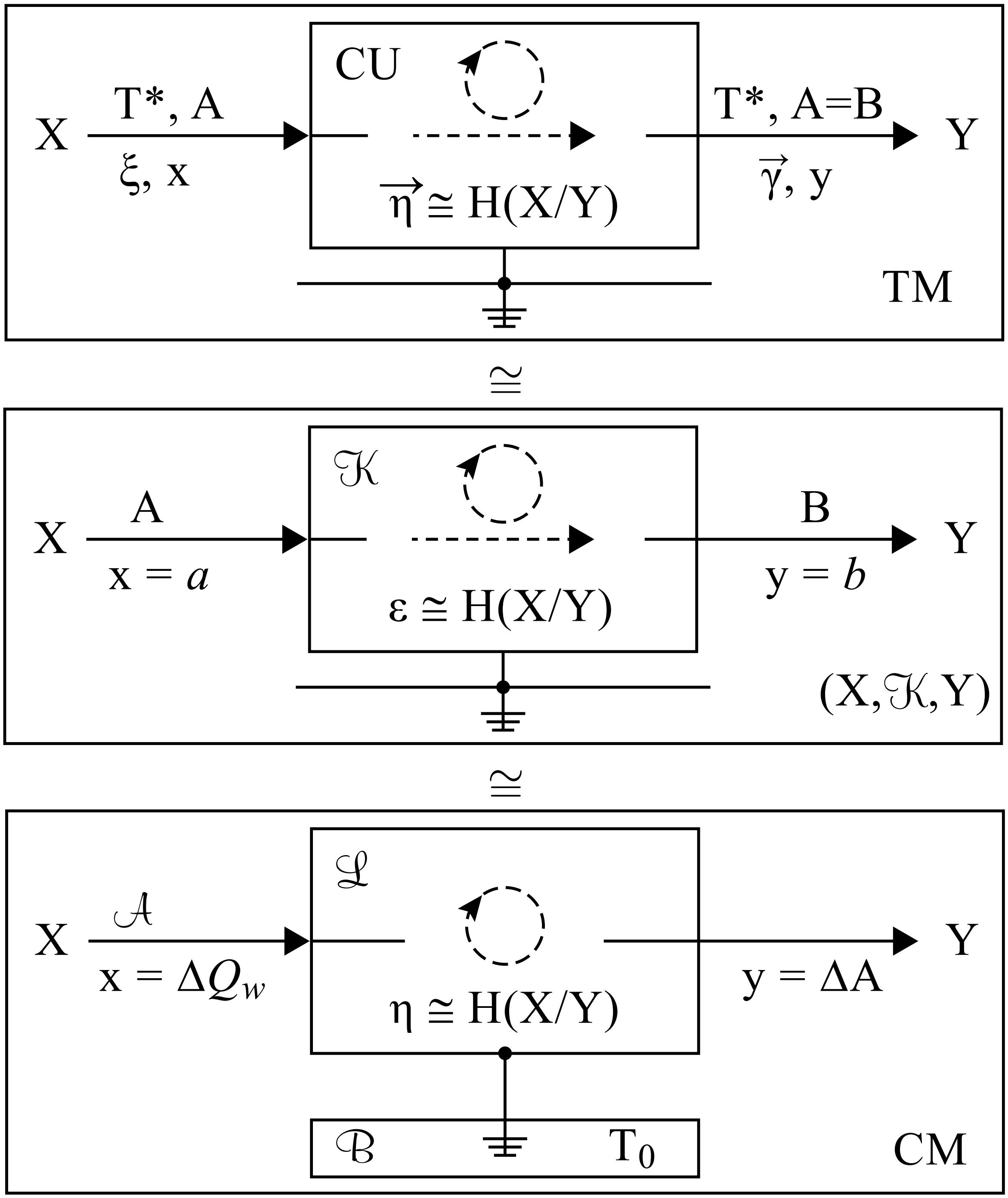
CU_{TM}'s step p ≅ $\mathcal{K}$'s transfer act i ≅ $\mathcal{L}$'s cycle $\mathcal{O}$ run i

$\mathcal{T}_{\mathcal{PA}}$ Inference ≅ Message Transfer ≅ Heat Transformation

states: $CU_{TM} \cong \mathcal{K} \cong \mathcal{L}$

config: $(\vec{\sigma}, \; s_i, \; \vec{\rho}) \cong [(X)^i, X^i|Y^i, (X)^{i+1}] \cong [\sum Q_W^i, (p^i, V^i, T^i)_{\mathcal{L}}, Q_W - \sum Q_W^i]$

$$TM \cong (X, \mathcal{K}, Y) \cong CM$$



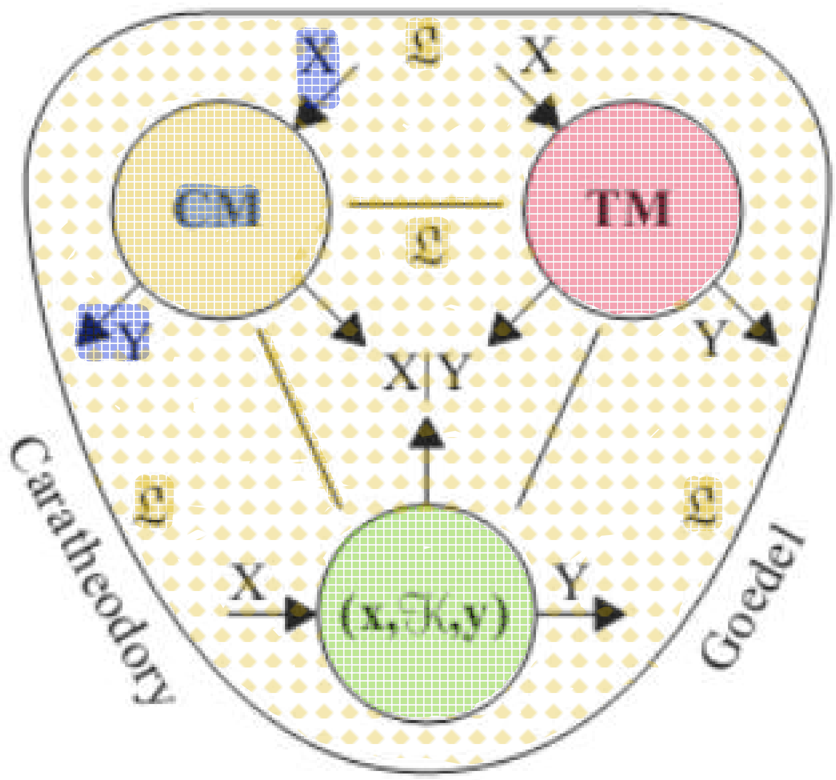
The TM's, $(X, \mathcal{K}, Y)$'s, CM's runs are considered in isolated systems for X, Y and X|Y energies.

