Theory of invisible cloaks

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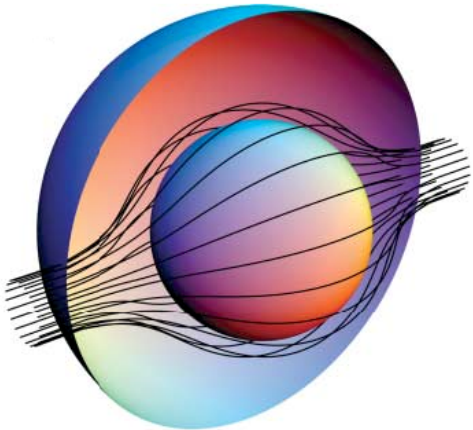
2021

Euclidean invisible cloak

[J. B. Pendry, D. Schurig, D. R. Smith, Science 312, 1780 (2006)]

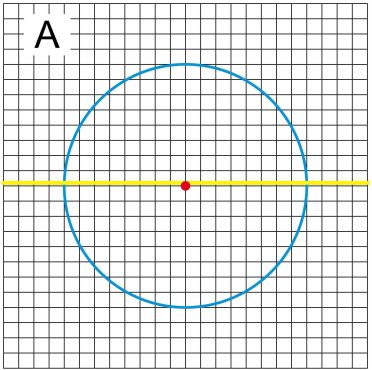


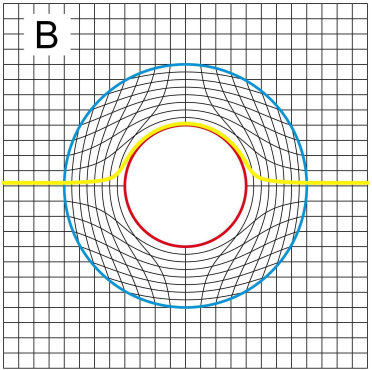
Euclidean invisible cloak

Transformation between virtual and physical space:

$(r, \theta, \varphi) \to (r', \theta, \varphi)$, where

$r'(0) = r_1, r'(1) = 1$, e.g. $r' = r_1 + (1 - r_1)r$





Euclidean invisible cloak

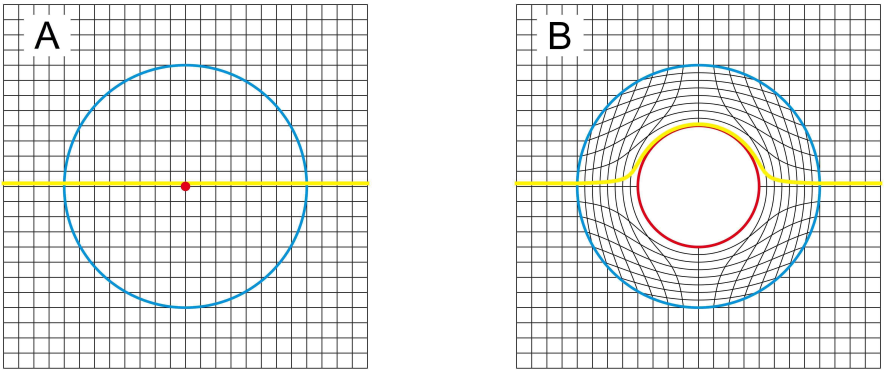
The resulting refractive index will be anisotropic (mapping is not conformal)

Remarkably, the optical transformation can be realised even for full electromagnetic field via permittivity and permeability tensors $\hat{\varepsilon}$ and $\hat{\mu}$

In general, it leads to impedance matching of the material to vacuum, i.e., $\hat{\varepsilon}_{\mathrm{r}} = \hat{\mu}_{\mathrm{r}}$

There are serious practical problems associated with this

Euclidean invisible cloak



Problem – speed of light goes to infinity as we approach the yellow circle

This is in principle allowed (even by relativity), but only for a single frequency $\Rightarrow$ the cloak would be extremely narrowband

