2) jak se při boostu transformují $\vec{E}$ a $\vec{B}$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & B_x \\ -E_z & -B_y & -B_x & 0 \end{pmatrix}, \quad c = 1, \quad \Lambda = \begin{pmatrix} \gamma & \gamma v_n \\ \gamma v^m & \delta^m_n + \frac{\gamma^2}{1+\gamma} v^m v_n \end{pmatrix}$$

$$F'_{\rho\sigma} = \Lambda^\mu_\rho F_{\mu\nu} \Lambda^\nu_\sigma$$

prozatím označím $\Lambda_{\sigma\nu} = \begin{pmatrix} \Lambda_{00} & \Lambda_{0i} \\ \Lambda_{j0} & \Lambda_{ij} \end{pmatrix}$

$$F'_{\rho\sigma} = \begin{pmatrix} \Lambda_{00} & \Lambda_{01} & \Lambda_{02} & \Lambda_{03} \\ \Lambda_{10} & \Lambda_{11} & \Lambda_{12} & \dots \\ \Lambda_{20} & \Lambda_{21} & \dots & \\ \Lambda_{30} & \Lambda_{31} & \dots & \end{pmatrix} \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & B_x \\ -E_z & -B_y & -B_x & 0 \end{pmatrix} \Lambda =$$

$$= \begin{pmatrix} -\Lambda_{0i}E_i & \Lambda_{00}E_x + \Lambda_{02}B_z - \Lambda_{03}B_y & \Lambda_{00}E_y - \Lambda_{01}B_z \mp \Lambda_{03}B_x & \Lambda_{00}E_z + \Lambda_{01}B_y + \Lambda_{02}B_x \\ \dots & & & \\ \dots & & & \end{pmatrix} \Lambda$$

nejdřív chci zjistit jen E'_x a pak to aplikovat i na ostatní složky, takže vypočítám z prvního násobení jen první řádek a ten pak vynásobím s druhým ~~...~~ sloupcem

chci zjistit jen E'_x

$$E'_x$$

$$= \begin{pmatrix} 0 & \Lambda_{01}(-\Lambda_{0i}E_i) + \Lambda_{11}(\Lambda_{00}E_x + \Lambda_{02}B_z - \Lambda_{03}B_y) + \Lambda_{21}(\Lambda_{00}E_y - \Lambda_{01}B_z - \Lambda_{03}B_x) \\ & + \Lambda_{31}(\Lambda_{00}E_z + \Lambda_{01}B_y + \Lambda_{02}B_x) \end{pmatrix}$$

toto celé je E'_x

2) pokračování

$$E'_x = \Lambda_{01}(-\Lambda_{0i}E_i) + \Lambda_{11}(\Lambda_{00}E_x + \Lambda_{02}B_z - \Lambda_{03}B_y) +$$

$$\Lambda_{21}(\Lambda_{00}E_y - \Lambda_{01}B_z - \Lambda_{03}B_x) + \Lambda_{31}(\Lambda_{00}E_z + \Lambda_{01}B_y + \Lambda_{02}B_x)$$

$$= \gamma v_1(-\gamma\vec{v}\vec{E}) + \left(1 + \frac{\gamma^2}{1+\gamma}v_1^2\right)(\gamma E_x + \gamma\cancel{v_2}B_z - \gamma\cancel{v_3}B_y) +$$

$$+ \frac{\gamma^2}{\gamma+1}v_2v_1(\gamma E_y - \gamma\cancel{v_1}B_z - \gamma\cancel{v_3}B_x) - \frac{\gamma^2}{1+\gamma}v_3v_1(\gamma E_z + \gamma\cancel{v_1}B_y + \gamma\cancel{v_2}B_x)$$

$$= -\gamma^2 v_1\vec{v}\vec{E} + \gamma E_x - \frac{\gamma^2}{1+\gamma}v_1v_i\gamma E_i + \gamma(v_2B_z - v_3B_y)$$

$$= v_1\vec{v}\cdot\vec{E}\left(-\gamma^2 - \frac{\gamma^2}{1+\gamma}\right) + \gamma E_x + \gamma(v_2B_z - v_3B_y)$$

$$= \left(\vec{v}\cdot(\vec{v}\cdot\vec{E})\cdot\gamma^2\left(-\frac{\gamma+2}{\gamma+1}\right) + \gamma\vec{E} + \gamma(\vec{v}\times\vec{B})\right)^1$$

