8. domácí úkol

1) Stavová rovnice Fermiho plynu

– disperzní relace $\varepsilon \sim p^s$, V, n-dim, ukažte, že platí:

a) $pV = \frac{s}{n} E$

$$g(E)\,dE = \frac{gV}{(2\pi)^n} \frac{(k(E))^{n-1}}{\left|\frac{dE}{dk}\right|} S_{d-1}\,dE \qquad k = \frac{p}{\hbar}$$

$$g(E)\,dE = \frac{gV}{(2\pi)^n} \frac{\left(\frac{p}{\hbar}\right)^{n-1}}{\hbar \frac{dp^s}{dp}} S_{n-1}\,dE = \frac{gV}{(2\pi)^n} \frac{1}{\hbar^n} \frac{p^{n-1}}{s \cdot p^{s-1}} S_{n-1}\,dE$$

$$= \frac{gV}{(2\pi\hbar)^n} \frac{p^{n-s}}{s} S_{n-1}\,dE = \left| p = E^{1/s} \right| = \frac{gV}{(2\pi\hbar)^n} \frac{1}{s} S_{n-1} E^{\frac{n}{s}-1}$$

$$\Omega = -kT \ln \Xi = -kT \sum_{a=1}^{\infty} \ln\left(1 + e^{-\frac{E_a - \mu}{kT}}\right) \simeq \left| \cdot e^{-\frac{E_a - \mu}{kT}} \right.$$

$$\simeq -kT \int_0^{\infty} \ln\left(1 + e^{-\frac{E_a - \mu}{kT}}\right) g(E)\,dE = \left| \begin{matrix} u = \ln\left(1 + e^{-\frac{E_a-\mu}{kT}}\right) & u' = \frac{1}{1 + e^{-\frac{E_a-\mu}{kT}}} \cdot \left(-\frac{1}{kT}\right) \\ v' = g(E) & v = \int_0^{E} g(E')\,dE' \end{matrix} \right|$$

$$= -kT \ln\left(1 + e^{-\frac{E_a - \mu}{kT}}\right) \int_0^{E} g(E')\,dE' - \int \left[\left(-\frac{1}{kT}\right) \frac{e^{-\frac{E_a-\mu}{kT}}}{1 + e^{-\frac{E_a-\mu}{kT}}} \int_0^{E} g(E')\,dE' \right] dE$$

kde $\int_0^E g(E')\,dE' = \int_0^E \frac{gV}{(2\pi\hbar)^n} \frac{1}{s} S_{n-1} E'^{\frac{n}{s}-1}\,dE' = \frac{gV}{(2\pi\hbar)^n} \frac{1}{\cancel{s}} S_{n-1} E^{\frac{n}{s}} \frac{\cancel{s}}{n}$

$$\Rightarrow \Omega = -\int \frac{e^{-\frac{E_a-\mu}{kT}}}{1 - e^{-\frac{E_a-\mu}{kT}}} \frac{gV}{(2\pi\hbar)^n} \frac{1}{n} S_{n-1} E^{\frac{n}{s}}\,dE =$$

$$= - \int \frac{1}{1+e^{\frac{E-\mu}{kT}}} \frac{gV}{(2\pi\hbar)^n} \frac{S_{n-1}}{n} E^{\frac{n}{s}}\, dE$$

$$N_F = \frac{1}{1+e^{\frac{E-\mu}{kT}}}$$

$$\langle E \rangle = \int E\, dN = \int_0^\infty E\, \rho(E)\, N_F(E)\, dE =$$

$$= \int \frac{1}{1+e^{\frac{E-\mu}{kT}}} \cdot \frac{gV}{(2\pi\hbar)^n} \frac{S_{n-1}}{s} E^{\frac{n}{s}-1} \cdot E\, dE$$

$$\langle E \rangle = -\frac{n}{s}\Omega = -\frac{n}{s}(-pV) = \underline{\underline{\frac{n}{s} pV}}$$

b) $pV^{1+\frac{s}{n}} = \text{konst.}$

$$p = -\frac{\Omega}{V} = +\frac{1}{V}\int \frac{1}{1+e^{\frac{E-\mu}{kT}}} \frac{gV}{(2\pi\hbar)^n} \frac{S_{n-1}}{n} E^{\frac{n}{s}}\, dE = \left| \begin{array}{l} t = \frac{E}{kT} \\ \text{substituce} \\ \text{a nějak} \\ \text{by se integrovalo} \end{array} \right| =$$