Cloak based on conformal mapping

Optical Conformal Mapping

Ulf Leonhardt

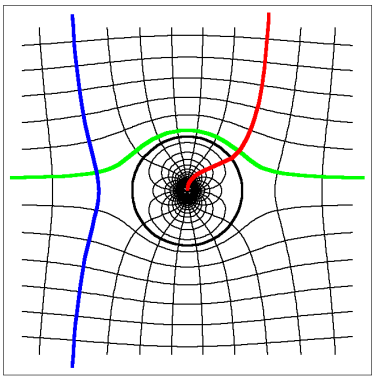
An invisibility device should guide light around an object as if nothing were there, regardless of where the light comes from. Ideal invisibility devices are impossible, owing to the wave nature of light. This study develops a general recipe for the design of media that create perfect invisibility within the accuracy of geometrical optics. The imperfections of invisibility can be made arbitrarily small to hide objects that are much larger than the wavelength. With the use of modern metamaterials, practical demonstrations of such devices may be possible. The method developed here can also be applied to escape detection by other electromagnetic waves or sound.

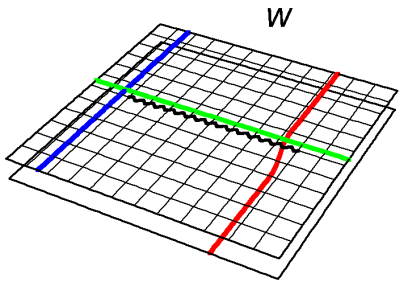
According to Fermat's principle (*1*), light rays take the shortest optical paths in dielectric media, where the refractive index n integrated along the ray trajectory defines the path length. When n is spatially varying, the shortest optical paths are not straight lines, but are curved. This light bending is the cause of many optical illusions. Imagine a situation where a medium guides light around a hole in it. Suppose that all parallel bundles of incident rays are bent around the hole and recombined in precisely the same direction as they entered the medium. An

Zhukowski mapping

$$w = z + \frac{1}{z}, \qquad z = \frac{w}{2} \pm \sqrt{\frac{w^2}{4} - 1}$$

z





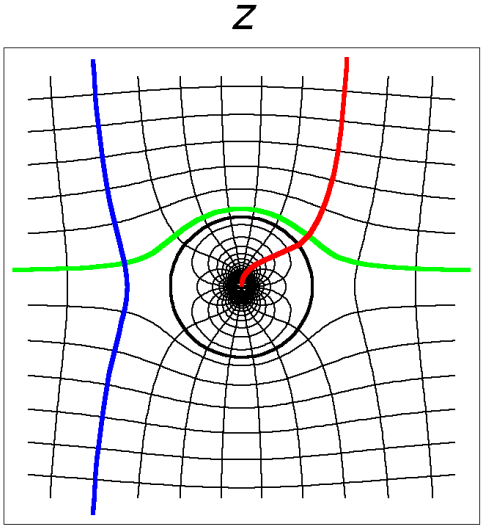
Zhukowski mapping

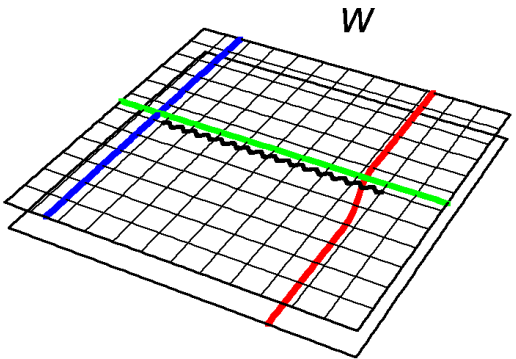
The mapping maps two points, z and $1/z$, to a single point w

For $|z| \gg 1$, $w = z + \frac{1}{z} \approx z$

For $|z| \ll 1$, $w = z + \frac{1}{z} \approx \frac{1}{z}$

Mapping from w to z is double-valued, the structure of Riemann sheets is very useful



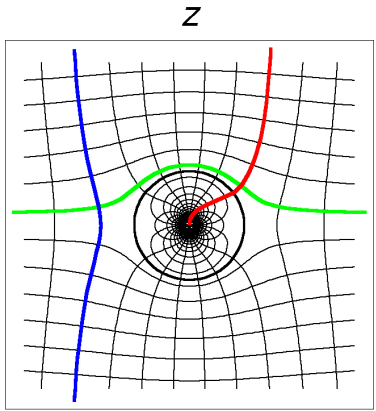


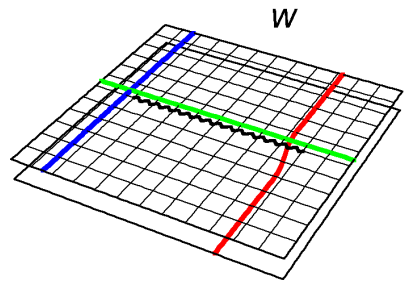
Upper sheet of w is mapped to the outside of the disk of z, lower sheet is mapped to the disk

Rays entering the lower Riemann sheet are lost

Can we bring them back?

