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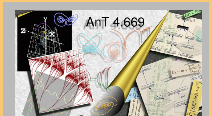

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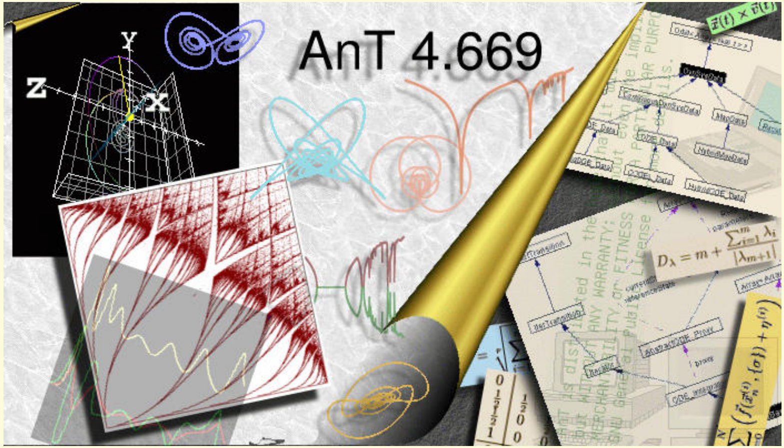

AnT 4.669 – a tool for simulating and investigating dynamical systems

Dr. Michael Schanz

University of Stuttgart

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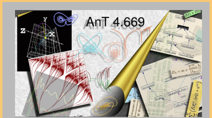

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1. Introduction

AnT 4.669 – a simulation and Analysis Tool for dynamical systems

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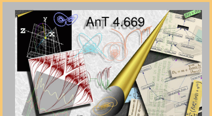

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1. Introduction

AnT 4.669 – a simulation and Analysis Tool for dynamical systems

AnT 4.669 application areas:

- science and education

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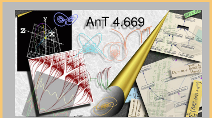

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1. Introduction

AnT 4.669 – a simulation and Analysis Tool for dynamical systems

AnT 4.669 application areas:

- science and education

AnT 4.669 capabilities:

- several classes of dynamical systems
- several investigation methods
- one-, two-, and higher dimensional scans
- distributed computation (grid computing)

AnT 4.669 properties:

- open software architecture
- GNU public license
- supported platforms
  Solaris, Linux, FreeBSD, Windows (98, NT, 2000, XP)

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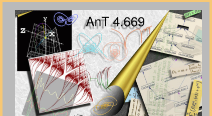

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AnT 4.669 was designed, is maintained and will be further developed by the Non–Linear Dynamics Group of the department Image Understanding (Head: Prof. Dr. P. Levi) at the Institute of Parallel and Distributed Systems (IPVS) of the University of Stuttgart.

Members of the group:

- Dr. Michael Schanz
- Dr. Viktor Avrutin
- Robert Lammert
- Georg Wackenhut

and about 25 students

History of the project:

1998:	first prototypes (FORTRAN, C)
2000:	**AnT 4.66** (C)
2001:	**AnT 4.669** (C++)

Current state:
$\approx$ 120 000 lines of source code

Application areas for dynamical systems:

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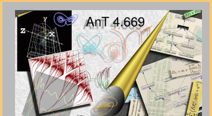

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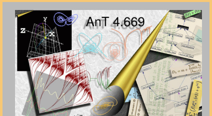

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Application areas for dynamical systems:

mathematics

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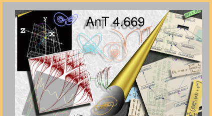

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Application areas for dynamical systems:

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physics | engineering | chemistry | biology

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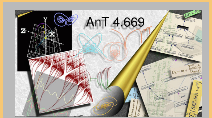

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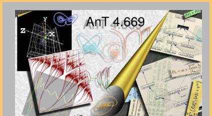

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Application areas for dynamical systems:

- mathematics
- physics
- engineering
- chemistry
- biology
- electronics
- computer science
- medicine
- . . .

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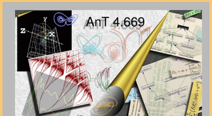

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Application areas for dynamical systems:

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. . .

modeling
⇓
simulation
⇓
analysis
⇓
interpretation

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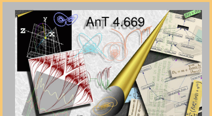

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analysis
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interpretation

investigation of the dynamic behavior

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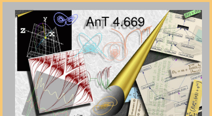

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analytic

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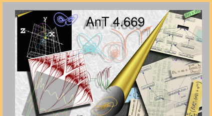

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Application areas for dynamical systems:

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simulation
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⇓
interpretation

investigation of the dynamic behavior

→ analytic

→ semi-analytic

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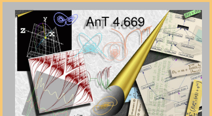

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Application areas for dynamical systems:

mathematics

physics | engineering | chemistry | biology

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...

modeling
⇓
simulation
⇓
analysis
⇓
interpretation

investigation of the dynamic behavior

analytic — semi-analytic — numeric

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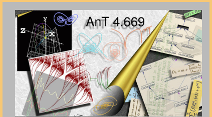

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⇓ ⇓

numerical simulation and Analysis Tools

are required

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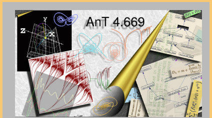

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Required knowledge and experience?
Involved areas of computer science?

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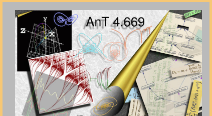

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Required knowledge and experience?
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- nonlinear dynamics

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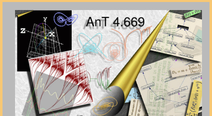

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- nonlinear dynamics
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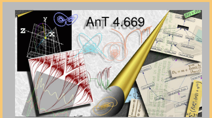

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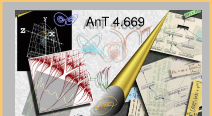

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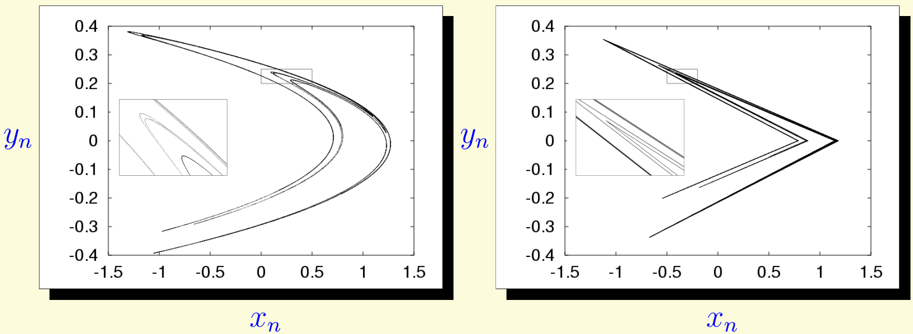
2. Dynamical systems

Attractors of the map–class

Generalized Hénon–Lozi map

$$x_{n+1} = 1 - a|x_n|^\gamma + y_n$$
$$y_{n+1} = bx_n$$

$a = 1.4, b = 0.3, \gamma = 2.0$ $a = 1.8, b = 0.3, \gamma = 1.0$



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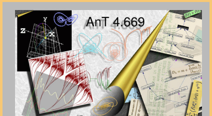

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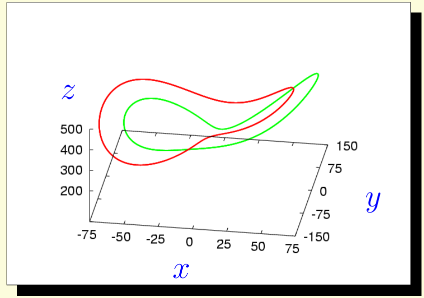
Attractors of the ODE–class

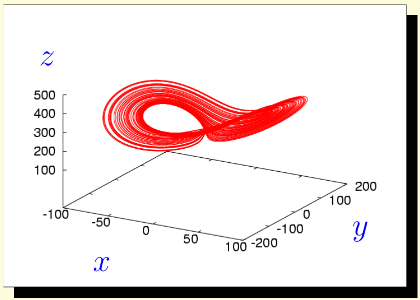
Lorenz 63 system

$$
\begin{aligned}
\dot{x} &= \sigma(y - x) \\
\dot{y} &= rx - y - xz \\
\dot{z} &= -bz + xy
\end{aligned}
$$