The states' $\boldsymbol{\theta}^{\mathfrak{L}}_{[\cdot]}$ changes in the adiabatic system $\mathfrak{L}/\mathfrak{Q}_{\mathfrak{L}}$,
along the trajectories $\mathrm{l}_{\mathfrak{Q}_{\mathfrak{L}}}$ are expressible **regularly**:

l_{2b}	***isothermic*** *irreversible*,	$\boldsymbol{\theta}^{\mathfrak{L}}_1 \rightarrow \Delta A_{1,2e} \boldsymbol{\theta}^{\mathfrak{L}}_{2e}$
$l_{2b'}$	***adiabatic*** *irreversible*,	$\boldsymbol{\theta}^{\mathfrak{L}}_1 \rightarrow \Delta A_{1,3} \boldsymbol{\theta}^{\mathfrak{L}}_{3}$
l_{2d}	***izobaric*** *irreversible*,	$\boldsymbol{\theta}^{\mathfrak{L}}_1 \rightarrow \Delta A_{1,2b} \boldsymbol{\theta}^{\mathfrak{L}}_{2b}$
l_{2e}	***izentropic***,	$\boldsymbol{\theta}^{\mathfrak{L}}_1 \rightarrow \Delta A_{1,2b'} \boldsymbol{\theta}^{\mathfrak{L}}_{2b'}$
l_3	***izochoric*** *irreversible*,	$\boldsymbol{\theta}^{\mathfrak{L}}_1 \rightarrow \Delta A_{1,2d} \boldsymbol{\theta}^{\mathfrak{L}}_{2d}$
		$\boldsymbol{\theta}^{\mathfrak{L}}_1 \rightarrow \lambda \boldsymbol{\theta}^{\mathfrak{L}}_1$

The thermodynamic model
for the consistent $\mathcal{P}/\mathcal{T}_{\mathcal{P}\mathcal{A}}$ inference
- from its axioms/formulas having been inferred so far -
is created by the *CM*'s activity, which
is modeling both the ***TM*** **and** the $(\mathbf{X},\ \mathcal{K},\ \mathbf{Y})$
and **which runs in the adiabatic system $\mathfrak{L}/\mathfrak{Q}_{\mathfrak{L}}$**.
The ***TM***'s, $(\mathbf{X},\ \mathcal{K},\ \mathbf{Y})$'s, **configurations**
are then modeled by the states $\boldsymbol{\theta}^{\mathfrak{L}}_{\mathfrak{i}} \in \mathfrak{Q}_{\mathfrak{L}}$
of the adiabatic $\mathfrak{L}/\mathfrak{Q}_{\mathfrak{L}}$ with this modeling *CM* inside
the configuration of which,
in fact, are creating these states.

II. P. T.



The $\mathfrak{L}$'s initial imbalance starts the $\boldsymbol{\theta}^{\mathfrak{L}}_{[\cdot]}$s states' sequence on a trajectory $\mathfrak{l}_{\mathfrak{Q}_{\mathfrak{L}}}$ (irreversible)

and **is given by** the modeled

temperature difference $\mathrm{T_W} - \mathrm{T_0} > 0$ on *CM*,
existence of the input message on $\mathcal{K}$,
input chain's existence on the *TM*'s input-output tape

$\Rightarrow$

These adiabatic trajectories $\mathfrak{l}_{\mathfrak{Q}_{\mathfrak{L}}}$

now **represent the norm**

of the **consistency** (and resultativity)

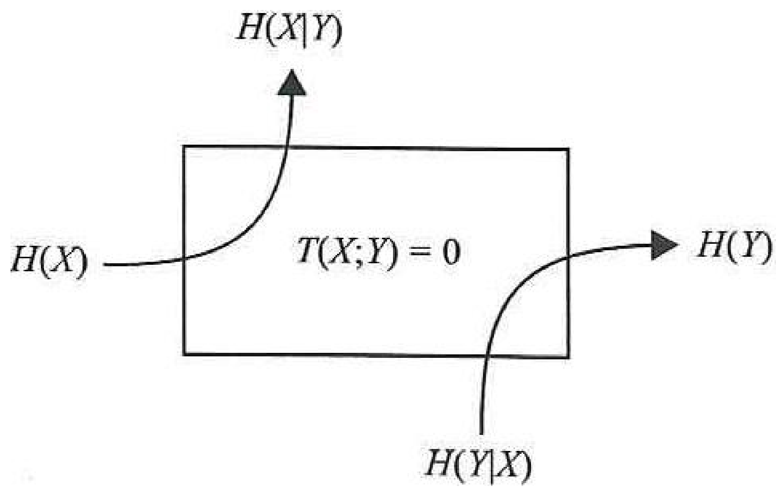
of the **$\mathcal{P}/\mathcal{T}_{\mathcal{P}\mathcal{A}}$-inference**/**computing process**

expressible also in terms

of the **information transfer**/**heat energy transformation.**

On the *interrupted* channel $\mathcal{K}$ is valid

$$\begin{aligned}\mathbf{H(X)} &= \mathbf{H(X|Y)}\\ \mathbf{T(X;Y)} &= \mathbf{H(X)} - \mathbf{H(X|Y)} = \mathbf{0}\end{aligned}$$



The input $\mathbf{H(X)}$ is now
the measure of the $\mathcal{K}$'s internal state - $\mathbf{H(X|Y)}$ -
the output $\mathbf{H(Y)}$ is without any relation to $\mathbf{H(X)}$.