Yes, by a perfect lens profile!





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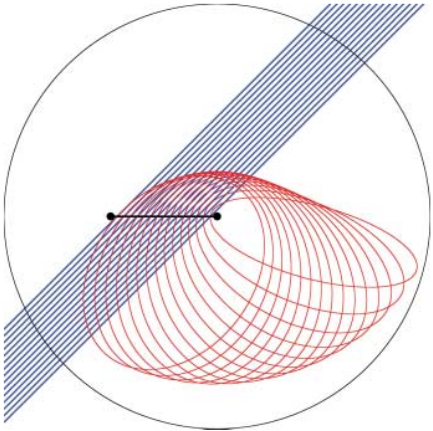
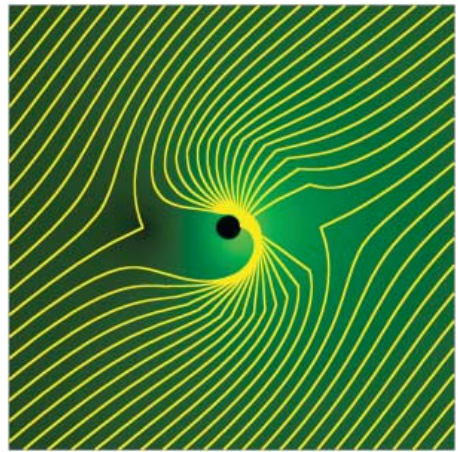




Problems

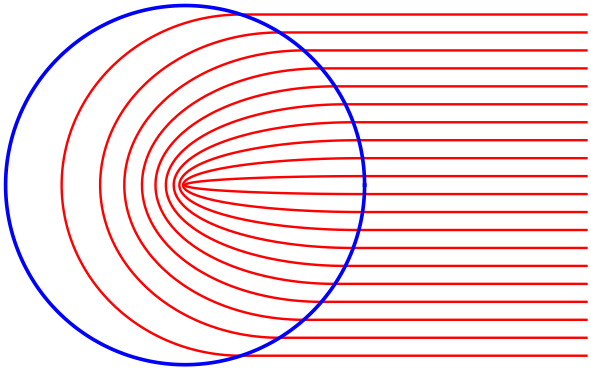
There are still problems with this invisible cloak:

- The refractive index on the lower Riemann sheet goes to zero at the circle – still infinite speed of light (even in physical space)
- There are also points where index goes to infinity due to the conformal mapping

Can we solve these problems?

Transmutation of singularities

By a suitable radial transformation, the infinity of refractive index can be elliminated



T. Tyc, U. Leonhardt, New J. Phys. 10, 115038 (2008)

Y. G. Ma, C. K. Ong, T. Tyc, U. Leonhardt, Nature Materials 8, 639 (2009)

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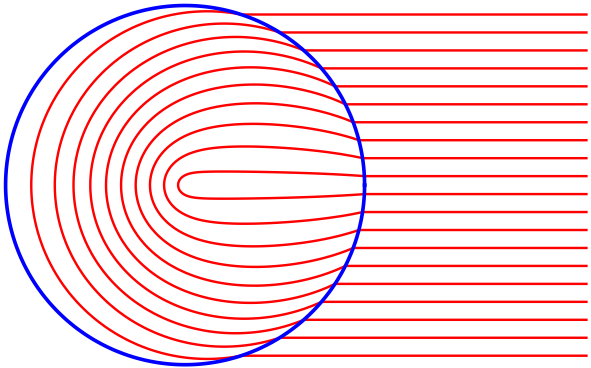