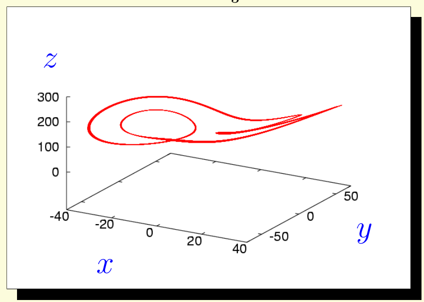
$\sigma = 16.0,\, r = 370.0,\, b = 4.0$



$\sigma = 16.0,\, r = 305.0,\, b = 4.0$



$\sigma = 10,\, r = 146.7981,\, b = \frac{8}{3}$



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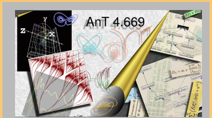

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Coexisting periodic attractors of the DDE–class

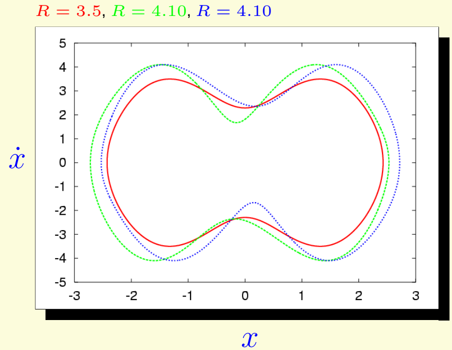
Phase Locked Loop (PLL) with time delay

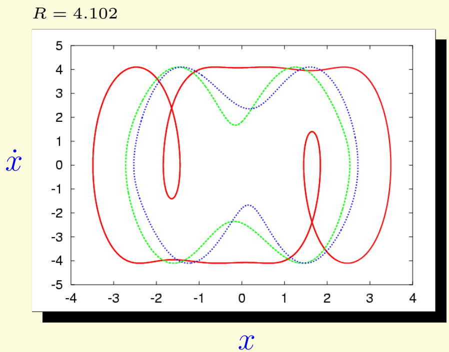
$$\dot{x}(t) \quad = \quad -R\sin(x(t-\tau))$$

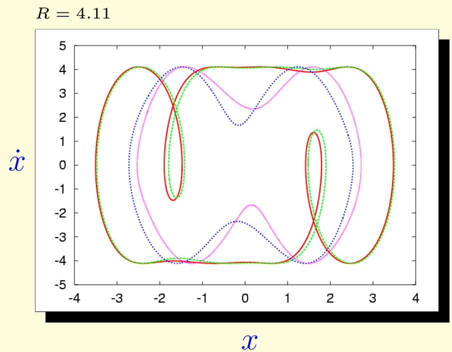
$\tau = 1.0$

four different constant initial functions

on the interval $[-\tau, 0]$:







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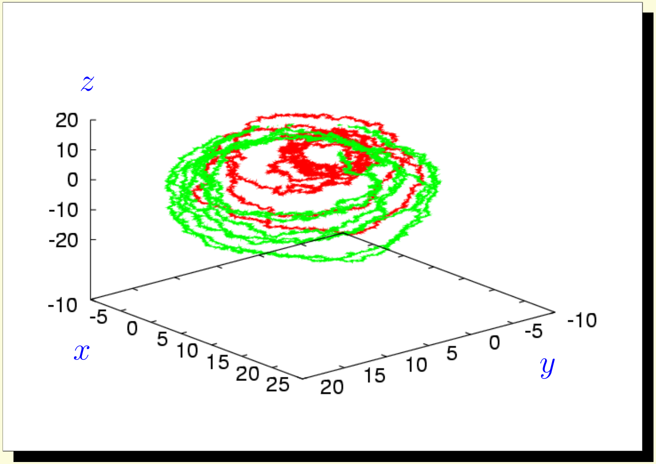
Dynamics of a stochastic system

Ornstein–Uhlenbeck process:

$$d\underline{x}_t = -\underline{\underline{M}}\,\underline{x}\,dt + \sigma\,d\underline{W}_t$$

$$\sigma = 1 \quad , \quad \underline{\underline{M}} = \begin{pmatrix} -10^{-4} & 0.1 & -0.2 \\ -0.1 & -10^{-4} & 0.2 \\ 0.5 & -0.5 & -10^{-4} \end{pmatrix}$$



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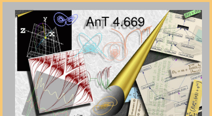

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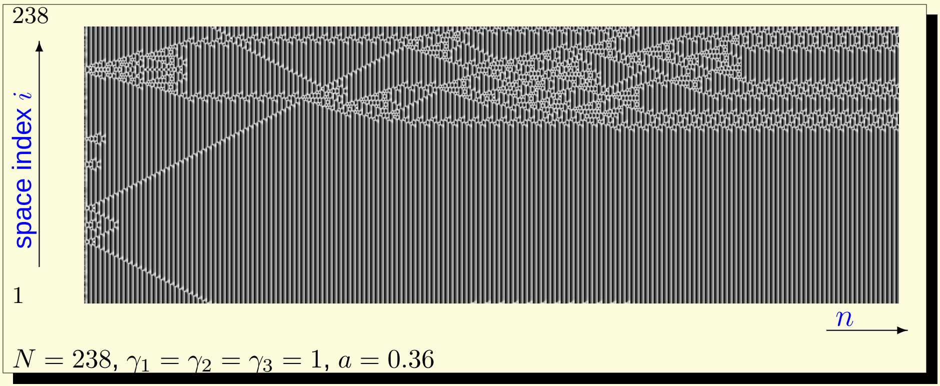
Transient and asymptotic dynamics of a coupled map lattice (CML)

Coupled piecewise–linear maps:

$$x^i_{n+1} = f(\varkappa^i_n) \qquad f(x) = \begin{cases} x + a & \text{if} \quad x < 1 \\ 0 & \text{if} \quad x \geq 1 \end{cases} \qquad i = 1..N$$

$$\varkappa^i_n = \frac{\gamma_1 \, x_n^{(i-1) \bmod N} + \gamma_2 \, x^i_n + \gamma_3 \, x_n^{(i+1) \bmod N}}{\gamma_1 + \gamma_2 + \gamma_3}$$



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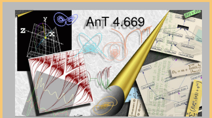
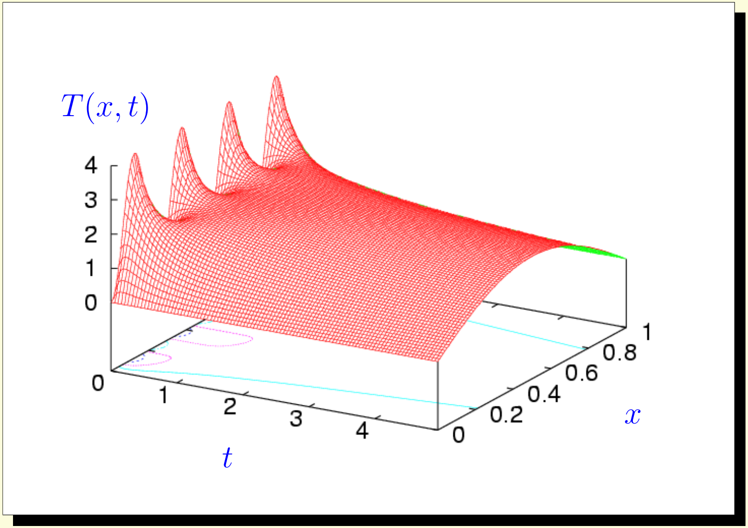


Transient dynamics of a partial differential equation (PDE)

Heat conduction equation:

$$\frac{\partial T(x,t)}{\partial t} = \kappa \frac{\partial^2 T(x,t)}{\partial x^2} \quad \text{with} \quad \kappa = 0.01$$



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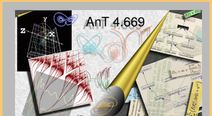

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Classes of dynamical systems supported by AnT 4.669

- basic
  - map
  - ODE
  - DDE
  - FDE
- composite
  - CML
  - CODEL
  - 1D–PDE
- hybrid
  - hybrid map
  - hybrid ODE
  - hybrid DDE
- stochastic
  - stochastic map
  - stochastic ODE
  - stochastic DDE
- etc.
  - recurrent map
  - external data

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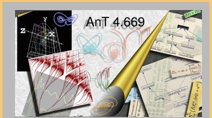

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$\Longrightarrow$ Support of 15 different classes of dynamical systems

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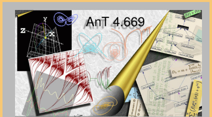

