Plšek

2. domácí úkol

1) vyjádřete entropii pomocí stat. sumy $\quad S = \frac{\langle E\rangle}{T} + k_B \ln Z = k_B T \left(\frac{\partial Z}{\partial T}\right)_V + k_B \ln Z$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V \quad , \quad F = -k_B T \ln Z$$

$$S = k_B \left(\frac{\partial (T \ln Z)}{\partial T}\right)_V = k_B \ln Z + k_B T \left(\frac{\partial (\ln Z)}{\partial T}\right)_V =$$

$$= k_B \ln Z + k_B T \frac{1}{Z}\left(\frac{\partial Z}{\partial T}\right)_V = \text{pravá strana}$$

$$\left(\frac{\partial Z}{\partial T}\right)_V = \left| Z = \sum_n e^{-\frac{E_n}{kT}} \right| = \sum_n \frac{\partial}{\partial T} e^{-\frac{E_n}{kT}} = \sum_n (-1)\cdot \frac{-E_n}{kT^2} e^{-\frac{E_n}{kT}}$$

$$= \sum_n \frac{E_n}{kT^2} e^{-\frac{E_n}{kT}}$$

$$\Rightarrow S = k_B \ln Z + k_B T \frac{1}{Z} \sum_n \frac{E_n}{kT^2} e^{-\frac{E_n}{kT}} = k_B \ln Z + \frac{1}{T} \overbrace{\sum_n \frac{E_n}{Z} e^{-\frac{E_n}{kT}}}^{\langle E\rangle}$$

$$S = k_B \ln Z + \frac{\langle E\rangle}{T} = \text{levá strana}$$

2) ověřte přímým výpočtem $c_v = \frac{1}{k_B T^2} \langle \Delta E \rangle^2$ pro 1atomový ideální plyn

$$c_v = \left(\frac{\partial E}{\partial T}\right)_V = \left| E = \sum_n \frac{1}{Z} E_n e^{-\frac{E_n}{kT}} \right| = \frac{\partial}{\partial T}\left(\sum_n \frac{1}{Z} E_n e^{-\frac{E_n}{kT}}\right) =$$

$$= \sum_n E_n \frac{\partial}{\partial T}\left(\frac{1}{Z} e^{-\frac{E_n}{kT}}\right) = \sum_n E_n \left[\left(\frac{\partial \frac{1}{Z}}{\partial T}\right)_V e^{-\frac{E_n}{kT}} + \frac{1}{Z}\frac{E_n}{kT^2} e^{-\frac{E_n}{kT}}\right]$$

$$= \sum_n E_n \left[-\frac{1}{Z^2} \frac{\partial\left(\sum_m e^{-\frac{E_m}{kT}}\right)}{\partial T} e^{-\frac{E_n}{kT}} + \frac{1}{Z}\frac{E_n}{kT^2} e^{-\frac{E_n}{kT}}\right] =$$

$$= \sum_n E_n \left[-\frac{1}{Z^2}\left(\sum_m \frac{E_m}{kT^2} e^{-\frac{E_m}{kT}}\right) \cdot e^{-\frac{E_n}{kT}} + \frac{1}{Z}\frac{E_n}{kT^2} e^{-\frac{E_n}{kT}}\right] =$$

$$= \sum_n E_n \left[-\frac{1}{Z} e^{-\frac{E_n}{kT}} \underbrace{\left(\sum_m \frac{1}{kT^2}\frac{E_m}{Z} e^{-\frac{E_m}{kT}}\right)}_{\frac{\langle E \rangle}{kT^2}} + \frac{1}{kT^2}\frac{E_n}{Z} e^{-\frac{E_n}{kT}}\right]$$

$$= \sum_n E_n^2 \frac{1}{Z}\frac{1}{kT^2} e^{-\frac{E_n}{kT}} - \sum_n E_n \frac{1}{Z}\frac{\langle E \rangle}{kT^2} e^{-\frac{E_n}{kT}} =$$

$$= \frac{1}{kT^2}\left[\underbrace{\sum_n \frac{E_n^2}{Z} e^{-\frac{E_n}{kT}}}_{\langle E^2 \rangle} - \langle E \rangle \cdot \underbrace{\sum_n \frac{E_n}{Z} e^{-\frac{E_n}{kT}}}_{\langle E \rangle}\right] =$$

$$= \frac{1}{kT^2}\left[\langle E^2 \rangle - \langle E \rangle^2\right] = \frac{1}{kT^2}\langle \Delta E \rangle^2$$