T. Tyc, U. Leonhardt, New J. Phys. 10, 115038 (2008)

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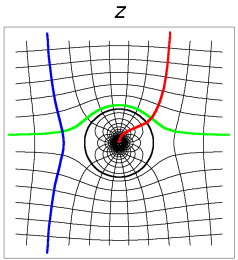
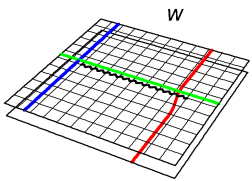
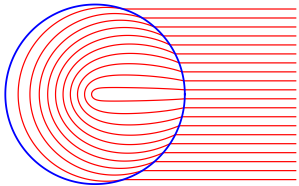
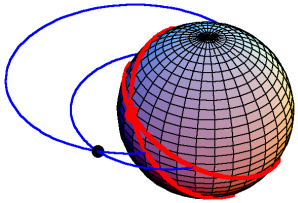
Non-Euclidean invisible cloak



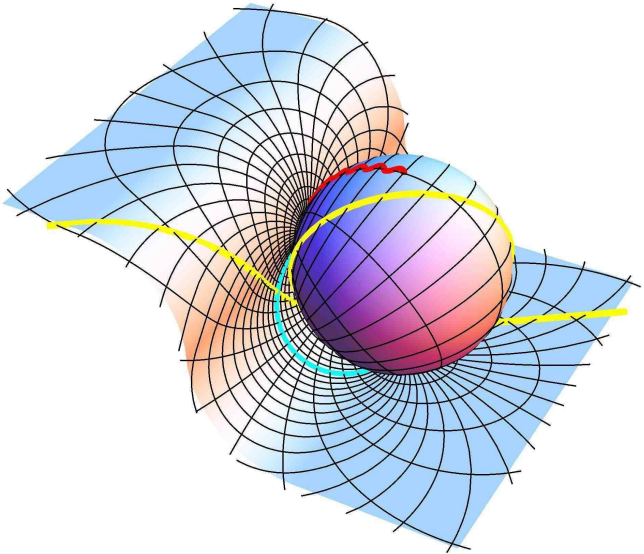




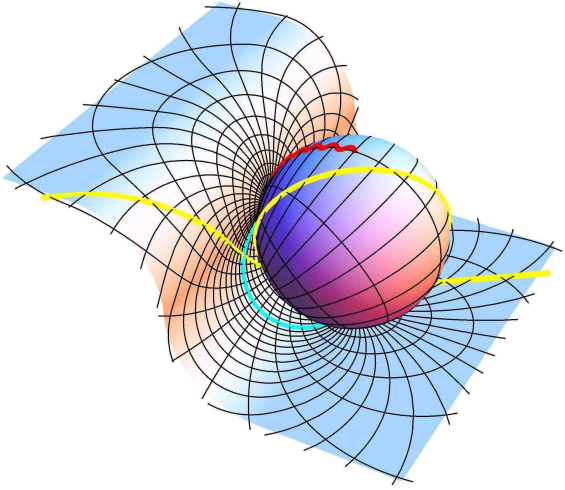


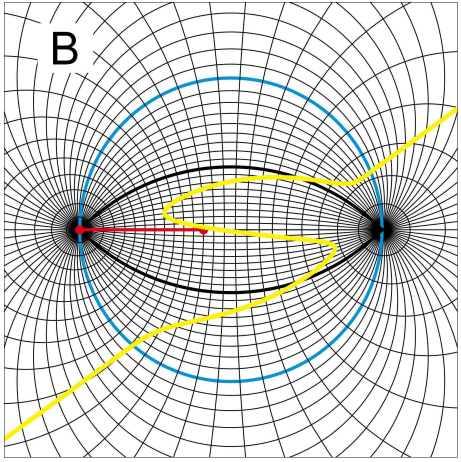
U. Leonhardt and T. Tyc, Science 323, 110 (2009)

