# Exam problems for the course
# Thermodynamics and Statistical physics
**spring semester 2006.**

This is the exam for the course in "Thermodynamics and Statistical physics" given at Masaryk University at the spring semester 2006. It makes up the first part of the course requirements, the second part being an oral exam. The answers to the problems can be written in English or Czech. **Do not leave out any part of the calculation! Motivate your assumptions and approximations carefully**. As a *minimum* requirement to pass the course I have 12 points. Good luck!

1. A system of ideal gas with constant specific heat $c_V = \frac{3}{2}Nk$ is taken from volume $V_0$ to volume $V_1$ through a process where the pressure varies according to a function $P = f(V)$. During this process the work performed on the system can be found as an integral

   $$W = -\int_{V_0}^{V_1} f(V)dV.$$

   Find an expression for the heat transferred to the system given as an integral over $V$. (**4p**)

2. In a magnetic system the first law of thermodynamics can be written $dE = TdS + HdM$ where $H$ is the external magnetic field and $M$ is the total magnetic moment induced. Given a material with constant specific heat $C_H$ and equation of state $M(T,H) = \chi H$ $\chi$ constant, find expressions for the total energy and entropy of the material as functions of $T$ and $H$. (Your answer will contain two unknown constants.) (**4p**)

3. An ideal monatomic gas of $N$ particles, each of mass $m$, is in thermal equilibrium at absolute temperature $T$. The gas is contained in a cubical box of side $L$, whose top and bottom sides are parallel to the earth's surface. The effect of the earth's uniform gravitational field on the particles should be considered, the acceleration due to gravity being $g$.

   (a) What is the average kinetic energy of a particle?

   (b) What is the average potential energy of a particle?

   (**4p**)

4. A homogeneous substance consisting of $N$ particles at temperature $T$ and pressure $p$ has a specific heat (measured at constant pressure) given by $C_p$. Its coefficient of volume expansion

   $$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

   is known as a function of temperature. Calculate how $C_p$ depends on the pressure at a given temperature; i.e., calculate

   $$\left(\frac{\partial C_p}{\partial P}\right)_T$$

   expressing the result in terms of $T$, $V$, $\alpha$ and derivatives of $\alpha$. (**4p**)

5. The equation of state of a gas can be written in the form

$$P = \frac{N}{V}kT\left(1 + B_2(T)\frac{N}{V}\right)$$

where $B_2(T)$ is an *increasing* function of the temperature. Find how the mean internal energy $E$ of the gas depends on its volume $V$, i.e., find an expression for $(\partial E/\partial V)_T$. Is it positive or negative? (**4p**)

6. Use that for an ideal gas we know that $E = \frac{3}{2}NkT$ to find an expression for the density of states $\Gamma(E)$. (Your answer will contain an unknown constant). (**4p**)

Useful facts:
Basic assumption: in an isolated system in equilibrium, all states are equally probable.
For a system in contact with a heat reservoir at temperature $T$, the probability to find a state with energy $E$ is proportional to $e^{-E/kT}$
Equation of state of an ideal gas: $PV = NkT$
First law of thermodynamics dE = TdS-PdV

$$\int dx x e^{-x} = -(1+x)e^{-x}$$

$$\int \ln x = x\ln x - x$$

$$\int_{-\infty}^{\infty} dx e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$$