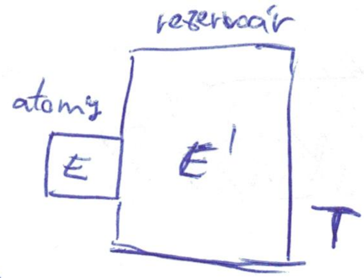
plšek

3. domácí úkol



1) dvouhladinový atom I

$N = n_1 + n_2$, E_1, E_2 v kontaktu s rezervoárem T

emise do rezervoáru $n_2 \to n_2 - 1$, $n_1 \to n_1 + 1$ pro $n_1, n_2 \gg 1$

a) změna entropie atomu

vzorec z netu $\Omega = \dfrac{(n_1+n_2)!}{n_1! \cdot n_2!}$

$$\Rightarrow S_i = k \ln \Omega = k \ln \frac{(n_1+n_2)!}{n_1!\, n_2!}$$

$$S_f = k \ln \frac{(n_1+n_2)!}{(n_1+1)!\,(n_2-1)!}$$

c) odvoďte Boltzmanův vztah

$$k \ln \frac{n_1+1}{n_2} = \frac{E_2-E_1}{T}$$

$$\frac{n_1+1}{n_2} = e^{\frac{E_2-E_1}{kT}}$$

$$\Rightarrow \frac{n_1}{n_2} \approx e^{\frac{E_2-E_1}{kT}}$$

$$S_f - S_i = k \left[\ln \frac{(\cancel{n_1+n_2})!}{n_1!\, n_2!} - \ln \frac{(\cancel{n_1+n_2})!}{(n_1+1)!\,(n_2-1)!} \right] =$$

$$S_f - S_i = k \ln \frac{(n_1+1)!\,(n_2-1)!}{n_1!\, n_2!} = k \ln \frac{n_1+1}{n_2}$$

$$\Delta S = k \ln \frac{n_1+1}{n_2}$$

b) změna entropie rezervoáru

$$\delta E' = E'_f - E'_i = E_2 - E_1$$

$$\delta E' = \delta Q' = T\,dS' \Rightarrow dS' = \frac{E_2 - E_1}{T}$$

$$E^{(0)} = E_i + E'_i = E_f + E'_f$$

$$= n_1 E_1 + n_2 E_2 + E'_i = \underbrace{(n_1+1)E_1 + (n_2-1)E_2}_{E_f} + \underbrace{E'_i + E_2 - E_1}_{E'_f}$$

2) N částic s energiemi E_1, E_2 a deg. g_1, g_2, T

a) partiční fcí $\quad Z_{atom} = \sum_n g_n e^{-\frac{E_n}{kT}} = g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}$

$$Z_N = \overbrace{Z_{atom} \cdot Z_{atom} \cdots Z_{atom}}^{N} = (Z_{atom})^N = \left(g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}\right)^N$$

b) střední hodnota E $\quad E = F + TS \qquad F = -kT \ln Z \;, \quad S = -\left(\frac{\partial F}{\partial T}\right)_V$

$$F = -NkT \cancel{\ln\left(g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}\right)}$$

$$S = +Nk \cancel{\ln\left(g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}\right)} + NkT \frac{\frac{E_1}{kT^2} g_1 e^{-\frac{E_1}{kT}} + \frac{E_2}{kT^2} g_2 e^{-\frac{E_2}{kT}}}{g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}}$$

$$E = F + TS = N \frac{E_1 g_1 e^{-\frac{E_1}{kT}} + E_2 g_2 e^{-\frac{E_2}{kT}}}{g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}}$$

$$\varepsilon = \frac{E}{N} = \frac{E_1 g_1 e^{-\frac{E_1}{kT}} + E_2 g_2 e^{-\frac{E_2}{kT}}}{g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}}$$

c) tepelná kapacita C_V $\quad C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{\partial}{\partial T}\left(\frac{E_1 g_1 e^{-\frac{E_1}{kT}} + E_2 g_2 e^{-\frac{E_2}{kT}}}{g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}}\right)$

= použil jsem wolfram =

$$= \frac{\left(\frac{E_1^2}{k^2T^4} - \frac{2E_1}{kT^3}\right) g_1 e^{-\frac{E_1}{kT}} + \left(\frac{E_2^2}{k^2T^4} - \frac{2E_2}{kT^3}\right) g_2 e^{-\frac{E_2}{kT}}}{g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}} - \frac{\left(\frac{E_1}{kT^2} g_1 e^{-\frac{E_1}{kT}} + \frac{E_2}{kT^2} g_2 e^{-\frac{E_2}{kT}}\right)^2}{\left(g_1 e^{-\frac{E_1}{kT}} + g_2 e^{-\frac{E_2}{kT}}\right)^2}$$