① $a_n = n \cdot \left(-\frac{1}{n}\right)^{\sin\frac{n\pi}{2}}$ 1

$\left\{ \sin\frac{n\pi}{2} \right\}_{n=1}^{\infty} = \{1, 0, -1, 0, 1, 0, -1, 0, \dots\}$

Vybrané posl. – sudé indexy $n = 2k \Rightarrow a_{2k} = n \cdot \left(-\frac{1}{n}\right)^0 = n \to \infty$

– indexy $n = 4k+1 \Rightarrow a_{4k+1} = n \cdot \left(-\frac{1}{n}\right)^1 = -1 \to -1$

– indexy $n = 4k+3 \Rightarrow a_{4k+3} = n \cdot \left(-\frac{1}{n}\right)^{-1} = -n^2 \to -\infty$

Hromadné body jsou $-1, +\infty, -\infty$ (0.9)

$\liminf\limits_{n\to\infty} a_n = -\infty$ (nejmenší hromadný bod)

$\limsup\limits_{n\to\infty} a_n = +\infty$ (největší hromadný bod) (0.1)

(1.0)

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② $F'(x) = \left[\frac{1}{2} x \sqrt{x^2-4} - 2 \ln\left(x + \sqrt{x^2-4}\right)\right]'$

$= \frac{1}{2}\sqrt{x^2-4} + \frac{1}{2}x \cdot \frac{\cancel{2}x}{2\sqrt{x^2-4}} - \frac{2}{x+\sqrt{x^2-4}} \cdot \left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-4}}\right)$

$= \frac{1}{2}\sqrt{x^2-4} + \frac{x^2}{2\sqrt{x^2-4}} - \frac{2 \cdot \frac{\cancel{\sqrt{x^2-4}+x}}{\sqrt{x^2-4}}}{\frac{\cancel{x+\sqrt{x^2-4}}}{1}}$

$= \frac{1}{2}\sqrt{x^2-4} + \frac{x^2}{2\sqrt{x^2-4}} - \frac{2}{\sqrt{x^2-4}}$

$= \frac{1}{2}\sqrt{x^2-4} + \frac{x^2-4}{2\sqrt{x^2-4}} = \frac{1}{2}\sqrt{x^2-4} + \frac{\sqrt{x^2-4}}{2} = \underline{\underline{\sqrt{x^2-4}}}$

$\Rightarrow F'(x) = f(x) \quad \forall x \in (2, \infty) \Rightarrow$ $\boxed{F(x) \text{ je primitivní k } f(x) \text{ na } (2, \infty)}$