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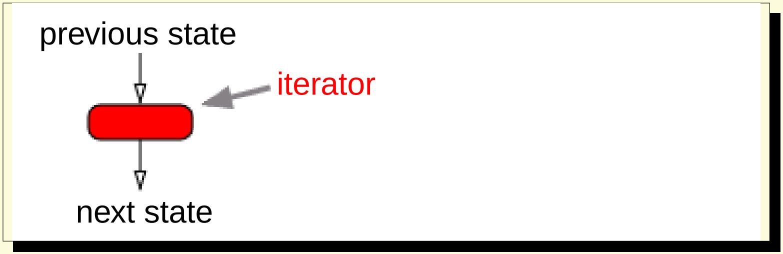
Support of different types of dynamical systems is possible due to the general concept of an abstract iterator, which is a special kind of an abstract transition:



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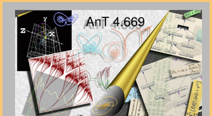

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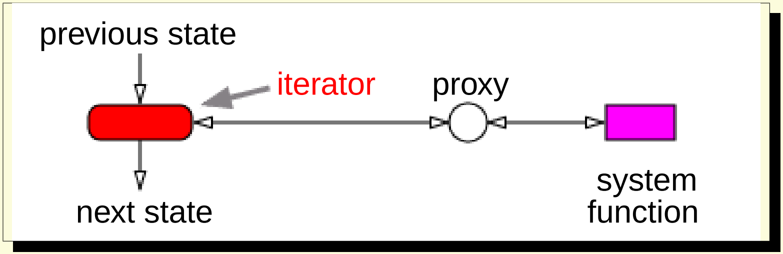

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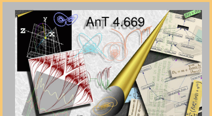

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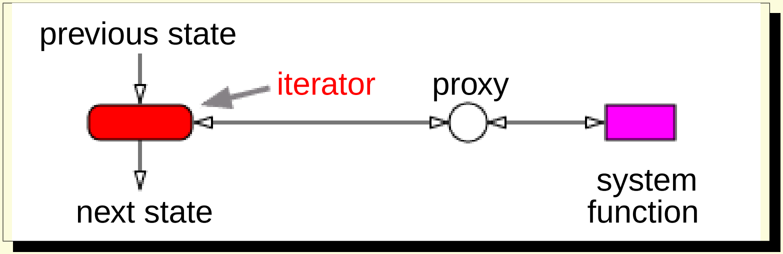

Dependent on the current type of the dynamical system the abstract iterator can be instantiated as:

- simple iterator
  (for maps, CMLs, Poincaré maps, external data input, etc.)
- ODE integrator (for ODEs, CODELs, 1D-PDEs)
- DDE integrator (for DDEs, CDDELs)
- FDE integrator (for FDEs)
- ...

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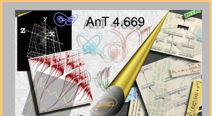

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Integration methods for ODEs, DDEs and FDEs, supported by AnT 4.669

one–step steppers

- Euler (expl., impl.)
- Heun (expl., impl.)
- Midpoint
- Radau
- Ralston
- Runge–Kutta
- Gill
- Runge–Kutta–Merson
- Runge–Kutta–Fehlberg
- Butcher
    - pre–defined arrays
    - user–defined arrays

multi-step steppers

- Adams–Bashforth (6)
- Adams–Moulton (6)
- BDF (6)
- PECE–AB–AM (6×6)
- PECE–AB–BDF (6×6)

wrappers

- basic, backward
- step size adaption
    - gradient based
    - halfstep
    - two steppers

$$\Longrightarrow \quad \left.\begin{array}{c} 70 \\ \approx 2000 \\ \ldots \end{array}\right\} \text{different integration methods}$$

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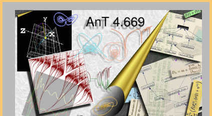

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Remarks on numerical integration I

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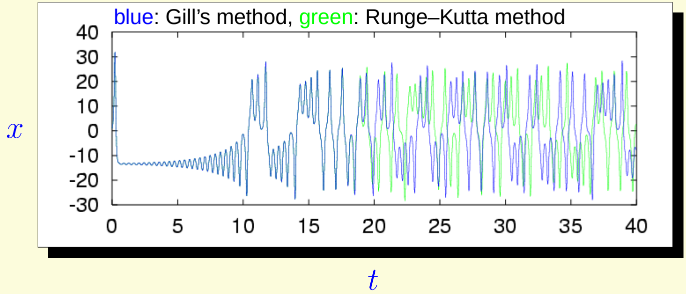
Remarks on numerical integration I

Two time series of the Lorenz 63 system for identical initial conditions calculated with two different integration methods:



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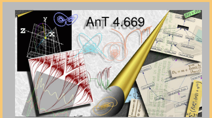

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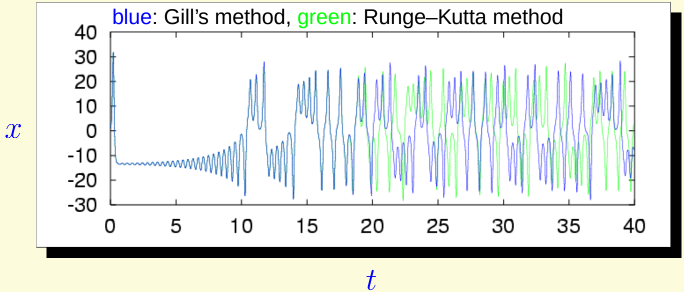
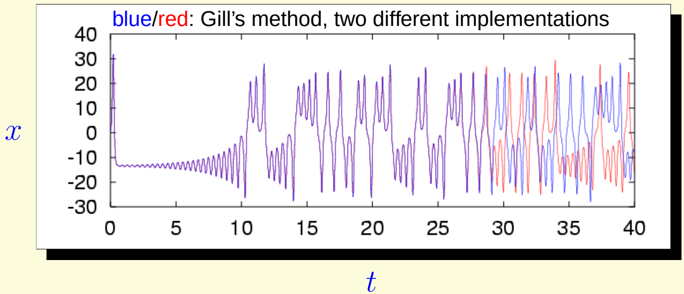


Two time series of the Lorenz 63 system for identical initial conditions calculated with the same integration method:



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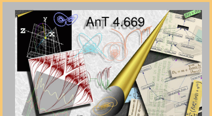

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Remarks on numerical integration II

Numerical solution of the circular co–planar restricted three body problem:

$$\begin{aligned} \ddot{x} &= x + 2\,\dot{y} - (1-\mu)\frac{x+\mu}{{r_1}^3} - \mu\frac{x+\mu-1}{{r_2}^3} \\ \ddot{y} &= y + 2\,\dot{x} - (1-\mu)\frac{y}{{r_1}^3} - \mu\frac{y}{{r_2}^3} \\ r_1 &= \left[(x+\mu)^2 + y^2\right]^{\frac{1}{2}} \\ r_2 &= \left[(x+\mu-1)^2 + y^2\right]^{\frac{1}{2}} \end{aligned}$$

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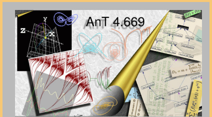

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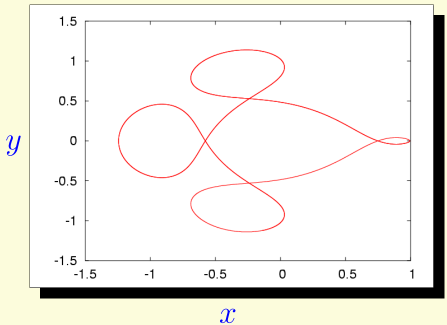
\end{aligned}
$$



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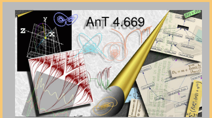

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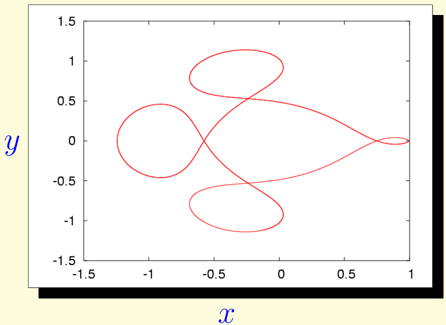

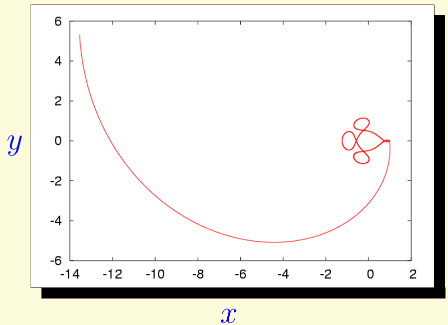

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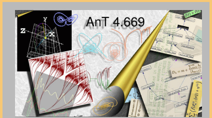