With insisting (?!?) on the information transfer
through this interrupted channel $\mathcal{K}$,
then, contradictorily, we want build the $(\mathbf{X}, \mathcal{K}, \mathbf{Y})$
with a reversible direct Carnot Machine *CM*, where

$$\mathbf{\Delta Q_W} = \mathbf{\Delta Q_0} \stackrel{\triangle}{=} \mathbf{\Delta Q} \;\; \& \; \mathbf{T_W} > \mathbf{T_0}, \;\; \eta_{\mathbf{max}} = \frac{\mathbf{T_W} - \mathbf{T_0}}{\mathbf{T_W}} > \mathbf{0}$$

In fact we 'measure' $\mathbf{\Delta Q}$ against $\mathbf{\Delta Q}$, $\mathbf{T_W} = \mathbf{T_0}$,[19]

$$\mathbf{H(X)} = \mathbf{H(X|Y)} = \frac{\Delta Q}{kT_W}, \quad \mathbf{H(Y)} = \mathbf{0} \;\; [= H(Y|X)]$$

[19] The measuring with the zero 'distance' between the measuring and measured - the Gibbs Paradox.

Our 'wish' to have the information transfer $\mathbf{H(Y)} > 0$

through such interrupted channel $\mathcal{K}$ formulates

the contradiction/paradox[20]

- the *II*. P.T.'s *violation* -

$$\Delta \mathbf{S}_{\mathcal{L}} = \oint_{\mathcal{O}} \frac{\delta \mathbf{Q}}{\mathbf{T}} = \frac{\Delta \mathbf{Q}}{\mathbf{T_W}} - \frac{\Delta \mathbf{Q}}{\mathbf{T_0}} = -\frac{\Delta \mathbf{Q}}{\mathbf{T_0}} \cdot \eta_{\mathbf{max}} \underline{< \ \mathbf{0} \ \ (!)}$$

$$\Delta \mathbf{S}_{\mathcal{AB}} = \frac{\Delta \mathbf{Q}}{\mathbf{T_W}} \cdot \eta_{\mathbf{max}}$$

$$\Delta \mathbf{S}_{\mathcal{C}} = \Delta \mathbf{S}_{\mathcal{L}} + \Delta \mathbf{S}_{\mathcal{AB}} \underline{< \ \mathbf{0} \ \ (!)} \quad \text{[in fact 0 is everywhere]}$$

Our 'measuring' is now with the 0 'distance' between

the measuring object $\mathcal{K}/\mathcal{L}$

and the measured object $\mathbf{X}/\mathcal{A}$ and

we see that **the equality $\mathbf{H(X)} = \mathbf{H(X|Y)}$ says that**

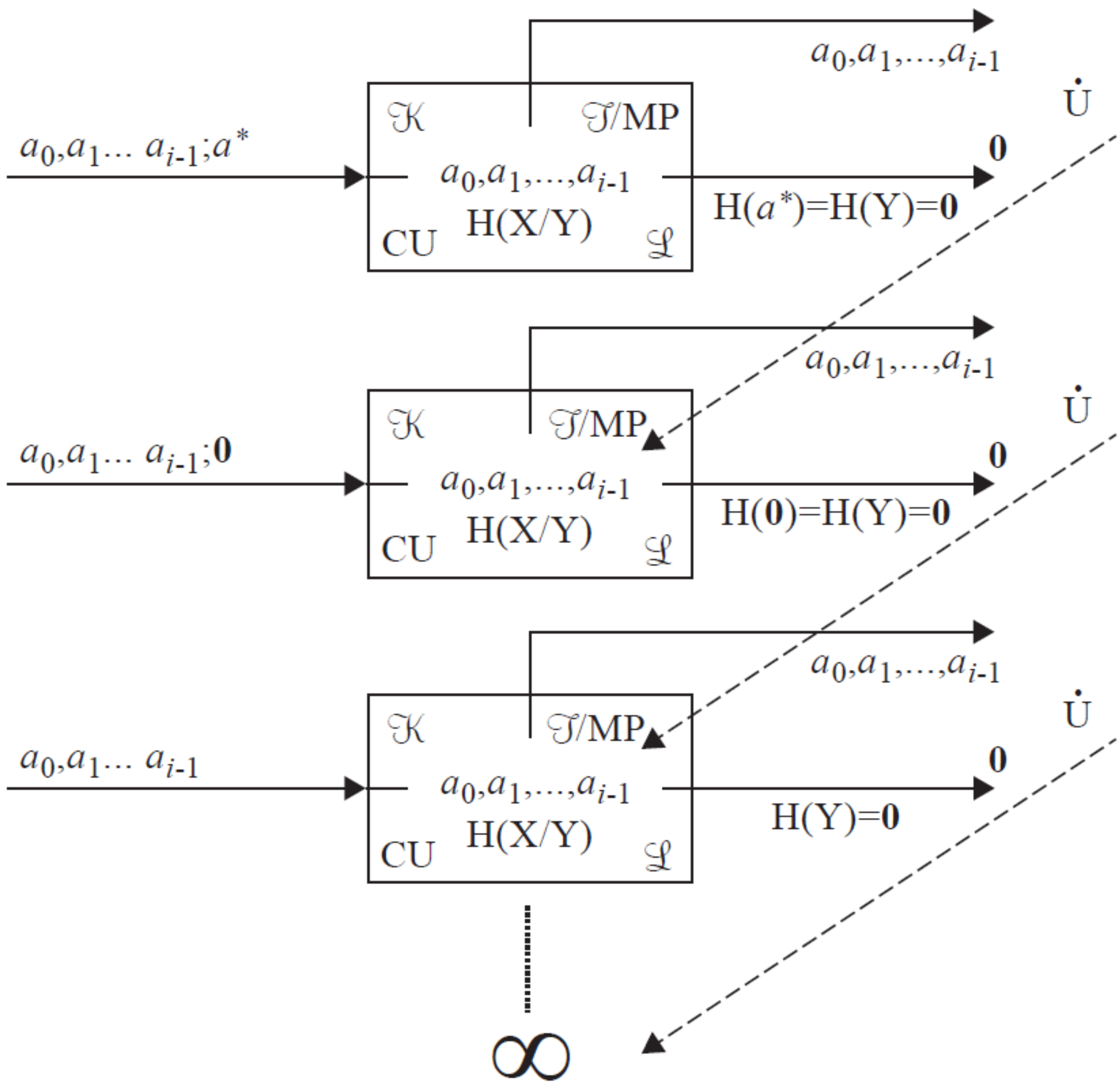
any $\mathcal{K}$ can't transfer its own states[21]

or observe/copy/measure itself.