$$= T^{\frac{n}{s}+1} f_\Omega\left(\frac{\mu}{kT}\right)$$

$$\frac{S}{V} = -\frac{1}{V}\left(\frac{\partial \Omega}{\partial T}\right)_{\mu, V} = \left(\frac{n}{s}+1\right) T^{\frac{n}{s}} f_\Omega\left(\frac{\mu}{kT}\right) + T^{\frac{n}{s}+1} \frac{\partial}{\partial T} f_\Omega\left(\frac{\mu}{kT}\right) =$$

$$= \left(\frac{n}{s}+1\right) T^{\frac{n}{s}} f_\Omega\left(\frac{\mu}{kT}\right) + T^{\frac{n}{s}+1} \cdot \left(-\frac{\mu}{kT^2}\right) f_\Omega\left(\frac{\mu}{kT}\right) =$$

$$= T^{\frac{n}{s}} \left[\left(\frac{n}{s}+1\right) f_\Omega\left(\frac{\mu}{kT}\right) - \frac{\mu}{kT} f_\Omega\left(\frac{\mu}{kT}\right)\right] = T^{\frac{n}{s}} f_S\left(\frac{\mu}{kT}\right)$$

$$\frac{N}{V} = -\frac{1}{V}\left(\frac{\partial \Omega}{\partial \mu}\right)_{kT} = T^{\frac{n}{s}+1} \frac{\partial}{\partial \mu} f_\Omega\left(\frac{\mu}{kT}\right) = T^{\frac{n}{s}+1} \frac{1}{kT} f_\Omega\left(\frac{\mu}{kT}\right)$$

$$= T^{\frac{n}{s}} f\left(\frac{\mu}{kT}\right)$$

8. domácí úkol

2/2

$$\frac{S}{V} = T^{\frac{n}{s}} f_S\left(\frac{\mu}{kT}\right)$$

$$\frac{N}{V} = T^{\frac{n}{s}} f_N\left(\frac{\mu}{kT}\right)$$

$$\frac{S}{N} = \text{konst.} = \frac{\frac{S}{V}}{\frac{N}{V}} = \frac{f_S\left(\frac{\mu}{kT}\right)}{f_N\left(\frac{\mu}{kT}\right)} \Rightarrow \frac{\mu}{T} = \text{konst.}$$

$$p = T^{\frac{n}{s}+1} f_\Omega\left(\frac{\mu}{kT}\right) \Rightarrow \frac{p}{T^{\frac{n}{s}+1}} = f_\Omega\left(\frac{\mu}{kT}\right) = \text{konst.}$$

$$\frac{N}{V} = T^{\frac{n}{s}} f_N\left(\frac{\mu}{kT}\right) \Rightarrow T^{\frac{n}{s}} \sim \frac{1}{V} \Rightarrow V^{-\frac{s}{n}} \sim T$$

$$\frac{p}{T^{\frac{n}{s}+1}} = \frac{p}{\left(V^{-\frac{s}{n}}\right)^{\frac{n}{s}+1}} \Rightarrow \underline{\frac{p}{V^{1+\frac{s}{n}}} = \text{konst.}}$$

c) pro $T \to \infty$ $c_V = \frac{n}{s} N$

$$c_V = \left(\frac{\partial E}{\partial T}\right)_V \qquad E = \frac{n}{s} V p$$

stavová rovnice pro fermiho plyn

$$\frac{pV}{NkT} = 1 + \frac{\lambda_T^3}{2^{\frac{5}{3}} g} \frac{N}{V}$$

$$pV = NkT \left(1 + \frac{\lambda_T^3}{2^{\frac{5}{3}} g} \frac{N}{V}\right)$$

$$E = \frac{n}{s} NkT \left(1 + \frac{\lambda_T^3}{2^{5/3} g} \frac{N}{V}\right)$$

$$c_V = \frac{n}{s} Nk \left(1 + \frac{\lambda_T^3}{2^{5/3} g} \frac{N}{V}\right) + \frac{n}{s} NkT \frac{N}{V} \frac{1}{2^{5/3} g} \frac{\partial}{\partial T}\left(\lambda_T^3\right)$$

$$\frac{N \lambda_T^3}{gV} \approx e^{\frac{\mu}{kT}}$$

což je pro $T \to \infty$ velmi malé

$$\Rightarrow \underline{c_V \approx \frac{n}{s} Nk}$$