1.

$$y' = \frac{1+y^2}{1+x^2}$$

$$y_0 = 1$$

$$\frac{y'}{1+y^2} = \frac{1}{1+x^2}$$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\arctan y = \arctan x + C$$

$$y = \tan(\arctan(x + C))$$

$$1 = \tan C$$

$$C = \frac{\pi}{4}$$

$$y = \tan\left(\arctan\left(x + \frac{\pi}{4}\right)\right)$$

$$\frac{x + \tan\frac{\pi}{4}}{1 - x\tan\frac{\pi}{4}} = \frac{x+1}{1-x}$$

2.

$$y' = \frac{y}{x}\left(1 + \ln\frac{y}{x}\right)$$

$$u = \frac{y}{x}$$

$$y = ux$$

$$u + u'x = u + u\ln u$$

$$pro: u \neq 0$$

$$\frac{du}{dx} = \frac{u\ln u}{x}$$

$$\frac{du}{u\ln u} = \frac{dx}{x}$$

$$\int\frac{du}{u\ln u} = \int\frac{dx}{x}$$

$$\ln(\ln u) = \ln|x| + \ln K$$

$$\ln(\ln u) = \ln Kx$$

$$\ln u = Kx$$

$$u = e^{Kx} + C$$

$$y = xe^{Kx} + C$$

Pokud je $u = 0$

pak $\frac{y}{x} = 0 \Rightarrow y = 0$

Ověření:

$$0 = 0(1 + \ln 0)$$

$$0 = nedefi$$

Další řešení nejsou.

3.

$$y' = 2xy + 2x^3$$

(1)

$$y' - 2xy = 0$$

$$\frac{dy}{y} = 2xdx$$

$$\ln y = x^2 + C$$

$$K = \pm C$$

$$\ln y = x^2 + \ln K$$

$$\ln y - \ln K = x^2$$

$$\ln \frac{y}{K} = x^2$$

$$y_0 = Ke^{x^2}$$

(2)

$$y = y_0 + y_p$$

$$y = C_{(x)} y_0$$

$$y = C_{(x)} K e^{x^2}$$

$$y = P_{(x)} e^{x^2}$$

$$DOSADIME:$$

$$P'_{(x)} e^{x^2} + P_{(x)} 2xe^{x^2} - P_{(x)} 2xe^{x^2} = 2x^3$$

$$P'_{(x)} e^{x^2} = 2x^3$$

$$P'_{(x)} = \frac{2x^3}{e^{x^2}}$$

$$\frac{dP}{dx} = \frac{2x^3}{e^{x^2}}$$

$$\int dP = \int \frac{2x^3}{e^{x^2}} dx$$

$$P = 2\int x^3 e^{-x^2} dx = \left| \begin{matrix} t = x^2 \\ dx = \dfrac{dt}{2x} \end{matrix} \right| = \int te^{-t} dt$$

$$P = -te^{-t} + \int e^{-t} dt$$

$$P = -e^{-x^2}\left(x^2 + 1\right)$$

$$y = Ke^{x^2}\left(-e^{-x^2}\left(x^2 + 1\right) + 1\right)$$

4.

$$y' - \frac{2x-1}{x^2}y = 1$$

(1)

$$\frac{dy}{dx} = \frac{2x-1}{x^2}y_0$$

$$pro: y \neq 0$$

$$\frac{dy}{y} = \frac{2x-1}{x^2}dx$$

$$\ln y_0 = 2\int \frac{1}{x}dx - \int \frac{1}{x^2}dx$$

$$\ln y_0 = \ln x^2 - x^{-1} + \ln C$$

$$\ln y_0 = \ln Cx^2 - \frac{1}{x}$$

$$\ln \frac{y}{Cx^2} = \frac{1}{x}$$

$$y = Ke^{\frac{1}{x}}x^2$$

$$y = Kx^2e^{\frac{1}{x}}$$

(2)

$$y_p = C_{(x)}x^2e^{\frac{1}{x}}$$

$$C'_{(x)}x^2e^{\frac{1}{x}} + C_{(x)}2xe^{\frac{1}{x}} - C_{(x)}e^{\frac{1}{x}} - C_{(x)}2xe^{\frac{1}{x}} + C_{(x)}e^{\frac{1}{x}} = 1$$

$$C'_{(x)}x^2e^{\frac{1}{x}} = 1$$

$$C'_{(x)} = \frac{1}{x^2e^{\frac{1}{x}}}$$

$$C_{(x)} = \int \frac{1}{x^2e^{\frac{1}{x}}}dx = \int x^{-2}e^{-x^{-1}}dx$$

$$\left| \begin{array}{l} t = -x^{-1} \\ dt = x^{-2}dx \end{array} \right|$$

$$= e^{-x^{-1}} + C$$

(3)

$$y = x^2e^{\frac{1}{x}}K + \frac{1}{e^{\frac{1}{x}}}x^2e^{\frac{1}{x}} = x^2\left(e^{\frac{1}{x}}K + 1\right)$$

Fyzikální úloha:

$\vec{F}_s \approx v$

$\vec{v} = (\dot{x},0,0)$

$ma = -b\dot{x} - mg$

$Homog.rovnice:$

$m\ddot{x} + b\dot{x} = 0$

$\dot{x} = v$

$m\dot{v} - bv$

$\frac{dv}{dt} = -\frac{dv}{m}$

$\int \frac{dv}{v} = -\frac{dt}{m}$

$\ln v = -\frac{b}{m}t + C$

$\ln\frac{v}{t} = -\frac{b}{m}t$

$v = e^{-\frac{b}{m}t}K$

$v_{(0)} = v_0$

$v_0 = e^0 K$

$K = v_0$

$v = v_0 e^{-\frac{b}{m}t} t$

Skutečné řešení se liší od řešení homogenní rovnice o konstantu C.

$v = v_0 K e^{-\frac{b}{m}t} + C$

$-\frac{b}{m}Ke^{-\frac{b}{m}t} + \frac{b}{m}\left(Ke^{-\frac{b}{m}t} + C\right) = -g$

$C = -g\frac{m}{b}$

$v_0 = K - \frac{mg}{b}$

$K = v_0 + \frac{mg}{b}$

$v = \left(v_0 + \frac{mg}{b}\right)\exp\left(-\frac{b}{m}t\right) - \frac{mg}{b}$