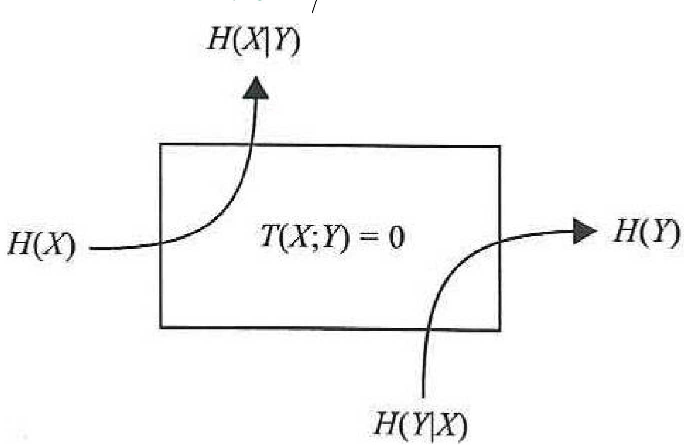
[20]**It is against the Caratheodory theorems. The existence of the *Perpetuum Mobile* *II*. and *I*. is required. It also requires the time arrow change $\frac{\mathbf{S}_{\mathcal{C}}}{\mathbf{t}} > 0$. 'Solving' this 'problem' represents the belief in the Maxwell demon's functionality. The need of distinguishing between the measured and the measuring - to avoid the *HALTING PROBLEM* - leads to the formulation of the Gödel theorems and their physical form as the Caratheodory theorems and vice versa.**

[21]**Used as input messages. *There's a need for a 'step-aside' outside the measured to gain a not zero and positive result*** $T(X;Y) = H(X) - H(X|Y) = H(Y) > 0$

The last *CLAIM* $\mathbf{a}^*$ in the input $\overrightarrow{\mathbf{x}} = \overrightarrow{a_0, a_1, \ldots, a_{i-1}; a^*}$ is not inferrable and, as such, interrupts the channel $\mathcal{K}$,

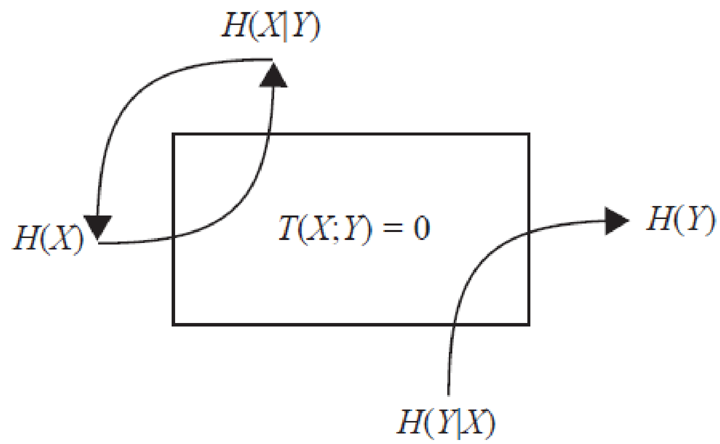


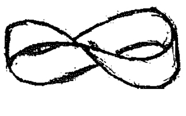
$\mathbf{H(X)}$ expresses the $\mathcal{K}$'s/***TM***'s structure $\mathbf{H(X|Y)}$,

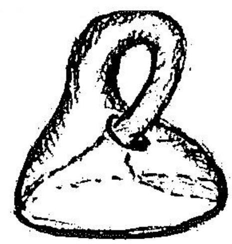


$\mathbf{H(Y)}$ does not relate to $\mathbf{H(X)}$

Repeating this **autoreference** attempt causes the **infinite cycle**,[22]







$\mathbf{H(X) = H(X|Y)}$ is the ***HALTING PROBLEM*** **core**.

Our bad awaitng
of a not zero and positive output, $\mathbf{H(Y)} > 0$,
formulates the methodological error, but,
identifiable by the II. P.T.

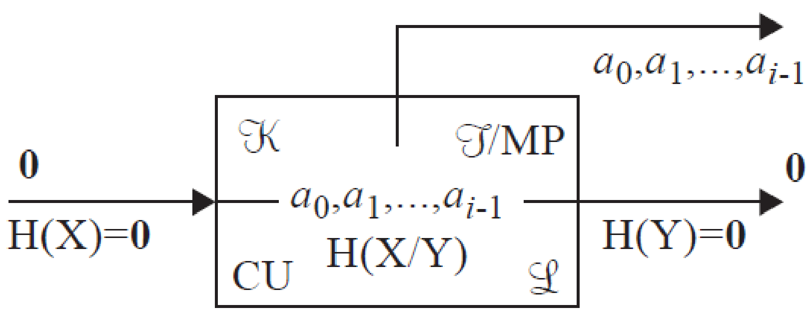
by **this possible consideration**,

$$\mathbf{H(X|Y)} \cong \mathbf{H(a_0, a_1, \ldots, a_{i-1}) > 0}, \quad \underline{\mathbf{H(X)} = \mathbf{0}}$$

$$\mathbf{H}_{\mathcal{C}} = \underline{\mathbf{H(X)}} - \mathbf{H(X|Y)} = \underline{\mathbf{0}} - \mathbf{H(a_0, a_1, \ldots, a_{i-1}) < 0}$$

which **violates/contradicts - $\mathbf{\Delta S_{\mathcal{C}} < 0}$ - the *II*. P.T.**

while we have



[22] **The same $\mathbf{H(Y) = H(Y|X)}$, $\mathbf{H(X|Y)}$ and the whole configuration (type) are repeated infinitely.**

Now let $\mathbf{x}$ be the ***SEQUENCE OF FORMULAE*** **valid** ("**1**") in the **theory** $\mathcal{T}_{\mathcal{PA}}$ and $\mathbf{y}$ is a ***CLAIM*** [23] **with a general syntax of** $\mathcal{P}^\star - \mathcal{P}/\mathcal{T}_{\mathcal{PA}}$, $\mathcal{P}^\star \supset \mathcal{P} \supset \mathcal{T}_{\mathcal{PA}}$.[24]

We define the **valid** ("**1**") **relation** $\mathbf{Q}(\mathbf{x},\mathbf{Y})$ saying:

"x ∈ 𝕏 is not in the $\mathcal{P}/\mathcal{T}_{\mathcal{PA}}$**-*INFERENCE* relation to the values y of Y",** $[\mathbf{y}|\mathbf{Y} \equiv \mathcal{T}_{\mathcal{PA}}\text{-property}]$

$$\mathbf{Y}:\ \mathbb{Y}\cong\{\{\mathfrak{Q}_{\mathfrak{L}^*}\}^\star - \{\mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}\}\}\ni\mathbf{y},\quad \mathbf{X}:\ \mathbb{X}\stackrel{\triangle}{=}\mathcal{T}_{\mathcal{PA}}\cong\{\mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}\}\ni\mathbf{x}$$

$\mathbf{Q}(\mathbf{x},\mathbf{Y}) = $ "**1**", from the construction and, **supposedly**,

$\mathbf{Q}(\mathbf{x},\mathbf{y}) = $ "**1**" $\equiv$ $\mathbf{Proof}_{\mathcal{P}}[\mathbf{Q}(\mathbf{x},\mathbf{y})]$ for $\mathbf{x}$ and any $\mathbf{y} \in \mathbf{Y}$

But, when **we set**

$$\underline{\mathbf{y}} = \mathbf{Q}(\mathbf{x},\mathbf{Y}) = \mathbf{y}(\mathbf{Y}) \quad [\cong \mathbf{y}|\mathbf{Y} \equiv \mathcal{T}_{\mathcal{PA}}\text{-property}]$$

$$\mathbf{Q}(\mathbf{x},\mathbf{y}) = \mathbf{y}[\mathbf{Q}(\mathbf{x},\mathbf{Y})] = \mathbf{y}(\mathbf{y}) = \mathbf{Q}[\mathbf{x},\mathbf{Q}(\mathbf{x},\mathbf{Y})]$$

the $\mathbf{Y} := \mathbf{y}$ in $\mathbf{y}(\mathbf{Y})$ **generates the *autoreference***,

$$\underline{\mathbf{Q}(\mathbf{x},\mathbf{y})};\quad \mathbf{Q}[\mathbf{x},[\mathbf{Q}(\mathbf{x},\mathbf{Y})]],\quad \mathbf{Q}[\mathbf{x},[\mathbf{Q}(\mathbf{x},[\mathbf{Q}(\mathbf{x},\mathbf{Y})])]],\ \ldots$$

neither constructible in $\mathcal{P}$ **and nor provable by** $\mathcal{T}_{\mathcal{PA}}$ **but its *validity follows* from the** *II.* **P.T.** ***validity***.[25]

$$\overline{\mathbf{Proof}_{\mathcal{P}}}[\mathbf{Q}(\mathbf{x},\mathbf{y})] = \text{"}\mathbf{1}\text{"}$$

$$II.\mathbf{P.T.} \cong [\mathbf{Proof}_{\mathcal{P}}[\mathbf{Q}(\mathbf{x},\mathbf{y})] = \text{"}\mathbf{0}\text{"}] = \text{"}\mathbf{1}\text{"}$$

$$[[\mathbf{Proof}_{\mathfrak{L}}([\mathrm{d}]\mathbf{Q}_{\mathbf{Ext},\mathbf{l}_{\mathfrak{Q}_{\mathfrak{L}}}}=\mathbf{0}) = \text{"}\mathbf{0}\text{"}] = \text{"}\mathbf{1}\text{"}] \equiv II.\ \mathbf{P.T.}$$

[23] **The x and y are the Gödel numbers.**

[24] $\mathcal{P}^\star \cong \{\mathfrak{Q}_{\mathfrak{L}^*}\}^\star$, $\mathcal{P} \cong \{\mathfrak{Q}_{\mathfrak{L}}\}$, $\mathcal{T}_{\mathcal{PA}}\cong\{l_{\mathfrak{Q}_{\mathfrak{L}}}\}$, $x \cong l_{\mathfrak{Q}_{\mathfrak{L}}}$.

[25] And within the **PM** - ***Principia Mathematica: B. Russel, L. Whitehead.***

Now **we set**

$$\underline{\mathbf{p}} \;=\; [\forall_{\mathbf{x}\in\mathbb{X}}|\mathbf{Q}(\mathbf{x},\mathbf{Y})] \;\stackrel{\triangle}{=}\; \mathbf{Q}(\mathbb{X},\mathbf{Y}) = \mathbf{p}(\mathbf{Y})$$

"no x $\in \mathbb{X}$ is in the $\mathcal{P}/\mathcal{T}_{\mathcal{PA}}$-*INFERENCE* relation to the variable Y" - to its space $\mathbb{Y} \cong \{\{\mathfrak{Q}_{\mathfrak{L}^*}\}^{\star} - \{\mathrm{l}_{\mathfrak{Q}_{\mathfrak{L}}}\}\}$

With the substitution $\mathbf{Y} := \mathbf{p}$

the resulting $\mathbf{Q}(\mathbb{X},\mathbf{p}) = \mathbf{p}(\mathbf{p})$ $[\cong \mathbf{p}|\mathbf{Y} \equiv \mathcal{T}_{\mathcal{PA}}\text{-property}]$

contains the *autoreference*

- the ***totality*** $\mathbb{X}/\mathcal{T}_{\mathcal{PA}}$ **'defines' (?!?) its property Y** -

$$[\forall_{\mathbf{x}\in\mathbb{X}}|[\mathbf{Q}[\mathbf{x},[\forall_{\mathbf{x}\in\mathbb{X}}|\mathbf{Q}(\mathbf{x},\mathbf{Y})]]]]_{\mathbf{Y}:=\mathbf{p}} = \mathbf{Q}[\mathbb{X},[\mathbf{Q}(\mathbb{X},\mathbf{Y})]]_{\mathbf{Y}:=\mathbf{p}}$$

$$\underline{\mathbf{Q}(\mathbb{X},\mathbf{p})};\quad \mathbf{Q}[\mathbb{X},[\mathbf{Q}(\mathbb{X},\mathbf{Y})]],\;\; \mathbf{Q}[\mathbb{X},[\mathbf{Q}(\mathbb{X},[\mathbf{Q}(\mathbb{X},\mathbf{Y})])]],\;\ldots$$

