3/

③ $\lim_{x\to\infty} (1-e^{-x})^{e^{2x}} = \lim_{x\to\infty} e^{\ln(1-e^{-x})^{e^{2x}}}$

$= e^{\lim_{x\to\infty} e^{2x}\ln(1-e^{-x})} = \boxed{e^{0} = 1}$ (0.6)

neboť je

$\lim_{x\to\infty} e^{2x}\ln(1-e^{-x})$ |typ $\infty\cdot 0$| $= \lim_{x\to\infty} \frac{\ln(1-e^{-x})}{e^{-2x}}$ |typ $\frac{0}{0}$|

$\overset{l'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1-e^{-x}}\cdot(e^{-x})\cdot(-1)}{e^{-2x}\cdot(-2)} = \lim_{x\to\infty} \frac{-e^{2x}\cdot e^{-x}}{(1-e^{-x})\cdot 2}$

$= \lim_{x\to\infty} \frac{-(e^{x})}{2(1-e^{-x})}$ |typ $\frac{-\infty}{2}$| $= -\infty$

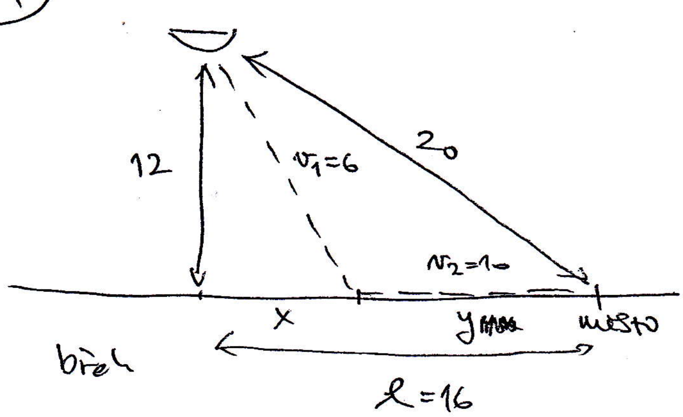
(0.4)

$\lim_{x\to 0} \frac{x}{\sin^3 x}$ |typ $\frac{0}{0}$| $= \lim_{x\to 0} \frac{1}{3\sin^2 x\cos x}$ |typ $\frac{1}{0^+}$| $= +\infty$

(1.0)

---

④



$l = \sqrt{20^2-12^2} = \sqrt{400-144} = \sqrt{256} = 16$

$y = 16 - x$

$s_1 = \sqrt{12^2+x^2} = \sqrt{144+x^2}$

$t_1 = \frac{s_1}{v_1} = \frac{\sqrt{144+x^2}}{6}$

$s_2 = y$, $t_2 = \frac{s_2}{v_2} = \frac{y}{10} = \frac{16-x}{10}$

$\boxed{t = t_1 + t_2 = \frac{\sqrt{144+x^2}}{6} + \frac{16-x}{10}}$ → min pro $x \in [0,16]$ (0.6)