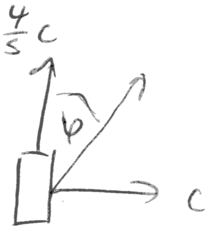
$$\vec{E}' = \vec{v}\cdot(\vec{v}\vec{E})\cdot\gamma^2\left(-\frac{\gamma+2}{\gamma+1}\right) + \gamma\vec{E} + \gamma(\vec{v}\times\vec{B})$$

dá se očekávat analogie i pro $\vec{B}$

$$\vec{B}' = \vec{v}\cdot(\vec{v}\vec{B})\cdot\gamma^2\left(-\frac{\gamma+2}{\gamma+1}\right) + \gamma(\vec{B} + \vec{v}\times\vec{E})$$

možná někde bude mínusko

3) cestující ve vlaku jede rychlostí $\frac{4}{5}c$ a svítí laserem, jaký je úhel mezi ním a kolejemi?



kalkulačka

$$x_\mu = (ct, 0, ct, 0) \qquad v = \frac{4}{5}c \Rightarrow \gamma = \frac{1}{\sqrt{1-(\frac{4}{5})^2}} = \frac{5}{3}$$

$$x_\nu = (ct', v_x t', v_y t', 0)$$

Lorentzův faktor pro rychlost ve směru x

$$x_\nu = \Lambda^\mu_\nu x_\mu = \begin{pmatrix} \gamma & \frac{4}{5}\gamma & 0 & 0 \\ \frac{4}{5}\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ 0 \\ ct \\ 0 \end{pmatrix} = \left(\gamma ct, \frac{4}{5}\gamma ct, ct, 0\right)$$

$$= \left(\frac{5}{3}ct, \frac{4}{3}ct, ct, 0\right)$$

$$(ct', v_x t', v_y t', 0) = \left(\frac{5}{3}ct, \frac{4}{3}ct, ct, 0\right) \qquad t' = \gamma t = \frac{5}{3}t$$

$$\left(\frac{5}{3}ct, \frac{5}{3}v_x t, \frac{5}{3}v_y t, 0\right) = \left(\frac{5}{3}ct, \frac{4}{3}ct, ct, 0\right)$$

$$\Rightarrow \frac{5}{3}v_x t = \frac{4}{3}ct \Rightarrow v_x = \frac{4}{5}c \qquad \operatorname{tg}\varphi = \frac{v_x}{v_y} = \frac{3}{4} \Rightarrow \varphi = \underline{\underline{36°52'11{,}63''}}$$

$$\frac{5}{3}v_y t = ct \Rightarrow v_y = \frac{3}{5}c$$

5) určete trajektorii částice e, m v mag. poli $\vec{B} = (0, 0, B)$

vyčteno ze sešitu

$$-mc\frac{du_\mu}{ds} = e F_{\mu\nu} u^\nu \qquad u_\mu = \gamma\left(1, -\frac{v^i}{c}\right), \; u^\nu = \gamma\left(1, \frac{v^j}{c}\right)$$

$$-m\gamma\frac{du_\mu}{dt} = e F_{\mu\nu} u^\nu \qquad F_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -B & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \text{pouze: } \begin{matrix} ①\; i=1, j=2 \\ ②\; i=2, j=1 \end{matrix}$$

①: $$-m\gamma\frac{d\left(-\gamma\frac{v_1}{c}\right)}{dt} = e(-B)\,v_2\frac{\gamma}{c}$$

použijeme γ = konst.