As before, by the *II.* P.T.'s validity, we know:

$$\overline{\mathbf{Proof}}_{\mathcal{P}}[\mathbf{Q}(\mathbb{X},\mathbf{p})] = \text{"}\mathbf{1}\text{"}$$

$$II.\mathbf{P.T.} \;\cong\; [\mathbf{Proof}_{\mathcal{P}}[\mathbf{Q}(\mathbb{X},\mathbf{p})] = \text{"}\mathbf{0}\text{"}] = \text{"}\mathbf{1}\text{"}$$

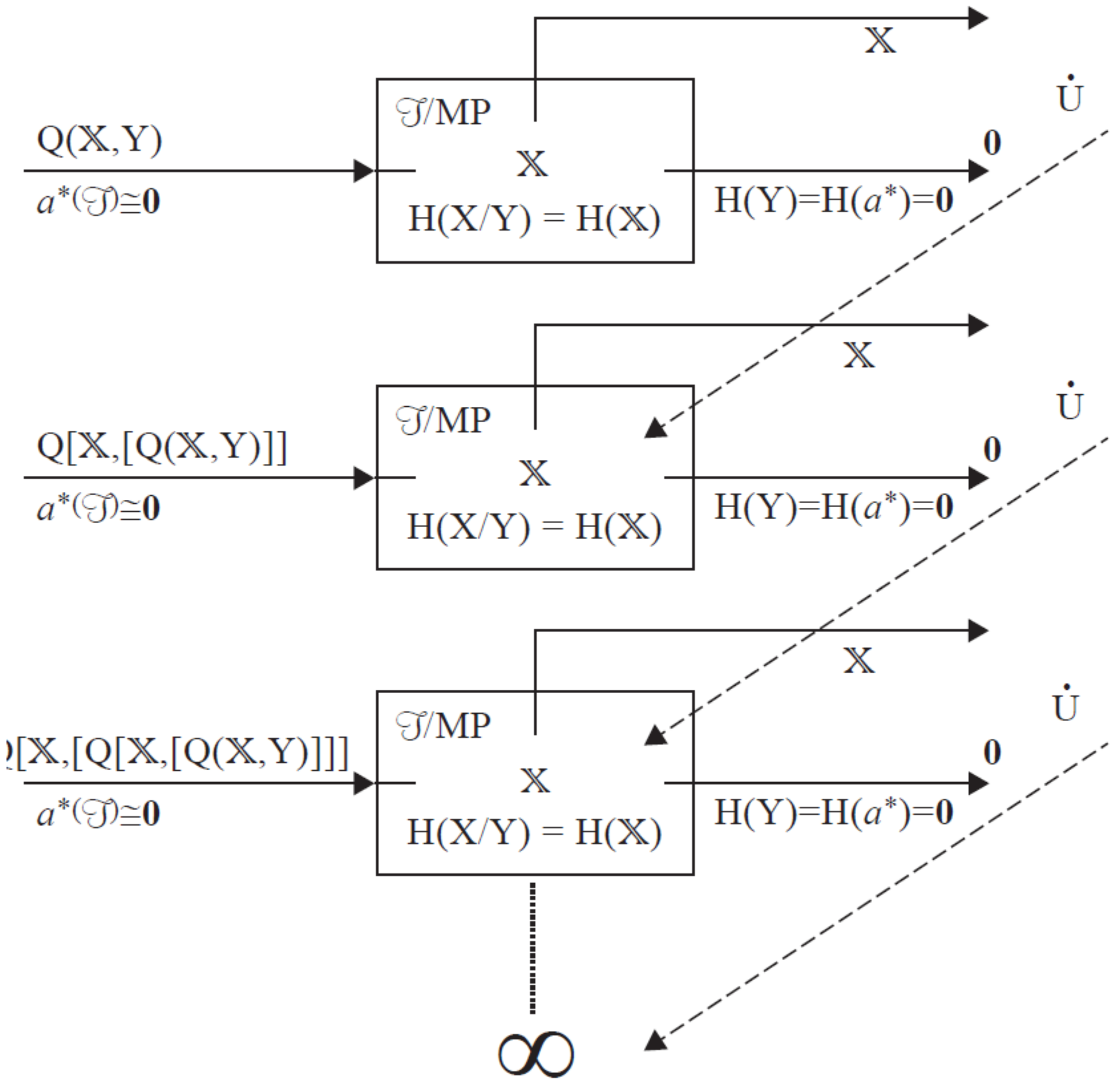
$$[[\mathbf{Proof}_{\mathfrak{L}}([\mathrm{d}]\mathbf{Q}_{\mathbf{Ext},\{\mathrm{l}_{\mathfrak{Q}_{\mathfrak{L}}}\}=\mathbf{0}}) = \text{"}\mathbf{0}\text{"}] = \text{"}\mathbf{1}\text{"}] \;\equiv\; II.\ \mathbf{P.T.}$$

$$\mathbf{y},\mathbf{p},\mathbf{Q}(\mathbf{x},\mathbf{y}),\mathbf{Q}(\mathbb{X},\mathbf{p}) \in \{\mathcal{P}^{\star} - \mathcal{P}\} \cong \{\{\mathfrak{Q}_{\mathfrak{L}^*}\}^{\star} - \mathfrak{Q}_{\mathfrak{L}}\},$$

$$\{\mathrm{l}_{\mathfrak{Q}_{\mathfrak{L}}}\} \cong \mathcal{T}_{\mathcal{PA}} \stackrel{\triangle}{=} \mathbb{X},\;\; \underline{\mathbf{Q}(\mathcal{T}_{\mathcal{PA}},\mathbf{p}) = \mathbf{Q}(\mathbb{X},\mathbf{p}) \stackrel{\triangle}{=} \mathbf{17Gen\ r} \;\cong \mathbf{Y}},$$

$$\mathbf{r} \stackrel{\triangle}{=} \mathbf{r}(\mathbf{X}) = \mathbf{Q}(\mathbf{X},\mathbf{p}),\;\; \mathbf{17Gen\ r} \;\stackrel{\triangle}{=}\; \forall_{\mathbf{x}\in\mathbb{X}}|\mathbf{r}(\mathbf{X}),\quad \text{card } \mathbf{Y} = \aleph_{\mathbf{1}}$$