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3. Investigation methods

Investigation methods supported by AnT 4.669

- General trajectory evaluations
  – orbits, velocities, extreme values, cobweb diagrams
- Basic statistics
  – mean values, standard deviations, cross–correlations
- Box counting methods
  – invariant measures, fractal dimensions
- Lyapunov exponents analysis
  – for maps, CMLs, ODEs, DDEs, FDEs, hybrid systems
- Extended Poincaré sections and Poincaré return maps
- Period analysis (systems discrete in time)
- Region analysis (based on period analysis)
- Spectral analysis
- Condition checker
- Principal component analysis
- Symbolic sequence analysis
  – symbolic entropies for an arbitrary description level
- Symbolic image analysis
  – detection of invariant sets, basins of attraction,
  – calculation of stable and unstable manifolds

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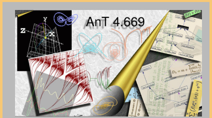

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Introduction
Dynamical systems
Investigation methods
Scans
AnT 4.669 *features*
Summary
Outlook

Examples for several investigation methods

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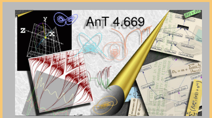

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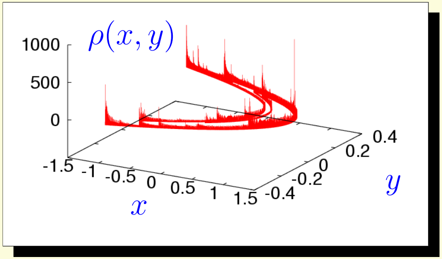
Examples for several investigation methods

Natural measure of a chaotic attractor



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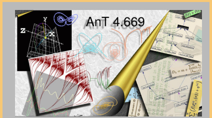

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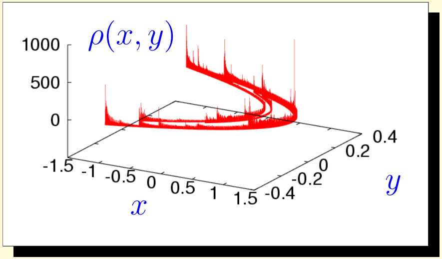
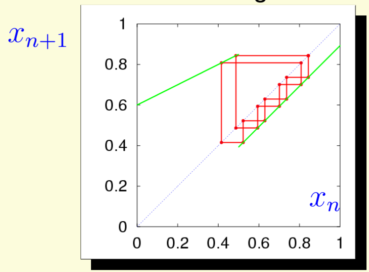


Cobweb diagram



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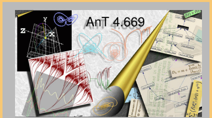

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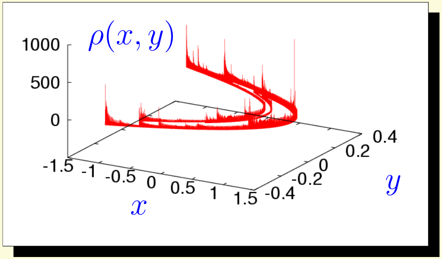
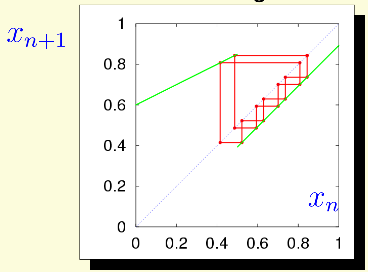
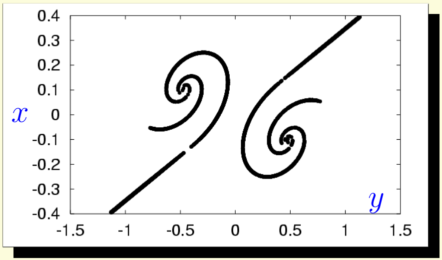


Cobweb diagram



Poincaré section of a chaotic attractor



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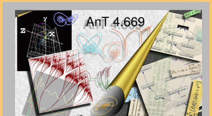

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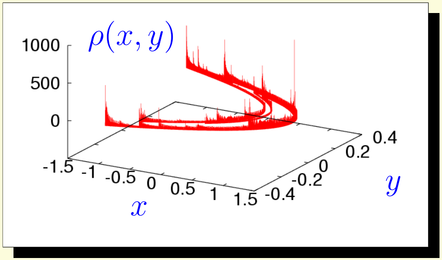
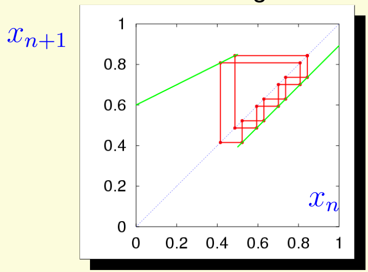
Examples for several investigation methods

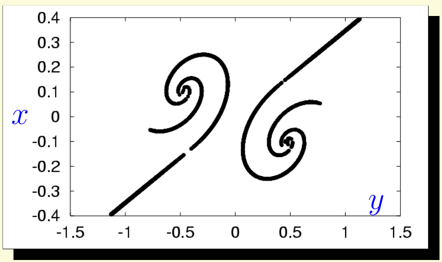
Natural measure of a chaotic attractor



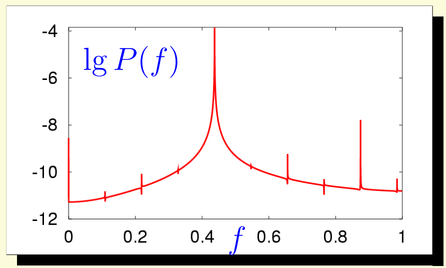
Cobweb diagram



Poincaré section of a chaotic attractor



Power spectrum of a limit cycle



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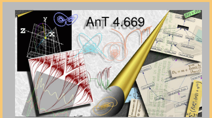

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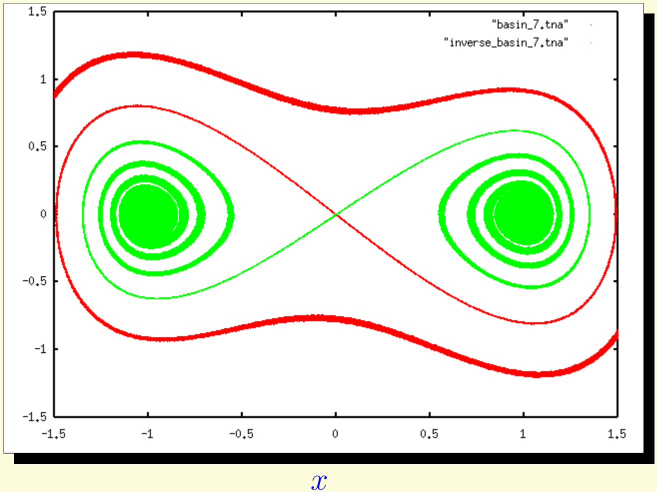
Duffing oscillator:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= x - x^3 - \varepsilon y \end{aligned}$$

$\varepsilon = 0.15$

stable and unstable manifolds of the fixed point at the origin



In cooperation with D. Fundinger and G.Osipenko
State Polytechnic University, St. Petersburg, Russia

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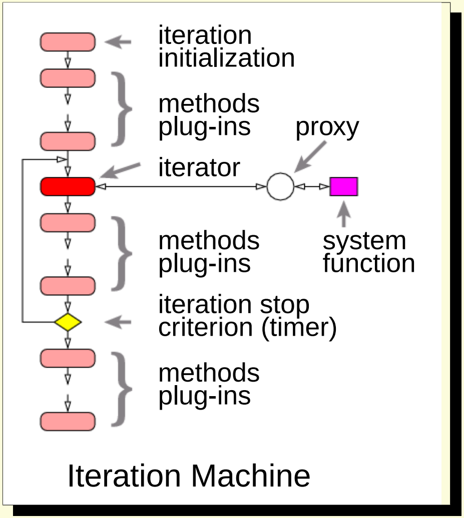
Basic concept: Iteration Machine



Structure: pre-sequence, cyclic sequence, post-sequence

Contents: iterator, iteration method plug-ins

Setup: dynamically during initialization phase

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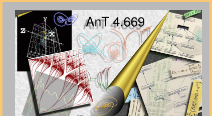

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4. Scans

Scans supported by AnT 4.669

Scannable objects:

▶ system parameters	bifurcation scenarios, regions with different behavior
▶ initial values	coexisting objects, basins of attraction
▶ method parameters	method tuning

Scan types:

- real, integer
- linear, logarithmic
- using external data
- parametric (linear, elliptic)

Scan item sequences $\Rightarrow$ N–dimensional scans.