Virtual space of the cloak



Virtual and physical space





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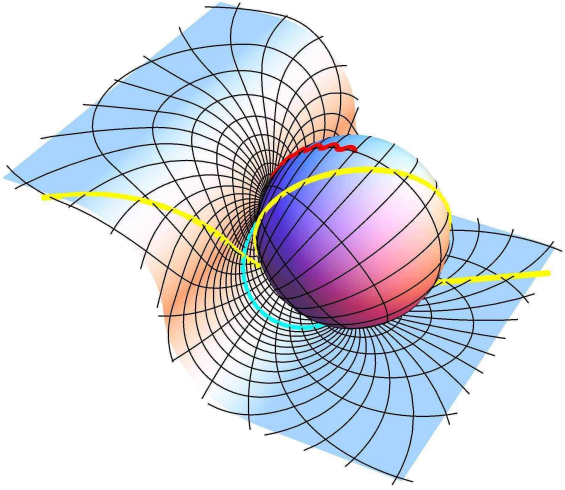
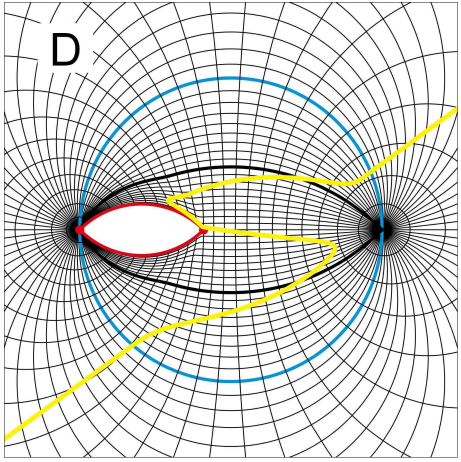
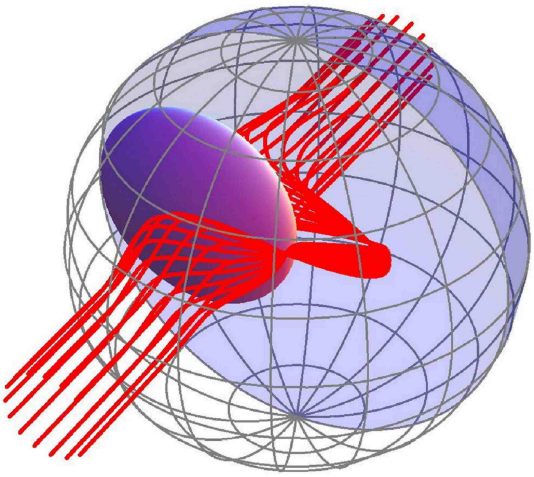
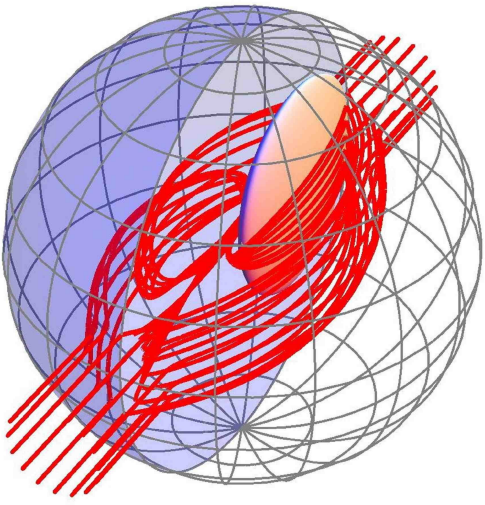




Ray trajectories in 3D





Still a problem...

Although phase velocity is nowhere infinite, it becomes superluminal at some places

Can there be a strictly sub-luminal cloak?

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Can there be a strictly sub-luminal cloak?

Yes!

Subluminal non-Euclidean cloak

New Journal of Physics
The open-access journal for physics

Invisibility cloaking without superluminal propagation

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[2] Faculty of Science, Kotlarska 2 and Faculty of Informatics, Botanicka 68a, Masaryk University, 61137 Brno, Czech Republic

