Exam problems for the course
Thermodynamics and Statistical physics
spring semester 2007.

This is the exam for the course in "Thermodynamics and Statistical physics" given at Masaryk University at the spring semester 2007. It makes up the first part of the course requirements, the second part being an oral exam. The answers to the problems can be written in English or Czech. **Do not leave out any part of the calculation! Motivate your assumptions and approximations carefully**. As a *minimum* requirement to pass the course I have 12 points. Good luck!

1. The heat of fusion of ice at 1 atm and $0°C$ is 333 kJ/kg and the heat of vaporization at 1 atm and $100°C$ is 2260 kJ/kg. Assuming an average heat capacity at constant pressure 1 atm of 4.19 kJ/kg/K between $0°C$ and $100°C$ for water, calculate the heat needed to take 1 kg of ice at $0°C$ to 1 kg of steam at $100°C$ and the increase of entropy in this process. (**4p**)

2. In a magnetic system the first law of thermodynamics can be written as $dE = TdS + HdM$ where H is the external magnetic field and M is the total magnetic moment induced. The equation of state of the system is $M(T, H)$. If the specific heat C_H happens to be *independent* of the external magnetic field H, what does that imply for dependence of M on T in the equation of state? (Hint: find an expression for $\frac{\partial^2 M}{\partial T^2}$) (**4p**)

3. Imagine a system of ideal gas kept at constant temperature T in a box with sides of length L. In such a system each particle moves independently of all the other particles. If there is also an external gravitational field each particle will carry a total energy

$$E = \frac{\mathbf{p}^2}{2m} + mgz$$

   To simplify things, imagine a system with only one particle. What is the probability to find the particle in the lower half of the box? (**4p**)

4. Find the increase in entropy for an ideal gas whose volume increases to twice its initial volume V in each of the following cases

   (a) When the resistance against expansion is removed suddenly so that the gas does not have to perform any work during the expansion but there is no heat added (so called free expansion).

   (b) When the expansion is done slowly and no heat is added.

   (c) When the expansion is done slowly but heat is added to keep the temperature constant all the time.

   (**4p**)

5. In a temperature range near absolute temperature T, the tension force F of a stretched plastic rod is related to its length L by the expression

$$F = aT^2 (L - L_0)$$

where a and L_0 are positive constants, L_0 being the unstretched length of the rod. When $L = L_0$, the heat capacity C_L of the rod (measured at constant L) is given by the relation $C_L = bT$, where b is a positive constant.

(a) Give a general expression for the work performed on the rod when stretching the rod an infinitesimal bit dL. What are the complementary thermodynamical variables? Write down the first law of thermodynamics for this system.

(b) Calculate the entropy $S(L, T)$ of the rod as a function of T and L.

(c) If one starts at $T = T_0$ and $L = L_0$ and stretches the thermally insulated rod quasi-statically until it attains length L_f, what is the final temperature T_f? Is T_f larger or smaller than T_i?

(**4p**)

6. For an ideal gas of Fermions at temperatures close to zero the equation of state takes the form

$$P = \left(3\pi^2\right)^{\frac{2}{3}} \frac{\hbar^2}{5m} \left(\frac{N}{V}\right)^{\frac{5}{3}} .$$

What is the chemical potential of this system as a function of T, V and N? Your answer will contain an unknown function of N and T. (**4p**)

Useful facts:
Basic assumption: for an isolated system in equilibrium, all states are equally probable.
For a system in contact with a heat reservoir at temperature T, the probability to find a state with energy E is proportional to $e^{-E/kT}$
Equation of state of an ideal gas: $PV = NkT$
First law of thermodynamics $dE = TdS - PdV + \mu dN$