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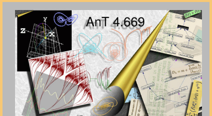

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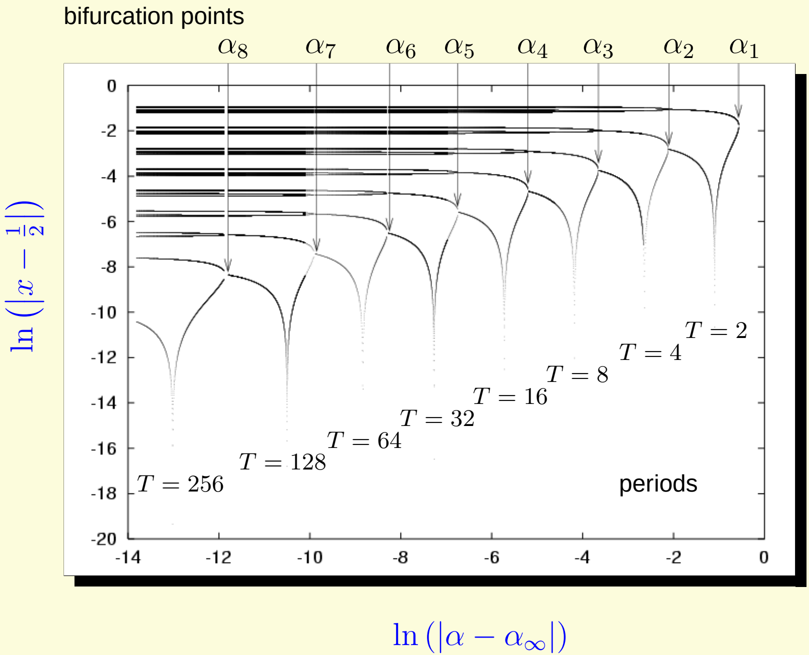
Parameter scan (1D)

logistic map: $x_{n+1} = \alpha x_n (1 - x_n)$



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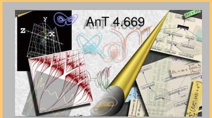

Initial value scan (2D)

Gingerbreadman map:

$$
\begin{aligned}
x_{n+1} &= 1 - y_n + |x_n| \\
y_{n+1} &= x_n
\end{aligned}
$$

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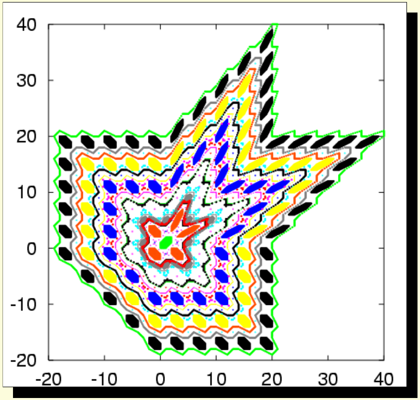
Initial value scan (2D)

Gingerbreadman map:

$$x_{n+1} = 1 - y_n + |x_n|$$
$$y_{n+1} = x_n$$

period analysis



y_0

x_0

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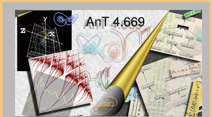

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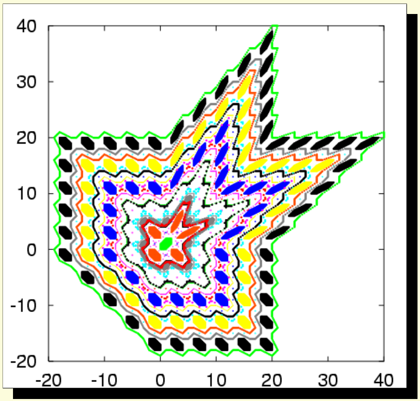
$$y_{n+1} = x_n$$

period analysis

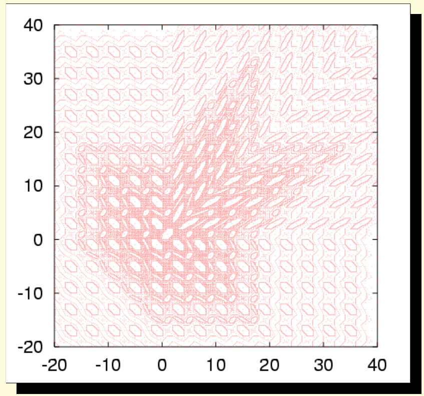
y_0



x_0

region analysis

y_0



x_0

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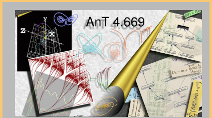

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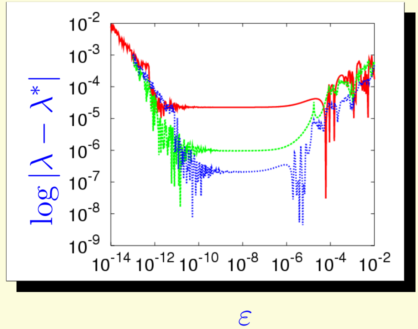
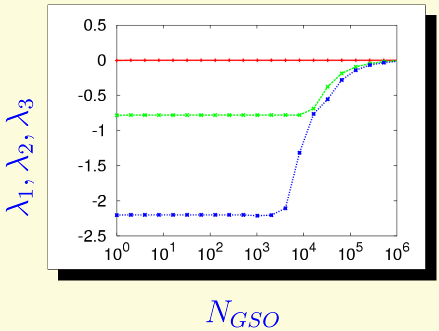
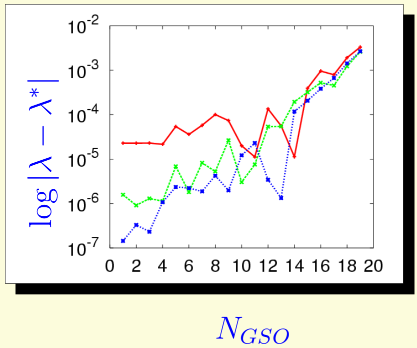
Scans of investigation method parameters

Parameter tuning for an investigation method: Lyapunov Exponents.

- norm ε of deviation vectors
- number N_{GSO} of steps between two Gram Schmidt orthonormalizations







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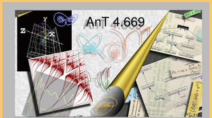

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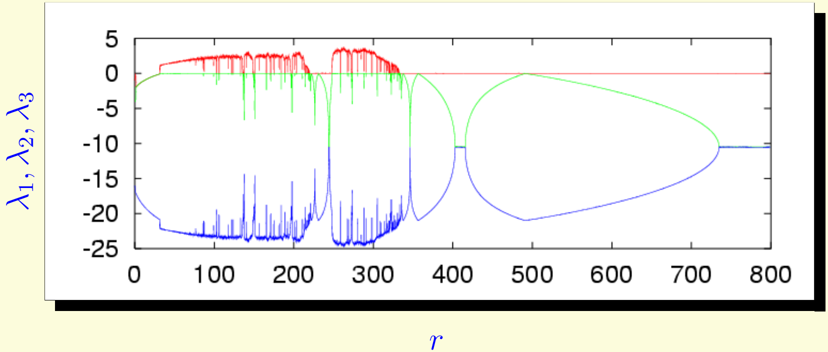
The three Lyapunov exponents of the Lorenz 63 system:



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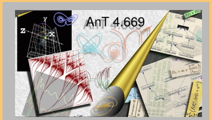

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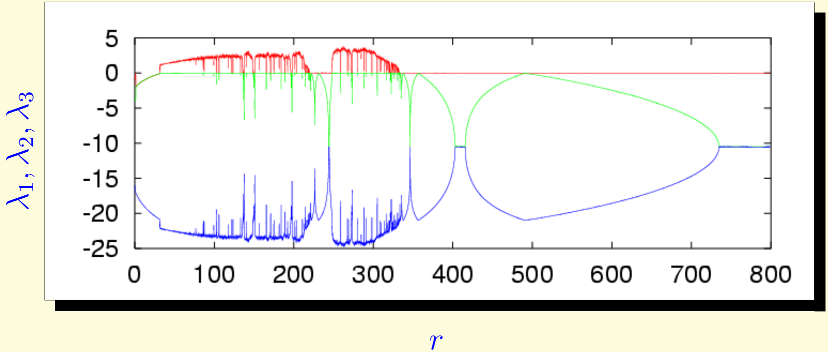
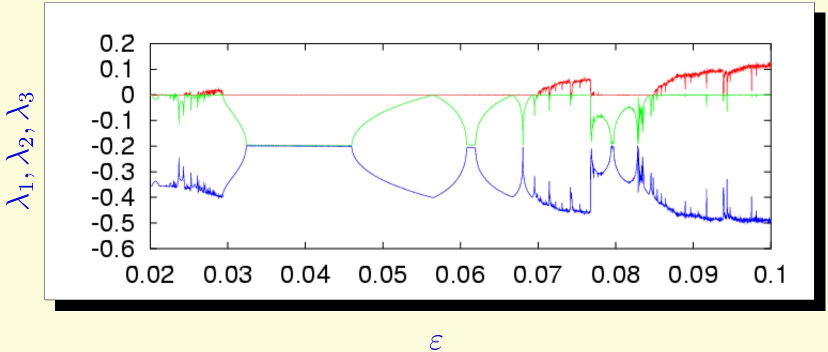


