SFT

DÚ 9

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2, Boseho–Einsteinova kondensace

$E \sim |p^\sigma|$ D dimense

hustota stavů v dimensi $d = D$ $k = \frac{p}{\hbar}$ $S_{D-1} = \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)}$

$$\rho_{(E)} = \frac{gV}{(2\pi)^D} S_{D-1} \frac{(k(E))^{D-1}}{\left|\frac{dE}{dk}\right|} = \frac{gV \cdot 2\pi^{\frac{D}{2}}}{(2\pi)^D \cdot \Gamma\left(\frac{D}{2}\right) \cdot \hbar^D} \cdot \frac{p_{(E)}^{D-1}}{\left|\frac{dE}{dp}\right|} = \frac{2\pi^{\frac{D}{2}} gV}{(2\pi\hbar)^D \Gamma\left(\frac{D}{2}\right)} \frac{p_{(E)}^{D-1}}{\left|\frac{d|p^\sigma|}{dp}\right|} =$$

$$= \frac{gV\, 2\pi^{\frac{D}{2}}}{(2\pi\hbar)^D \Gamma\left(\frac{D}{2}\right) \sigma} \frac{p_{(E)}^{D-1}}{p_{(E)}^{\sigma-1}}$$

$$\boxed{p_{(E)}^{D-\sigma} \sim E^{\frac{D}{\sigma}-1}}$$

termodynamický potenciál: $\Omega = -\int_0^\infty \rho_{(E)} \cdot E \cdot \frac{1}{e^{\frac{E-\mu}{kT}}-1}\, dE =$

$$= \frac{-gV\, 2\pi^{\frac{D}{2}}}{(2\pi\hbar)^D \Gamma\left(\frac{D}{2}\right) \sigma} \int \frac{E^{\frac{D}{\sigma}}}{e^{\frac{E-\mu}{kT}}-1}\, dE$$

$\mu \to 0$

počet částic: $N = \int_0^\infty dE \frac{\rho_{(E)}}{e^{\frac{E-\mu}{kT}}-1} = \frac{gV\, 2\pi^{\frac{D}{2}}}{(2\pi\hbar)^D \Gamma\left(\frac{D}{2}\right) \cdot \sigma} \cdot \int_0^\infty \frac{E^{\frac{D}{\sigma}-1}}{e^{\frac{E-\mu}{kT}}-1}\, dE$

pro $\int_0^\infty \frac{1}{e^{\frac{E}{kT}}-1}\, dE = \infty \to$ nastane B.-E. kondensace

aby nastala v tomto případě, musí platit: $\frac{D}{\sigma} - 1 = 0 \to \frac{D}{\sigma} = 1 \to \boxed{D = \sigma}$