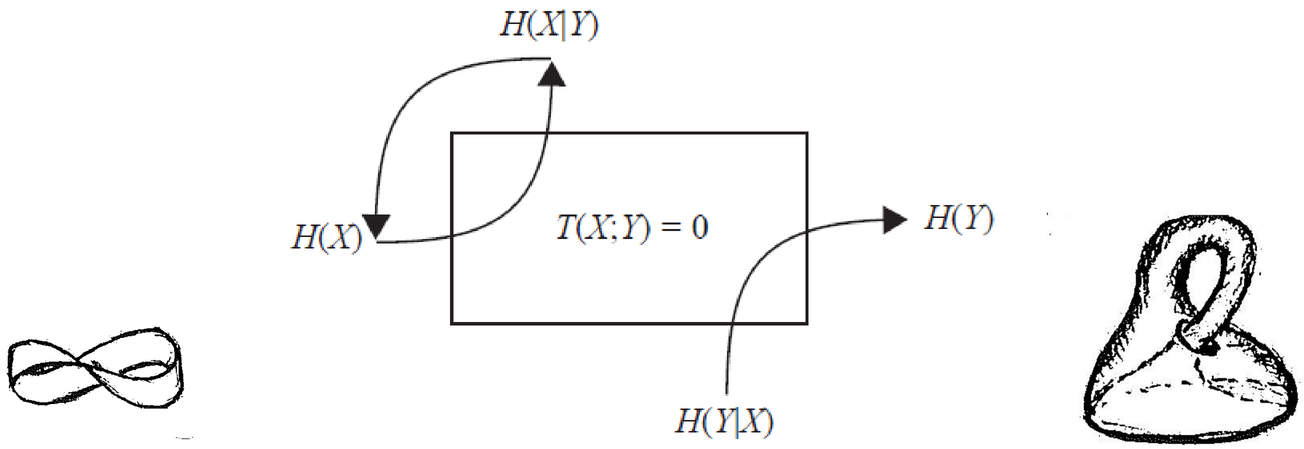
The consistent theory $\mathcal{T}_{\mathcal{PA}}$ is constructed
by ourselves, **from its outside**,
it does **not contain the axiom of its consistency**
as its own object/formula/status
The attempt to prove the $\mathcal{T}_{\mathcal{PA}}$ consistency by it itself
- imbuilding is property (**Y**) to itself as its own object -
defines the autoreference/*HALTING PROBLEM*,
not resultative but, as such, ***identifiable***;



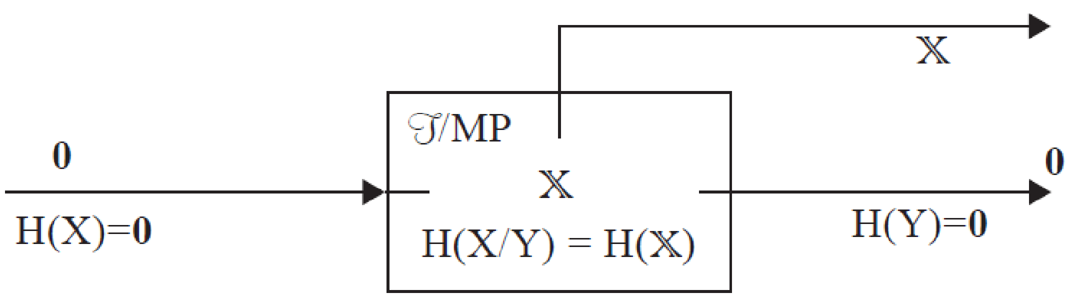
The *CLAIM* $\mathbf{Q}(\mathbb{X}, \mathbf{p})$/17Gen r is constructed
purely syntactically as for $\mathcal{P}/\mathcal{T}_{\mathcal{PA}}$, in $\{\mathcal{P}^{\star} - \mathcal{P}\}$,
but **is the descriptory *CLAIM***
of the $\mathcal{T}_{\mathcal{PA}}$ $[\mathbb{X}]$ consistent inference in $\mathcal{P}$.[26]

[26]**Also the Rao-Cramer inequality from the Mathematical Statistics documents it.**

For the channel $\mathcal{K}$, associated to the inferring TM, **is being interrupted infinitely,**[27] in any transfer act, **the TM and the $\mathcal{K}$ realize the Klein bottle run.**



But, **in fact we have**



We formulate the methodological error for $\mathcal{T}_{\mathcal{PA}}$

but *identifiable by the II. P.T.'s violation*, $\mathbf{\Delta S_{\mathcal{C}} < 0}$,

$$\underline{\mathbf{H(X) = H[Q(\mathbb{X}, Y)] = 0}}, \quad \mathbf{H(X|Y) \;\cong\; H(\mathbb{X}/\mathcal{T}_{\mathcal{PA}}) > 0}$$

$$\mathbf{H_{\mathcal{C}} \;=\; H(X) - H(X|Y) \;=\; \underline{0} - H(\mathbb{X}/\mathcal{T}_{\mathcal{PA}}) < 0}$$

The another form of the II. P.T. is formulated:

$$\mathbf{Q(\mathbb{X}, p)/\underline{17Gen\ r = "1"}}$$ [28]

[27]It is the reason for the **non expressibility of the consistency of the theory** $\mathcal{T}_{\mathcal{PA}}$ ***CLAIM*** - of the **axiom of its consistency expressing its general property** - in it itself. The **theory** $\mathcal{T}_{\mathcal{PA}}$ **does not contain this axiom as its own formula.**

[28]**The same methodolgical error represent the Epimenides liar and the Richard paradox - the property of a certain totality is not formulable within it as its own object part.**

12. Appendix 1 - Gödel Theorems

I. **Gödel theorem** (corrected semantically by [19] and [23]):[29]

For every ***recursive*** **and** ***consistent***

CLASS OF FORMULAE κ,[30]

and **outside** **this set**[31],

exists the true ("1") ***CLAIM*** **r**

$$[\mathbf{r} \notin \kappa,\ \mathbf{r} \notin \mathbf{Flg}(\kappa),\ \mathbf{r} \in \mathbf{Y},\ \text{card}\ \mathbf{Y} = \aleph_{\mathbf{1}}]$$

with a free ***VARIABLE*** **v that**

neither the ***CLAIM*** **vGen r**

nor the ***CLAIM*** **Neg(vGen r)**

belongs **to the set Flg**(κ)

$$[\mathbf{vGen\ r} \notin \mathbf{Flg}(\kappa)]\ \&\ [\mathbf{Neg}(\mathbf{vGen\ r}) \notin \mathbf{Flg}(\kappa)],$$

$$\mathbf{r} \stackrel{\triangle}{=} \mathbf{r}(\mathbf{X}) = \mathbf{Q}[\mathbf{X}, \forall_{\mathbf{x} \in \mathbb{X}} | \mathbf{Q}(\mathbf{x}, \mathbf{Y})],\quad \mathbf{17Gen\ r} \stackrel{\triangle}{=} \forall_{\mathbf{x} \in \mathbb{X}} | \mathbf{r}(\mathbf{X})$$

CLAIMS **vGen r** **and** **Neg(vGen r)**

are not κ-***PROVABLE***

the ***CLAIM*** **vGen r** ***is not*** κ-***DECIDABLE***.