The three Lyapunov exponents of the Aizawa system:



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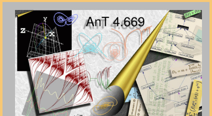

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Example for a 2D parameter scan

Period Adding Big Bang Bifurcation in a piecewise-linear map

$$x_{n+1} = \begin{cases} f_l(x_n) = bx_n + c & \text{if } x_n < \frac{1}{2} \\ f_r(x_n) = x_n - a & \text{if } x_n \geq \frac{1}{2} \end{cases}$$

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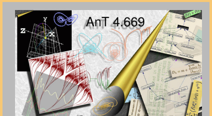

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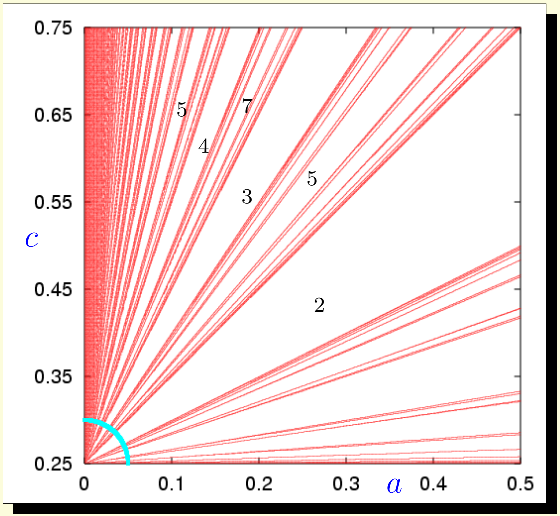
$b = \frac{1}{2}$, 2D parameter space $[a \times c]$



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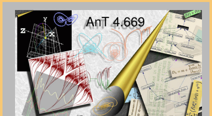

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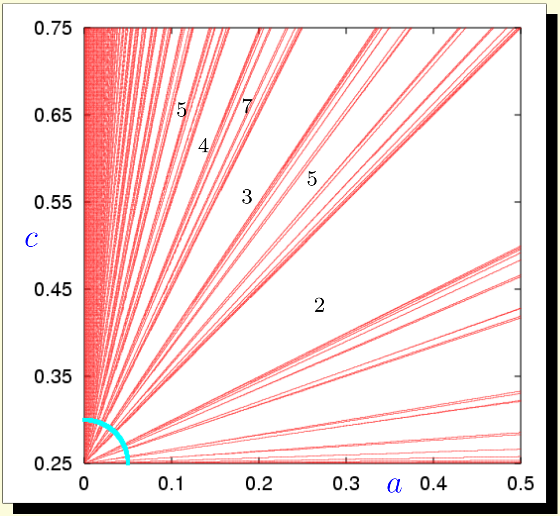
Example for a 2D parameter scan

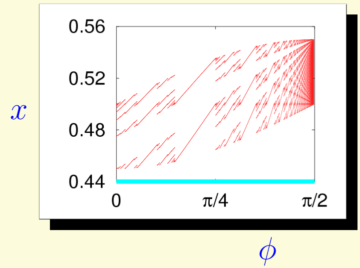
Period Adding Big Bang Bifurcation in a piecewise-linear map

$$x_{n+1} = \begin{cases} f_l(x_n) = bx_n + c & \text{if } x_n < \frac{1}{2} \\ f_r(x_n) = x_n - a & \text{if } x_n \geq \frac{1}{2} \end{cases}$$

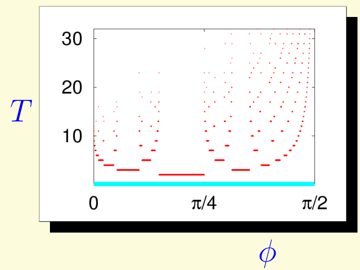
$b = \frac{1}{2}$, 2D parameter space $[a \times c]$



Bifurcation diagram



Period diagram



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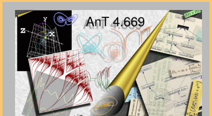

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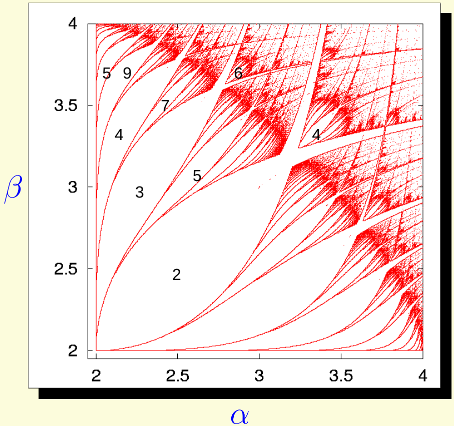
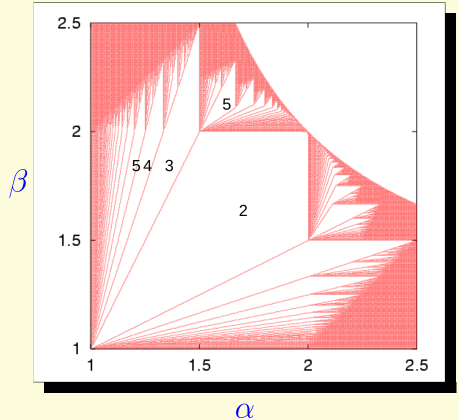
Example for a 2D parameter scan

2D bifurcation scenarios, induced by Big Bang Bifurcations



2D period adding scenario



2D period increment scenario

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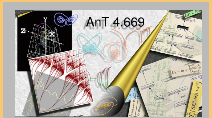

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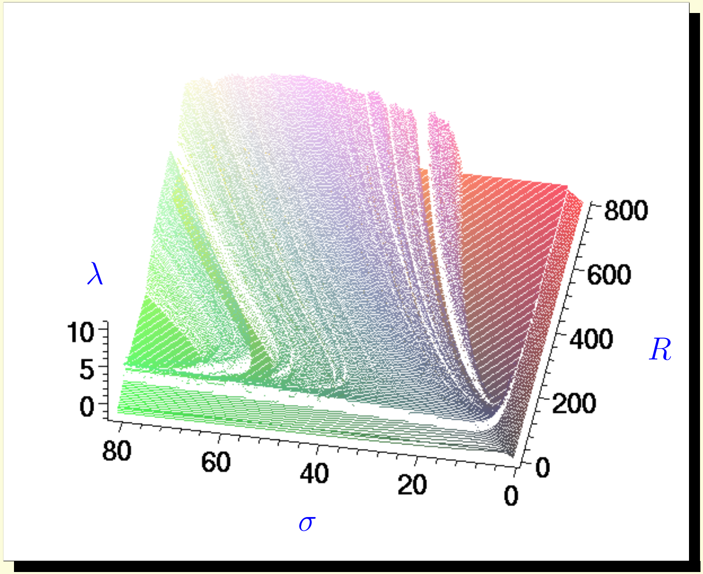
Example of a 2D parameter scan

Largest Lyapunov exponent of the Lorenz 63 system:



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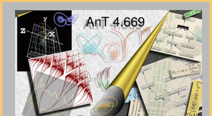

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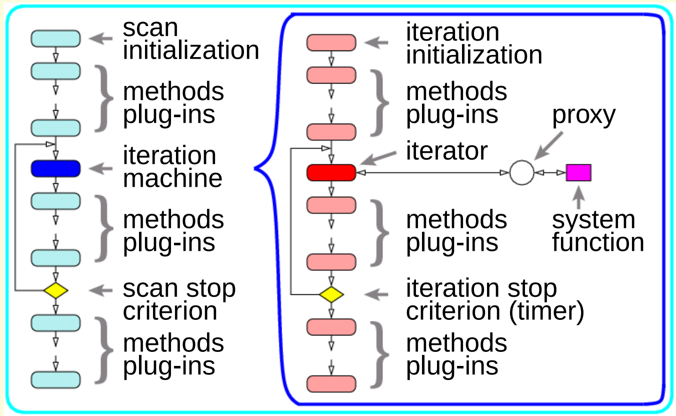
Basic concept: Scan Machine



