Pšek

2. Domácí úkol - oprava č. 2

2) dokažte $c_v = \frac{1}{kT^2}\langle \Delta E \rangle^2$ pro 1 atomový ideální plyn

$$c_v = \left(\frac{\partial E}{\partial T}\right)_V = \frac{3}{2}k \qquad E = \frac{3}{2}kT$$

$$\frac{1}{kT^2}\langle \Delta E\rangle^2 = \frac{1}{kT^2}\left(\langle E^2\rangle - \underbrace{\langle E\rangle}_{\frac{9}{4}k^2T^2}\right) = \frac{1}{kT^2}\left(\frac{15}{4}k^2T^2 - \frac{9}{4}k^2T^2\right) = \frac{3}{2}k$$

$$\langle E^2\rangle = \int E^2(p)\,\rho(p)\,d^3p\,d^3q = \int \frac{p^4}{4m^2}\,\frac{1}{(2\pi mkT)^{3/2}}\,e^{-\frac{p^2}{2mkT}}\,d^3p =$$

$$= (2\pi mkT)^{-3/2}\cdot\frac{1}{4m^2}\cdot 4\pi\int_0^\infty p^6 e^{-\frac{p^2}{2mkT}}\,dp = (2\pi mkT)^{-3/2}\,\frac{\pi}{m^2}\cdot\frac{15\sqrt{\pi}}{16}(2mkT)^{7/2}$$

$$= (2\pi mkT)^2\cdot\frac{\cancel{\pi}}{\cancel{m^2}}\cdot\cancel{\pi}^{-3/2}\,\cancel{\pi}^{1/2}\cdot\frac{15}{16} = \frac{15}{4}k^2T^2$$