$$-m\gamma\frac{d\left(\gamma\frac{v_1}{c}\right)}{dt} = eBv_2\frac{\gamma}{c} \qquad \cancel{d_t\left(\gamma\frac{v_1}{c}\right) = \frac{v_1}{c}d_t\gamma + \gamma d_t\frac{v_1}{c}} \qquad \cancel{d_t\gamma = \frac{d}{dt}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{\gamma^3}{c}\ldots\frac{dv_i}{dt}}$$

příklad 6c

$$-m\gamma^2\frac{1}{c}\frac{dv_1}{dt} = eBv_2\frac{\gamma}{c}$$

$$-m\gamma\frac{dv_1}{dt} = eBv_2$$

Ze zákona zachování energie

$$m\gamma c^2 = E$$ — vypůjčil jsem si od Sama

$$\Rightarrow \gamma = \frac{E}{mc^2} = \text{konst.}$$

$$-\cancel{m}\frac{E}{\cancel{m}c^2}\frac{dv_1}{dt} = eBv_2$$

$$\frac{dv_1}{dt} = -\frac{c^2eB}{E}v_2 \quad \Big/ \frac{d}{dt}$$

②: ~~...~~ analogicky: $$\frac{dv_2}{dt} = \frac{c^2eB}{E}v_1 \quad \Big/ \frac{d}{dt}$$

$$\frac{d^2v_1}{dt^2} = -\frac{c^2eB}{E}\frac{dv_2}{dt} = -\Bigg(\underbrace{\frac{c^2eB}{E}}_{\omega}\Bigg)^2 v_1 \Rightarrow v_1(t) = C_1e^{\omega t} + C_2e^{-\omega t} = A\cos\omega t + B\sin\omega t$$

$$\frac{dv_2}{dt} = \frac{c^2eB}{E}v_1 = \omega v_1 \Rightarrow v_2(t) = \omega\int v_1\,dt = \omega\left(\frac{A}{\omega}\sin\omega t - \frac{B}{\omega}\cos\omega t\right)$$

$$v_2(t) = A\sin\omega t - B\cos\omega t$$

$$x(t) = \int v_1\,dt = \frac{A}{\omega}\sin\omega t - \frac{B}{\omega}\cos\omega t$$

$$y(t) = \int v_2\,dt = -\frac{A}{\omega}\cos\omega t - \frac{B}{\omega}\sin\omega t$$

$\Rightarrow$ pohyb po kružnici

6) dokažte: $x^\mu = (ct, x, y, z)$, $\partial_\mu = \left(\frac{\partial}{c\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$, $P^{\mu\nu} := g^{\mu\nu} - u^\mu u^\nu$

a) $u^\mu p_\mu = mc$

$$u^\mu = \frac{dx^\mu}{ds} = \frac{dx^\mu}{c\,dt}\frac{c\,dt}{ds} = \gamma\frac{dx^\mu}{c\,dt} = \gamma\left(1, \frac{v_x}{c}, \frac{v_y}{c}, \frac{v_z}{c}\right) = \gamma\left(1, \frac{v^i}{c}\right)$$

$$p^\mu = mcu^\mu = mc\gamma\left(1, \frac{v^i}{c}\right) \Rightarrow p_\mu = mc\gamma\left(1, -\frac{v^i}{c}\right)$$

$$u^\mu p_\mu = mcu^\mu u_\mu = mc\gamma^2\left(1 - \frac{v^2}{c^2}\right) = mc\frac{1-\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = \underline{\underline{mc}}$$

b) $\partial_\mu x_\nu = g_{\mu\nu}$ $\partial_\mu = \left(\frac{\partial}{c\partial t}, \frac{\partial}{\partial x_i}\right)$ $x_\nu = (ct, -x^i)$