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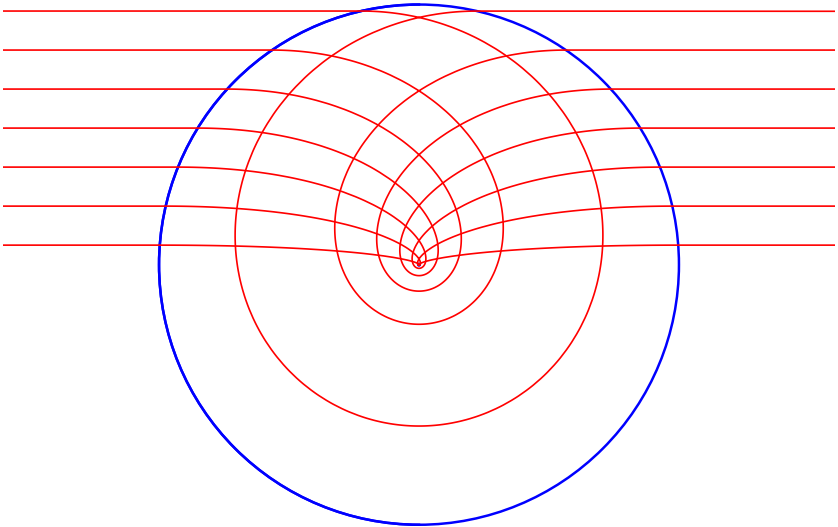
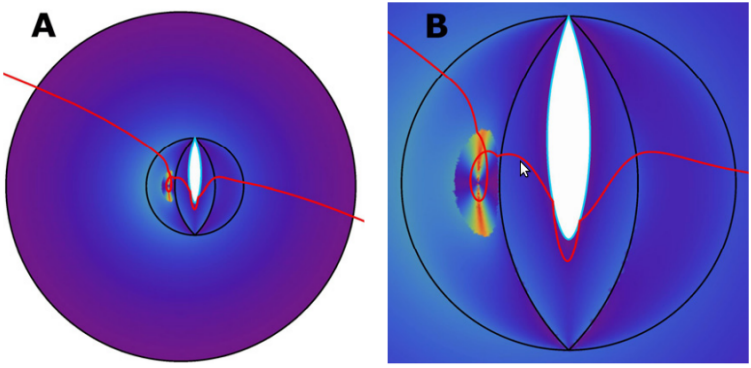
Invisible lens

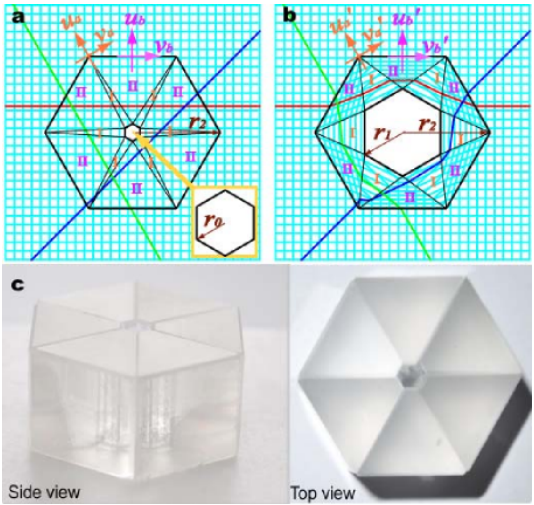


Subluminal non-Euclidean cloak

We combine non-Euclidean cloak with the invisible lens, speed of light $< c$ everywhere

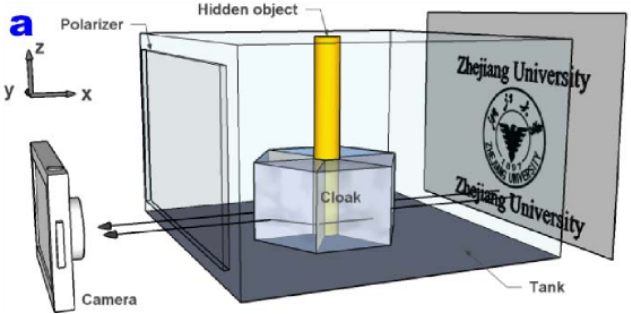


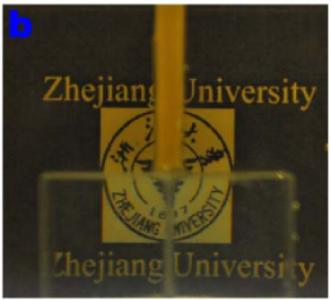
Another example of invisible cloak



H. Chen, B. Zheng, Scientific Reports 2, 255 (2012)

Mnohoúhelníkový plášť





Disadvantages of invisible cloaks

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Very limited use

Disadvantages of invisible cloaks

How many elements would there be in 1 m^3 of the cloak material?

One element: $\frac{1}{5000}$ mm

I.e., 5 000 000 elements per metre

I.e., $5\,000\,000^3 = 125\,000\,000\,000\,000\,000\,000 = 1{,}25 \,.\, 10^{20}$ elements per cubic metre

There have passed cca. 15 billions years since Big Bang $\approx 4.4 \times 10^{17}$ seconds

Manufacturing 300 elements every second since Big Bang till today