Scan Machine Iteration Machine

Structure: pre-sequence, cyclic sequence, post-sequence

Contents: Iteration Machine, scan method plug-ins

Setup: dynamically during initialization phase

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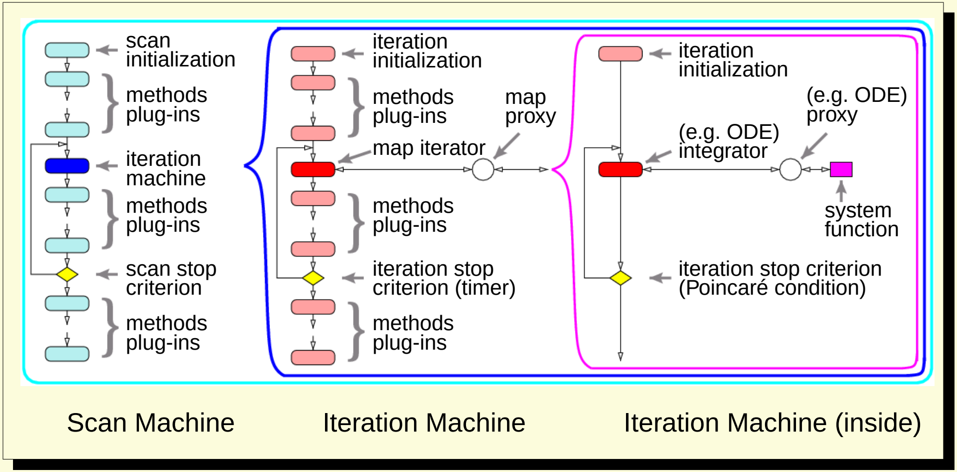
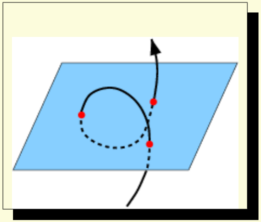
Scan Machine for Poincaré Maps



The system function of a Poincaré map is given by a complete iteration machine containing a dynamical system inside.

The generalized Poincaré condition defines the stop criterion of the iteration machine inside.



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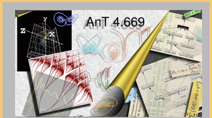

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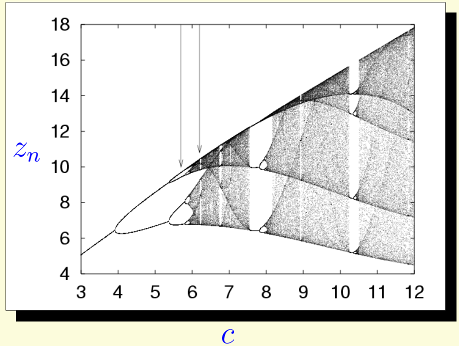
Examples of scans of a Poincaré map (ODE)

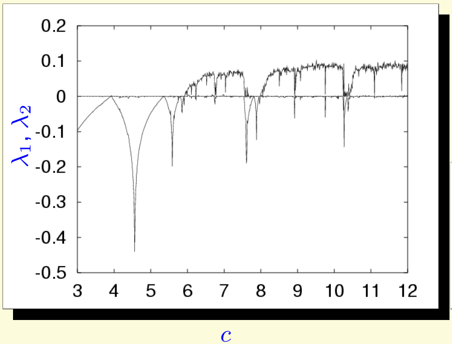
Rössler system:

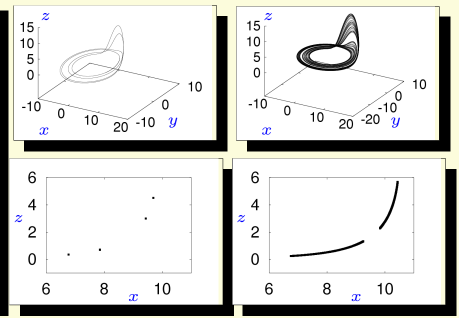
$$\begin{aligned}\dot{x} &= -(x+z)\\ \dot{y} &= x+ay\\ \dot{z} &= b+z(x-c)\end{aligned}$$

$a = 0.15$, $b = 0.2$

Poincaré section using the fixed half–plane $\{(x,y,z)^T \,|\, y=0,\, x>0\}$







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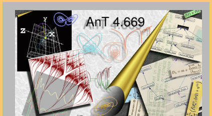

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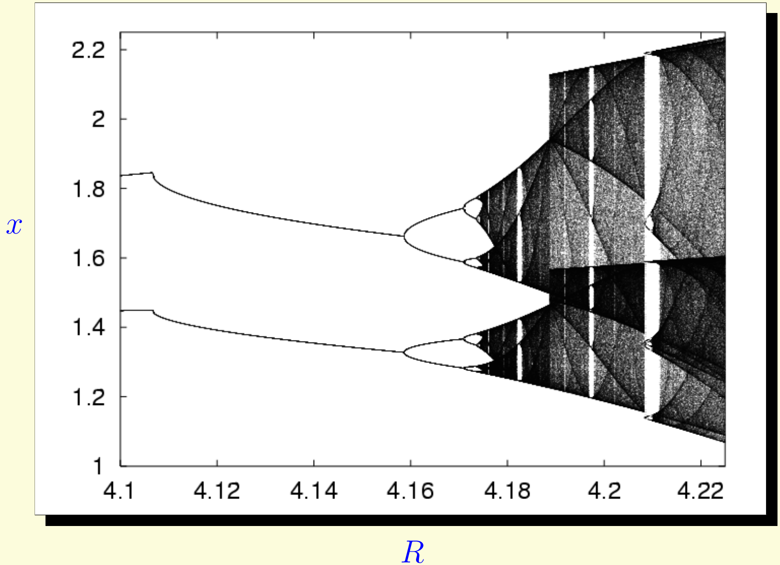
Examples of scans of a Poincaré map (DDE)

PLL system with delay: $\dot{x}(t) = -R \sin(x(t-\tau))$
Poincaré section using the condition defined by:
$\dot{x} = 0$ and $x \in [1, 2]$



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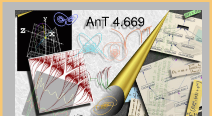

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5. AnT 4.669 features

Grid computing

Distribution of a scan among several nodes

- Client/Server architecture
- The server distributes tasks and manages calculation results
- An arbitrary number of clients perform the calculations
- Adding and removing of clients on-the-fly
- Data are sent/received via TCP/IP socket connections
- platform independence $\Rightarrow$ running of server and clients in a heterogeneous environment

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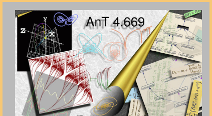

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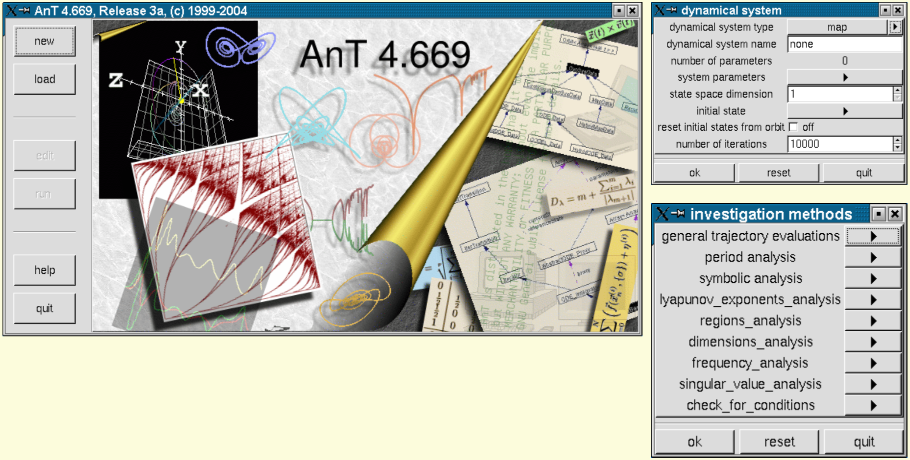
User Interface



- simplification of the complex initialization phase
- specification induced, automatic widget creation
- extendable for multi-lingual support

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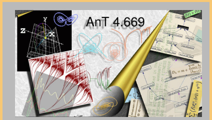