$$\partial_\mu x_\nu = \partial_\mu g_{\mu\nu} x^\mu = \partial_\mu x^\mu g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \underline{\underline{g_{\mu\nu}}}$$

c) $u^\mu \frac{du_\mu}{ds} = 0$ $\frac{du_\mu}{ds} = \frac{du_\mu}{c\,dt}\frac{c\,dt}{ds} = \gamma\frac{du_\mu}{c\,dt}$, $u^\mu = \gamma\left(1, \frac{v^i}{c}\right)$

$$u^\mu\frac{du_\mu}{ds} = u^\mu\frac{\gamma}{c}\frac{du_\mu}{dt} = \frac{\gamma}{c}\left(\gamma, \gamma\frac{v^i}{c}\right)\begin{pmatrix} \frac{d\gamma}{dt} \\ \frac{-d\gamma\frac{v^i}{c}}{dt} \end{pmatrix} = \frac{\gamma^2}{c}\left(\frac{d\gamma}{dt} - \frac{v^i}{c^2}\frac{d\gamma v^i}{dt}\right)$$

$$= \frac{\gamma^2}{c}\left[\frac{d\gamma}{dt} - \frac{v^i}{c^2}\left(v^i\frac{d\gamma}{dt} + \gamma\frac{dv^i}{dt}\right)\right] = \frac{\gamma^2}{c}\left[\frac{d\gamma}{dt}\left(1 - \frac{v^2}{c^2}\right) - \frac{v^i}{c^2}\gamma\frac{dv^i}{dt}\right] =$$

$= \Big|$ $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, $v = \sqrt{v^2} = \sqrt{v^i v_i}$ – půjčil jsem si od Sama

$$\frac{d\gamma}{dt} = -\frac{1}{2}\frac{-\frac{2v}{c^2}}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{dv}{dt} = \frac{\frac{v}{c^2}}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{dv}{dt} = \frac{\sqrt{v^i v_i}}{c^2\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{d\sqrt{v^i v_i}}{dt} =$$

$$= \frac{\gamma^3}{c^2}\sqrt{v^i v_i}\cdot\left(\frac{1}{2}\left(v^i v_i\right)^{-1/2} 2v^i\frac{dv^i}{dt}\right) = \frac{\gamma^3}{c^2}v^i\frac{dv^i}{dt} \Big|$$

$$= \frac{\gamma^2}{c}\left[\frac{\gamma^3}{c^2}v^i\frac{dv^i}{dt}\left(1-\frac{v^2}{c^2}\right) - \frac{\gamma}{c^2}v^i\frac{dv^i}{dt}\right] = \left|\left(1-\frac{v^2}{c^2}\right) = \gamma^{-2}\right| =$$

$$= \frac{\gamma^2}{c}\left[\frac{\gamma}{c^2}\cancel{v^i\frac{dv^i}{dt}} - \frac{\gamma}{c^2}\cancel{v^i\frac{dv^i}{dt}}\right] = \underline{0}$$

d) $P^{\mu\nu} u_\mu = 0 \qquad P^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

6) pokračování

$$P^{\mu\nu}u_\mu = \left(g^{\mu\nu} - u^\mu u^\nu\right)u_\mu = g^{\mu\nu}u_\mu - u^\mu u_\mu u^\nu =$$

$$= \left| \begin{array}{l} g^{\mu\nu}u_\mu = u_\nu \\ u^\mu u_\mu = \gamma\left(1, \frac{v^2}{c}\right)\gamma\begin{pmatrix}1\\ -\frac{v^2}{c}\end{pmatrix} = \gamma^2\left(1 - \frac{v^2}{c^2}\right) = \gamma^2\gamma^{-2} = 1 \end{array} \right|$$

