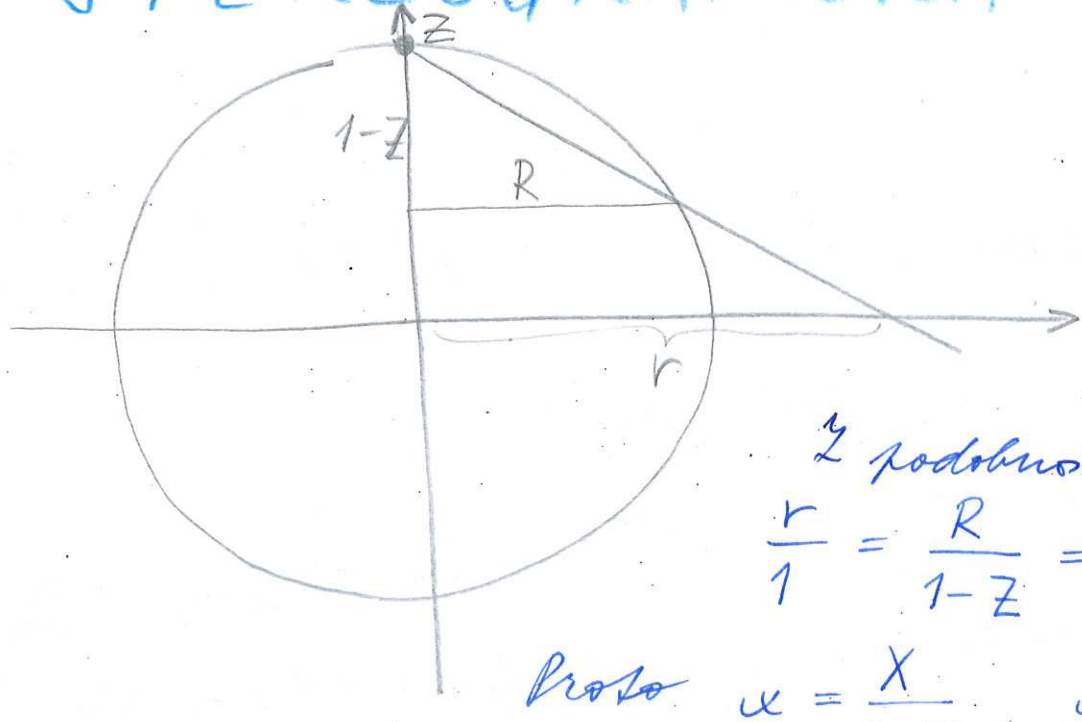
STEREOGRAFICKÁ PROJEKCE



Z podobnosti trojúhelníků:

$$\frac{r}{1} = \frac{R}{1-Z} \Rightarrow r = \frac{R}{1-Z}$$

Proto $x = \dfrac{X}{1-Z}$, $y = \dfrac{Y}{1-Z}$

Víme, že platí $X^2+Y^2+Z^2=1$,

Počítejme

$$x^2+y^2+1 = \frac{X^2+Y^2+1-2Z+Z^2}{(1-Z)^2} = 2\frac{1-Z}{(1-Z)^2}$$

$$= \frac{2}{1-Z}$$

Proto

$$Z = 1 - \frac{2}{x^2+y^2+1} = \frac{x^2+y^2-1}{x^2+y^2+1}$$

$$X = x(1-Z) = \frac{2x}{x^2+y^2+1}$$

$$Y = y(1-Z) = \frac{2y}{x^2+y^2+1}$$

Pomocí sférických souřadnic:

$X = \sin\theta\cos\varphi$

$Y = \sin\theta\sin\varphi$

$Z = \cos\theta$

$$x = \frac{X}{1-Z} = \frac{\sin\theta\cos\varphi}{1-\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}\cos\varphi = \operatorname{cotg}\frac{\theta}{2}\cos\varphi$$

$$y = \operatorname{cotg}\frac{\theta}{2}\sin\varphi$$

Poloha obrazu B bodu A

Bod A .. (θ, φ)

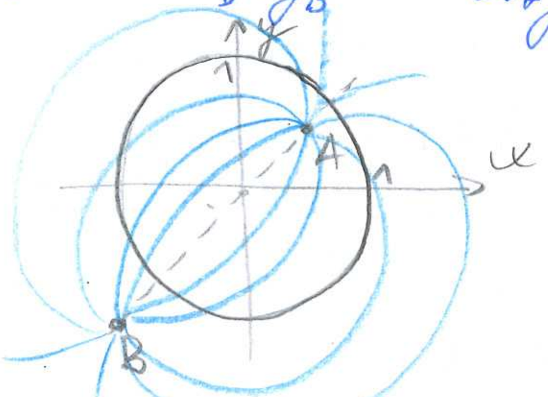
Bod B .. protilehlý k A .. $(\pi-\theta, \varphi+\pi)$

$$(x,y)_A = \left(\cot g \frac{\theta}{2} \cos\varphi, \cot g \frac{\theta}{2} \sin\varphi\right)$$

$$(x,y)_B = \left(\cot g \frac{\pi-\theta}{2} \cos(\varphi+\pi), \cot g \frac{\pi-\theta}{2} \sin(\varphi+\pi)\right) = \left(-\tan\frac{\theta}{2}\cos\varphi, -\tan\frac{\theta}{2}\sin\varphi\right)$$

Součin vzdálenosti zobrazených bodů od počátku:

$$\sqrt{x_A^2+y_A^2}\cdot\sqrt{x_B^2+y_B^2} = \cot g\frac{\theta}{2}\tan\frac{\theta}{2} = 1$$



Element délky na sféře

$$\frac{\partial X}{\partial x} = 2\frac{-x^2+y^2+1}{(1+x^2+y^2)^2}, \quad \frac{\partial X}{\partial y} = \frac{-4xy}{(1+x^2+y^2)^2}, \quad \frac{\partial Y}{\partial x} = \frac{-4xy}{(1+x^2+y^2)^2}$$

$$\frac{\partial Y}{\partial y} = 2\frac{x^2-y^2+1}{(x^2+y^2+1)^2} \quad \frac{\partial Z}{\partial x} = \frac{4x}{(x^2+y^2+1)^2} \quad \frac{\partial Z}{\partial y} = \frac{4y}{(x^2+y^2+1)^2}$$

$$ds^2 = dX^2+dY^2+dZ^2 = \left(\frac{\partial X}{\partial x}dx + \frac{\partial X}{\partial y}dy\right)^2 + \left(\frac{\partial Y}{\partial x}dx + \frac{\partial Y}{\partial y}dy\right)^2 + \left(\frac{\partial Z}{\dots}\right)^2$$

$$= \dots = \left(\frac{2}{1+x^2+y^2}\right)^2 (dx^2+dy^2)$$

Pokud zadefinujeme v rovině (x,y) index lomu $n=\frac{2}{1+r^2}$, bude $ds = n\,dl$, tedy délka křivky na sféře je rovna přímo optické dráze v rovině (x,y).