1) Načrtněte definiční obor funkce

$f(x,y) = \arcsin \frac{x}{y^2} + \arcsin(1-y)$

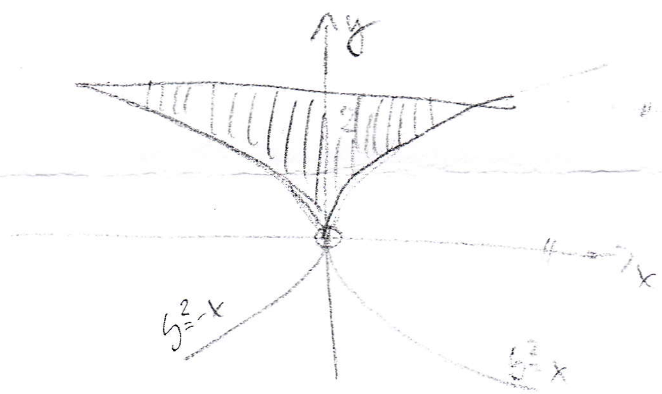
Řeš.:

$y \neq 0$

$-1 \leq \frac{x}{y^2} \leq 1$ & $-1 \leq 1-y \leq 1$

$-y^2 \leq x \leq y^2$ & $-2 \leq -y \leq 0$

$0 \leq y \leq 2$



(1)

2) Rozhodněte o spojitosti funkce

$$f(x,y)= \begin{cases} \frac{x^2y^2}{x^2+y^2} & [x,y]\neq[0,0] \\ 0 & [x,y]=[0,0] \end{cases}$$

v bodě $[0,0]$.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} \left| \begin{matrix} x=r\cdot\cos\varphi \\ y=r\cdot\sin\varphi \end{matrix} \right| = \lim_{r\to 0^+} \frac{r^4\cos^2\varphi\sin^2\varphi}{r^2} =$$

$$= \lim_{r\to 0^+} r^2\cos^2\varphi\sin^2\varphi = 0$$

je spojitá.

3) Vypočtěte parciální derivace 1. a 2. řádu pro (A)

$$z(x,y) = \frac{\cos x^2}{y}$$

$$z_x = \frac{-2x\sin x^2}{y}, \quad z_y = -\frac{\cos x^2}{y^2}$$

√

$$z_{xx} = -\frac{1}{y}\left(2\sin x^2 + 4x^2\cos x^2\right)$$

$$z_{xy} = \frac{2x\sin x^2}{y^2} = z_{yx}$$

$$z_{yy} = \frac{2\cos x^2}{y^3}$$

4) Diferenciální rovnost

$$x z_{xx} + y z_{xy} + z_x = 0$$

transformujte do nových proměnných $u = x+y$, $v = \frac{y}{x+y}$

$u_x = 1$ $v_x = -\frac{y}{(x+y)^2}$

$u_y = 1$ $v_y = \frac{x}{(x+y)^2}$

$$z_x = z_u \cdot u_x + z_v v_x = z_u + z_v \cdot \frac{-y}{(x+y)^2}$$

$$z_{xx} = z_{uu} \cdot 1 + z_{uv} \cdot \frac{-y}{(x+y)^2} + z_v \cdot \frac{2y}{(x+y)^3} + z_{vu} \cdot 1 \cdot \frac{-y}{(x+y)^2} + z_{vv} \cdot \frac{y^2}{(x+y)^4}$$

$$z_{xy} = z_{uu} \cdot 1 + z_{uv} \cdot \frac{x}{(x+y)^2} + z_v \cdot \frac{-x+y}{(x+y)^3} + z_{vu} \frac{-y}{(x+y)^2} + z_{vv} \frac{-xy}{(x+y)^4}$$

$\Downarrow$

$$0 = x \cdot \left(z_{uu} + z_{uv} \frac{-y}{(x+y)^2} + z_v \frac{2y}{(x+y)^3} + z_{vu} \frac{-y}{(x+y)^2} + z_{vv} \frac{y^2}{(x+y)^4} \right) +$$

$$+ y \left(z_{uu} + z_{uv} \frac{x}{(x+y)^2} + z_v \frac{-x+y}{(x+y)^3} + z_{vu} \frac{-y}{(x+y)^2} + z_{vv} \frac{-xy}{(x+y)^4} \right) +$$

$$+ z_u + z_v \cdot \frac{-y}{(x+y)^2} = (x+y) z_{uu} + \frac{-xy - y^2}{(x+y)^2} z_{vu} + z_u$$

$\Downarrow$

$$u z_{uu} - v \cdot z_{vu} + z_u = 0$$

1) Vypočtěte lokální extrémy funkce

$$f(x,y,z) = \frac{y^2}{4x} + \frac{z^2}{y} + \frac{2}{z} + x$$

$f_x = -\frac{y^2}{4x^2} + 1 = 0 \longrightarrow \left(\frac{y}{2x}\right)^2 = 1 \Rightarrow y = 2x$ nebo $y = -2x$

$f_y = \frac{2y}{4x} - \frac{z^2}{y^2} = 0$

$f_z = \frac{2z}{y} - \frac{2}{z^2} = 0$

$f_y = 0 \Rightarrow z^2 = 4x^2$

$z = 2x$ nebo $z = -2x$

$f_z = 0 \Rightarrow 2 - \frac{1}{2x^2} = 0 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm 1/2$

$f_z = 0 \Rightarrow -2 - \frac{1}{2x^2} = 0$

nelze

$f_y = 0 \Rightarrow -1 - \frac{z^2}{y^2} = 0 \Rightarrow$ neexistuje

stacionární body: $[1/2, 1, 1]$ a $[-1/2, -1, -1]$

$f_{xy} = \frac{y}{2x^3}$ $\quad f_{xy} = -\frac{y}{2x^2}$ $\quad f_{xz} = 0$

$f_{yy} = \frac{1}{2x} + \frac{2z^2}{y^3}$ $\quad f_{yz} = -\frac{2z}{y^2}$ $\quad f_{zz} = \frac{2}{y} + \frac{4}{z^3}$

$[1/2, 1, 1]$: $\begin{vmatrix} 4 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 6 \end{vmatrix} = 32 > 0$ je poz. def. $\Rightarrow$ lok. minimum $\quad (>0, \; 8>0)$

$[-1/2, -1, -1]$: $\begin{vmatrix} 4 & 2 & 0 \\ 2 & -3 & 2 \\ 0 & 2 & -6 \end{vmatrix} = -32 < 0$ je neg. def. $\Rightarrow$ lok. maximum $\quad (<0, \; -8>0)$