$$= u_\nu - u_\nu = \underline{0}$$

e) $P^{\mu\nu}P_{\nu\rho} = \delta^\mu_\rho - u^\mu u_\rho = P^\mu_\rho$

$$P^{\mu\nu}P_{\nu\rho} = \left(g^{\mu\nu} - u^\mu u^\nu\right)\left(g^{\nu\rho} - u^\nu u^\rho\right) = g^{\mu\nu}g^{\nu\rho} - g^{\mu\nu}u^\nu u^\rho - g^{\nu\rho}u^\mu u^\nu$$

$$+ u^\mu u^\nu u^\nu u^\rho =$$

$$= g^\mu_\rho - u_\mu u^\rho - u^\mu u_\rho +$$

$$u_\nu g_{\mu\nu} \quad u^\mu_\mu g_{\mu\nu} u^\nu u^\rho$$

$$= g^\mu_\rho - u_\mu u^\rho - u^\rho \cancel{g^\rho_\mu} u_\rho + u_\mu u_\rho u^\nu u^\rho$$

$$= g^\mu_\rho - u_\mu u^\rho = \underline{P^\mu_\rho}$$

8) ukažte, že $F_{\mu\nu}(*F^{\mu\nu}) = 2\partial_\mu(\varepsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma)$, kde $*F^{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$(\partial_\mu A_\nu - \partial_\nu A_\mu)\left[\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho)\right] =$$

$$= \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\left[(\partial_\mu A_\nu)(\partial_\rho A_\sigma) - (\partial_\mu A_\nu)(\partial_\sigma A_\rho) - (\partial_\nu A_\mu)(\partial_\rho A_\sigma) + (\partial_\nu A_\mu)(\partial_\sigma A_\rho)\right] =$$

prohození indexů $\Rightarrow \ominus$ | prohození indexů $\Rightarrow \ominus$ | prohození indexů $\ominus\ominus = \oplus$

$$= \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\left[(\partial_\mu A_\nu)(\partial_\rho A_\sigma) + (\partial_\mu A_\nu)(\partial_\rho A_\sigma) + (\partial_\mu A_\nu)(\partial_\rho A_\sigma) + (\partial_\mu A_\nu)(\partial_\rho A_\sigma)\right]$$

$$= 2\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(\partial_\rho A_\sigma) = 2\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu A_\nu)(\partial_\rho A_\sigma) + \overbrace{2\varepsilon^{\mu\nu\rho\sigma} A_\nu \partial_\mu \partial_\rho A_\sigma}^{0 \text{ - antisymetrie epsilontenzoru + symetrie derivací}}$$

$$= \underline{\underline{2\varepsilon^{\mu\nu\rho\sigma}\partial_\mu(A_\nu \partial_\rho A_\sigma)}}$$

7) ukažte, že výraz je invariantní vůči Lor. transformacím

$$W^2 - \frac{|\vec{S}|^2}{c^2} \geq 0 \quad , \quad W = \frac{1}{2}\left(\varepsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right) , \quad \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

- fyzikální význam nerovnosti: aby rychlost nebyla imaginární? :-)

výsledek příkladu č. 2 ze cvičení 21. 5. 18:

$$v_{1,2} = \frac{Wc^2}{|\vec{S}|} \pm \sqrt{\frac{W^2c^4}{|\vec{S}|^2} - c^2}$$

— je jedno jaká bude rychlost, ten člen bude pořád pod odmocninou

$\Rightarrow$ pro jakoukoliv rychlost (Lor. transformaci) bude výraz stále roven nebo větší od nuly

:-)

9) podélné elmag. vlnění?

Neexistuje! (A co když tu je...?)

podélné vlnění $\vec{E} \parallel \vec{k}$, $\vec{B} \parallel \vec{k}$, ale $\vec{E}$ a $\vec{B}$ musí $\vec{E} \perp \vec{B}$ ~~[illegible]~~

ale zároveň musí být $\vec{E}$ i $\vec{B}$ kolmé na směr šíření $\Rightarrow$ nelze najít takový boost, aby se všechny 3 směry promítli do jednoho ~~[illegible]~~.