SFT

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DÚ 7

2) a) hustota energie ε $\quad \varepsilon = \frac{E}{V} \rightarrow E = \varepsilon \cdot V$

pro tří-rozměrný vesmír $\quad p = \frac{\varepsilon}{3}$

1. věta termodynamická: $dE = TdS - pdV$

$$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

$$\left(\frac{\partial E}{\partial V}\right)_T = \left(\frac{\partial \varepsilon \cdot V}{\partial V}\right)_T = \varepsilon \cdot \left(\frac{\partial V}{\partial V}\right)_T = \varepsilon$$

$$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p = \varepsilon \qquad \text{dos.}: p = \frac{\varepsilon}{3}$$

$$\varepsilon = -\frac{\varepsilon}{3} + T\left(\frac{\partial p}{\partial T}\right)_V \longrightarrow \frac{4}{\cancel{3}}\varepsilon = \frac{T}{\cancel{3}}\left(\frac{\partial \varepsilon}{\partial T}\right)_V \longrightarrow \frac{d\varepsilon}{dT} \rightarrow 4\varepsilon = T\frac{d\varepsilon}{dT}$$

$$\frac{dT}{T} = \frac{d\varepsilon}{4\varepsilon} \longrightarrow \int \frac{dT}{T} = \int \frac{d\varepsilon}{4\varepsilon}$$

$$\ln T = \frac{1}{4} \cdot \ln(\varepsilon \cdot K') \Big/ \cdot 4 \qquad K' - \text{konst.} \qquad K = K'^{-1} = \text{konst.}$$

$$\ln T^4 = \ln(\varepsilon \cdot K') \longrightarrow T^4 = \varepsilon \cdot K' \longrightarrow \underline{\varepsilon = K \cdot T^4}$$

tenzor energie – napětí

$$\begin{pmatrix} T^{00} & T^{01} & \dots & T^{03} \\ T^{10} & \cdot & & \\ \vdots & & \cdot & \\ T^{30} & & & T^{33} \end{pmatrix}$$

index 0 – čas

1, 2, 3 – souřadnice

zde je representován tlak

$$n=3 \rightarrow 3p - e = 0 \rightarrow p = \frac{e}{3} = \boxed{\frac{e}{n} = p}$$

1D případ $\quad p = \frac{e}{1}$

$$e = -e + T\frac{de}{dT} \longrightarrow 2e = T\frac{de}{dT}$$

$$e = K \cdot T^2$$

2D případ $\quad p = \frac{e}{2} \qquad e = K \cdot T^3$

n – rozměrný případ

$$e = -\frac{e}{n} + T\left(\frac{dp}{dT}\right)_V = -\frac{e}{n} + \frac{T}{n}\frac{de}{dT}$$

$$\underbrace{e + \frac{e}{n}} = \frac{T}{n}\frac{de}{dT}$$

$$\frac{ne + e}{n} = \frac{e(n+1)}{n}$$

$$\frac{e(n+1)}{\cancel{n}} = \frac{T}{\cancel{n}}\frac{de}{dT}$$

$$\boxed{\begin{gathered} e = K \cdot T^{n+1} = K \cdot T^{\alpha} \\ \alpha = n+1 \end{gathered}}$$