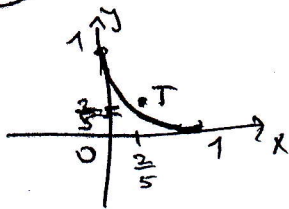


(7)

Asteroides

$$x(t) = \cos^3 t, \quad y = \sin^3 t, \quad t \in [0, \pi/2]$$



$$x' = 3 \cos^2 t (-\sin t)$$

$$y' = 3 \sin^2 t \cos t$$

$$H = d = \int_0^{\pi/2} \sqrt{x'^2 + y'^2} dt = \int_0^{\pi/2} \sqrt{9 \cos^4 t \sin^2 t + 9 \sin^4 t \cos^2 t}$$

$$= \int_0^{\pi/2} 3 \cos t \sin t dt = \frac{3}{2} \int_0^{\pi/2} \sin 2t dt$$

$$= \frac{3}{2} \left[\frac{-\cos 2t}{2} \right]_0^{\pi/2} = \frac{3}{2} \left[\frac{-\cos \pi}{2} + \frac{1}{2} \right] = \frac{3}{2}$$

$$S_x = \int_0^{\pi/2} y \sqrt{x'^2 + y'^2} dt = \int_0^{\pi/2} \sin^3 t \cdot 3 \cos t \sin t dt = 3 \int_0^{\pi/2} \sin^4 t \cos t dt$$

$$\left. \begin{array}{l} u = \sin t \\ du = \cos t dt \\ t=0 \Rightarrow u=0 \\ t=\pi/2 \Rightarrow u=1 \end{array} \right| = 3 \int_0^1 u^4 du = 3 \left[\frac{u^5}{5} \right]_0^1 = \frac{3}{5} \quad (0.6)$$

$$S_y = \int_0^{\pi/2} x \sqrt{x'^2 + y'^2} dt = \int_0^{\pi/2} \cos^3 t + 3 \cos t \sin t dt = 3 \int_0^{\pi/2} \cos^4 t \sin t dt$$

$$\left. \begin{array}{l} u = \cos t \\ du = -\sin t dt \\ t=0 \Rightarrow u=1 \\ t=\pi/2 \Rightarrow u=0 \end{array} \right| = 3 \int_1^0 u^4 (-du) = 3 \int_0^1 u^4 du = \frac{3}{5} \quad (0.6)$$

$$\frac{S_x}{H} = \frac{S_y}{H} = \frac{\frac{3}{5}}{\frac{3}{2}} = \frac{6}{15} = \frac{2}{5} \Rightarrow T = \left[\frac{2}{5}, \frac{2}{5} \right]$$

(0.2)

(2.0)