1. Let $s \in (-l/2, l/2)$ be a length parameter on the rigidly rotating string with $s = 0$ chosen to be the fixed center of the string. Let $\epsilon(s)$ denote the energy per unit length as a function of s

   i. Show that $\epsilon(s) = T_0/\sqrt{1-(2s/l)^2}$. Plot $\epsilon(s)$ as a function of s. Note that $\epsilon(s)$ has integrable singularities at the string endpoints, and confirm that the total energy is $\frac{\pi}{2} l T_0$.

   ii. For what points on the string is the local energy density equal to the average energy density?

   iii. Calculate the energy $E(s)$ carried by the string on the interval $[-s, s]$. For what value of $2s/l$ is $E(s)$ half of the total energy? 90% of the energy?

   iv. Calculate the total space-time momentum and angular momentum of the string. Is there a relation between the angular momentum and the mass square of the string?

2. Show that the action for a relativistic point particle

$$S = m \int dt \sqrt{-X^\mu(t) X_\mu(t)}$$

is invariant under infinitesimal reparametrizations of the world line parameter t.

3. The quantum Virasoro algebra is given by

$$[L_m, L_n] = (m-n)L_{m+n} + A(m)\delta_{m+n}$$

where $A(m)$ is a function of m that we did not explicitly calculate in the lectures. Use the constraints on $A(m)$ that can be obtained from the requirement that the Virasoro algebra should be a Lie algebra to calculate $A(m)$. A Lie algebra satisfies

$$[A, B] = -[B, A] \text{ antisymmetry}$$

$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0 \text{ Jacobi}$$

   i. What does the antisymmetry of the bracket tell you about $A(m)$? What is $A(0)$?

ii. Consider now the Jacobi identity for generators L_m, L_n and L_k with $m+n+k=0$. Show that

$$(m-n)A(k) + (n-k)A(m) + (k-m)A(n) = 0$$

iii. Use this to show that $A(m) = am$ and $A(m) = bm^3$, for constants a and b, are solutions.

iv. Consider the solution of the Jacobi identity with $k=1$. Show that knowledge of $A(1)$ and $A(2)$ determine all $A(m)$.

4. Consider two infintely long $D1$-branes stretched on the (x^2, x^3) plane. The first brane is defined by $x^3 = 0$, and the second brane is at an angle γ measured counterclockwise from the x^2 axis. Let the open string coordinates be $X^2(\tau,\sigma)$, and $X^3(\tau,\sigma)$, and consider only open strings which begin on the first brane and end on the second brane. Determine the boundary conditions satisfied by X^2 and X^3 at $\sigma = 0$ and $\sigma = \pi$.