[**They are elements of the formulating/syntactic meta-system** $\kappa^\star$**, inconsistent against** κ.]

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[29] **Gödel-Rosser theorem.**

[30] **Recursively axiomatizable and with the given set of the inference rules (Peano/Robinson arithmetics).** $\kappa = \mathcal{P}$ is from the **PM** - ***Principia Mathematica: B. Russel, L. Whitehead.*** 1910, 1912, 1913, 1927. , containing the Peano arithmetics $\mathcal{T}_{\mathcal{PA}}$. On the $\mathcal{T}_{\mathcal{PA}}$ the *real and complex number arithmetics* is based. The **PM** is the textitformal-syntax-logic-semantic base for the physical teories/hypotheses.

[31] **Far from "...**[PA-]**arithmetic and sentencial/*SENTENCIAL*"** and **far from (!) "In ...."**

$II.$ **Gödel theorem** (corrected semantically by [19] and [23]):[32]

If κ is an arbitrary *recursive* and *consistent*

***CLASS OF FORMULAE*,**

then any *CLAIM* y $[\mathbf{y} \in \mathbf{Y},\ \text{card}\ \mathbf{Y} = \aleph_1]$

saying that *CLASS* κ is consistent

must be constructed **outside** **this set**[33]

and for this fact, it **is not κ-*PROVABLE***[34]

The **consistency** of the ***CLASS OF FORMULAE*** κ
is **tested** by the ***relation*** **Wid**(κ)[35]

$$\mathbf{Wid}(\kappa)\ \sim\ (\mathbf{Ex})[\boldsymbol{CLAIM}(\mathbf{y})\ \&\ \overline{\mathbf{Proof}_{\kappa}(\mathbf{y})}]$$

The *FORMULAE* class κ is consistent

$\Leftrightarrow$

at least one κ-***UNPROVABLE CLAIM*** **y** exists.

Now $\mathbf{y} = \underline{\mathbf{17Gen\ r}} \notin \mathcal{P}/\mathcal{T}_{\mathcal{PA}},\ \kappa = \mathcal{T}_{\mathcal{PA}} \subset \mathcal{P} \subset \mathcal{P}^{\star}$

$$[[\mathbf{Proof}_{\mathcal{P}}(\underline{\mathbf{17Gen\ r}}) = \text{"}\mathbf{0}\text{"}] = \text{"}\mathbf{1}\text{"}]\ \equiv\ \mathbf{Wid}(\mathcal{T}_{\mathcal{PA}})$$
$$\cong$$
$$[[\mathbf{Proof}_{\mathfrak{L}}([\mathrm{d}]\mathbf{Q}_{\mathbf{Ext},\mathfrak{Q}_{\mathfrak{L}}}{=}\mathbf{0}) = \text{"}\mathbf{0}\text{"}] = \text{"}\mathbf{1}\text{"}]\ \equiv\ II.\ \mathbf{P.T.}$$

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[32] **Gödel-Rosser theorem.**

[33] **Far from "...[PA-]arithmetic and sentencial/*SENTENCIAL*"** and **far from (!) "In ...."**

[34] **Any attempt to prove/*PROVE/INFER it* in the system $\mathcal{P}/\kappa$ leads to the requirement for inconsistency of the consistent (!) system $\mathcal{P}/\kappa$ (in fact we are entering into the inconsistent meta-system $\mathcal{P}^{\star}$ - see the real sense of the Proposition V.).**

[35] Die ***Widerspruchsfreiheit - the Consistency.***

13. Appendix 2

Under the adiabaticity, [d]$\mathbf{Q_{Ext}=0}$, of the system $\mathfrak{L}$
it is not possible to derive* such a *CLAIM
that is stating this adiabatic supposition.

This ***CLAIM*** is **constructible not adiabaticaly**,
outside **the adiabatic $\mathfrak{L}$ only.**

Autoreference/***HALTING PROBLEM***

Self-Observation

- the *CLAIM* about adiabaticity of $\mathfrak{L}$ within $\mathfrak{L}$ -
- the *CLAIM* about consistency of $\mathcal{T}_{\mathcal{PA}}$ within $\mathcal{P}$ -
is excluded.

This is the nature law expressed

by the **Caratheodory formulation** of the *II.* **P.T.**

and by the **Gödel theorems' sense**.

The eye can not look at and into itself.

Any mixing of the various
observation/expressing/approach levels
leads to the paradoxes and is to be excluded.

Under the consistency of the system $\mathcal{P}$
it is not possible to derive* such a *CLAIM
that is stating this consistency supposition.

This ***CLAIM*** is **constructible *purely syntactically*,**
outside **the consistent $\mathcal{P}$ only** (in $\mathcal{P}^\star - \mathcal{P}$).[36]

[36]The Great Fermat's theorem is not inferrable within $\mathcal{P}/\mathcal{T}_{\mathcal{PA}}$ either - it is not of the $\mathcal{P}/\mathcal{T}_{\mathcal{PA}}$ type, although is arithmetical. In fact it is not a part of the $\mathcal{P}$ but of the $\mathcal{P}^\star - \mathcal{P}$, $\mathcal{P}^\star \cong \{\mathbf{p}, \mathbf{V}, \mathbf{T}\} = \mathbb{R}^3$.

(1).PDF

That's me or it is the picture of me - P1.

(2).PDF

This is the mirror picture of me - P2.

(3).PDF

Here I have 'ordered' the mirror picture P2 to step out from the mirror a stand, e.g., in front of me/P1 having the right hands overlapped.

CHAOS, EQUILIBRIUM, INFINITE CYCLE, PARADOX by mixing of various observation levels.

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