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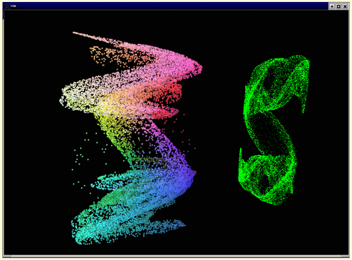
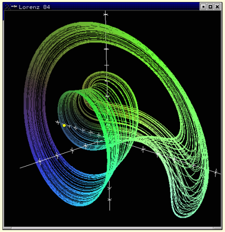
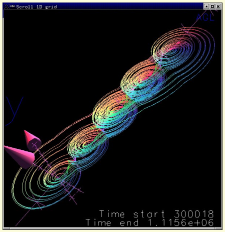
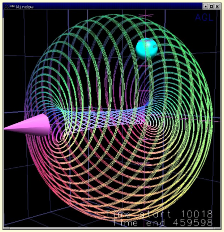
Visualization

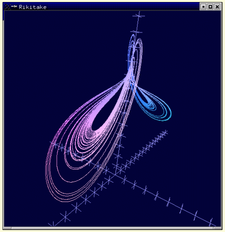
- time series, space-time plots, phase portraits
- translation, scaling, rotation
- multiple views
- based on OpenGL standard

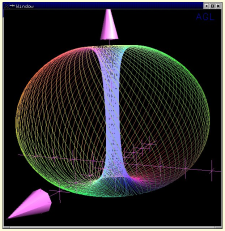


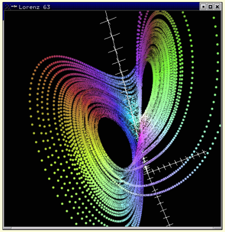


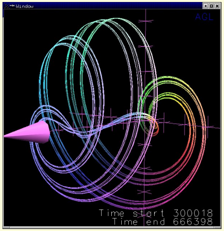


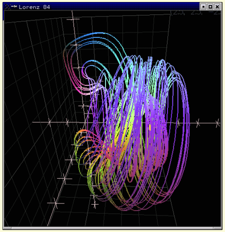












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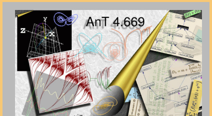

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Web interface

Challenge: **AnT 4.669** is designed as a desktop–application

Architecture:

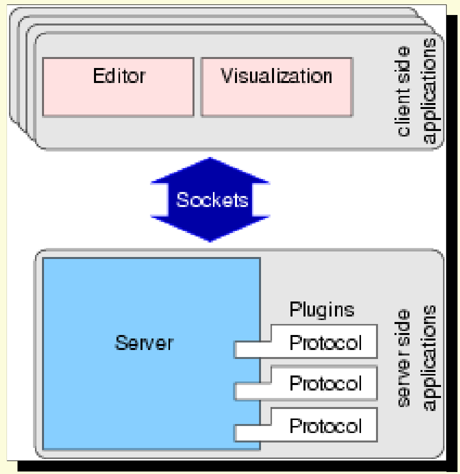
- Separation between the computation engine and the graphical user interface

Target solution:

- The computation engine is on the server side
- Configuration input and visualization are on the client side

Three new applications:

- Server
- Configuration editor
- Visualization client



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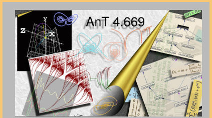

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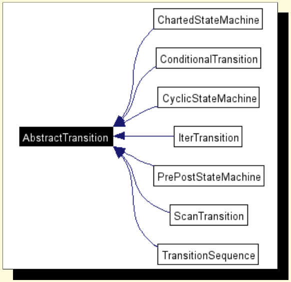
6. Summary

Transition concept



Reasons for building transition subclasses:

1. creating new transition structures
2. implementing specific functionality

The transition concept is used in **AnT 4.669** extensively.

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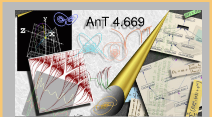

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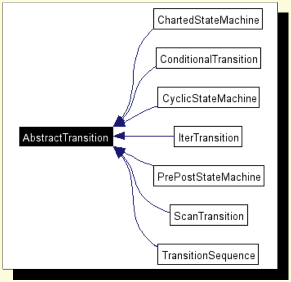

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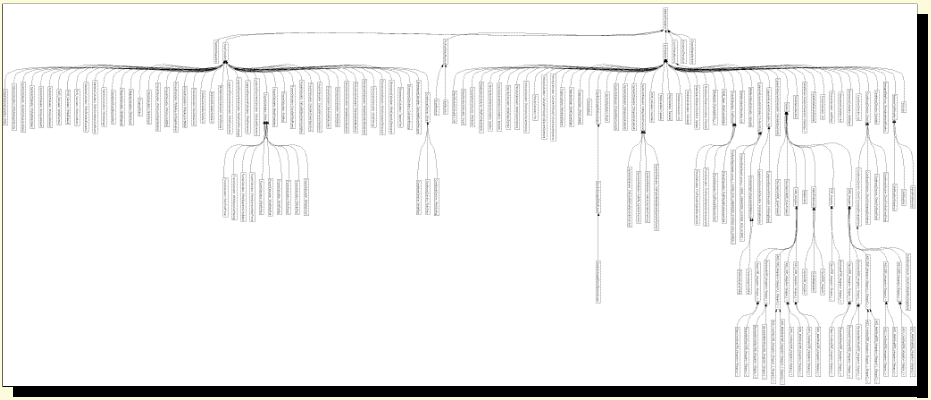

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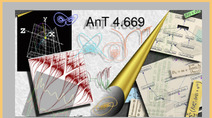

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Involved areas of computer science?

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- numerics
- scientific computing

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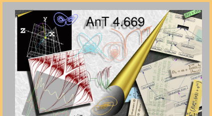

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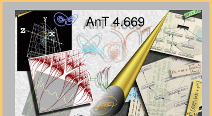

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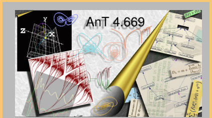

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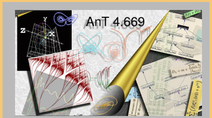

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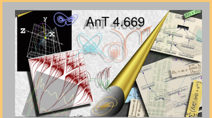

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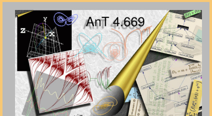

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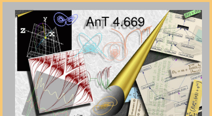

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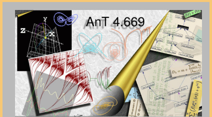

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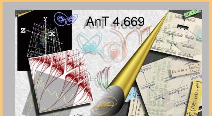

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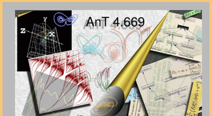

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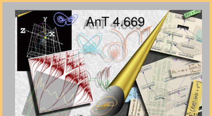

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7. Outlook

- Extension of integration methods:
  - usage of third party ODE and DDE integrators
  - implementation of symplectic integrators

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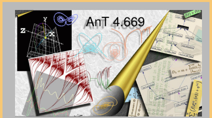

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- Extension of PDE solvers
  - Implementation of 2D–PDEs
  - Implementation of adaptive grid methods for PDEs

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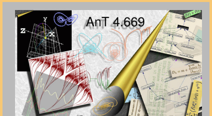

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7. Outlook

- Extension of integration methods:
  - usage of third party ODE and DDE integrators
  - implementation of symplectic integrators
- Extension of PDE solvers
  - Implementation of 2D–PDEs
  - Implementation of adaptive grid methods for PDEs
- Extension of investigation methods:
  - continuation method
  - local divergence rates
  - . . .

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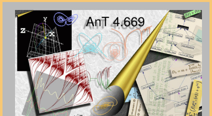

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  - ’attractor flight’
  - more sophisticated coloring schemes

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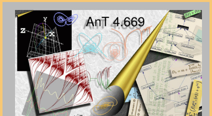

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  - more sophisticated coloring schemes
- Implementation of new system classes
  - DAEs, ImDEs, IDEs, PIDEs and PDDEs

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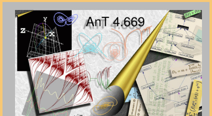

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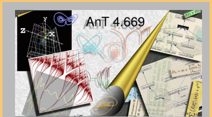

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System function implementation

Example of a system function for an ODE

```
#define sigma parameters[0]    #define X currentState[0]
#define r     parameters[1]    #define Y currentState[1]
#define b     parameters[2]    #define Z currentState[2]

bool lorenz63
 (const Array<real_t>& currentState,
  const Array<real_t>& parameters,
        Array<real_t>& rhs)
{
  rhs[0] = sigma * (Y - X);
  rhs[1] = X * (r - Z) - Y;
  rhs[2] = - b * Z + X * Y;

  return true;
}

extern "C"
{ void connectSystem ()
  {  ODE_Proxy::systemFunction = lorenz63; }
}
```

$$\dot{x} = f(x, p)$$

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= -bz + xy \end{aligned}$$