

A)

$$\int \sin^2(x) \cos x dx = \int f'(x) g x dx$$

metoda per partes:

$$f = \sin^2 x \quad g' = \cos x$$

$$f' = 2 \sin x \cos x \quad g = \sin x$$

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$$\int \sin^2(x) \cos x dx = \sin^3 x - 2 \int \sin^2(x) \cos x dx$$

$$3 \int \sin^2(x) \cos x dx = \sin^3 x$$

$$\int \sin^2(x) \cos x dx = \frac{1}{3} \sin^3 x + C$$

B)

$$\int \frac{x+1}{x^2+2x+1} dx = \int \frac{x+1}{(x+1)(x+1)} dx$$

Substituce:

$$x+1 = t$$

$$dx = dt$$

$$\int \frac{dt}{t} = \ln|t|$$

zpět do substituce:

$$t = x+1$$

$$\int \frac{x+1}{x^2+2x+1} dx = \ln|x+1| + C$$

C)

$$\int \frac{\sqrt{\ln x}}{x} dx$$

Metoda per partes:

$$v = (\ln x)^{0,5} \quad u' = \frac{1}{x}$$

$$v' = \frac{1}{2x\sqrt{\ln x}} \quad u = \ln x$$

$$\int \frac{\sqrt{\ln x}}{x} dx = \ln x \cdot \sqrt{\ln x} - \frac{1}{2} \int \frac{\ln x}{x\sqrt{\ln x}} dx$$

$$\int \frac{\sqrt{\ln x}}{x} dx = \ln x \cdot \sqrt{\ln x} - \frac{1}{2} \int \frac{\sqrt{\ln x}}{x} dx$$

$$2 \int \frac{\sqrt{\ln x}}{x} dx = 2 \ln x \cdot \sqrt{\ln x} - \int \frac{\sqrt{\ln x}}{x} dx$$

$$3 \int \frac{\sqrt{\ln x}}{x} dx = 2 \ln x \cdot \sqrt{\ln x}$$

$$\int \frac{\sqrt{\ln x}}{x} dx = \frac{2 \ln x \cdot \sqrt{\ln x}}{3} + C$$