Dvou-tekutinový model plazmatu

Rovnice kontinuity

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = S_e$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = S_i$$

Hustota náboje
Hustota proudu

$$\rho = e(n_i - n_e)$$

$$\mathbf{j} = e(n_i \mathbf{u}_i - n_e \mathbf{u}_e)$$

Rovnice pro přenos hybnosti

$$m_e n_e \left[\frac{\partial}{\partial t}\mathbf{u}_e + (\mathbf{u}_e \cdot \nabla)\mathbf{u}_e\right] = -n_e e(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \nabla p_e + m_e n_e \mathbf{g} - m_e \nu_{ei}(\mathbf{u}_e - \mathbf{u}_i) - m_e \mathbf{u}_e S_e$$

$$m_i n_i \left[\frac{\partial}{\partial t}\mathbf{u}_i + (\mathbf{u}_i \cdot \nabla)\mathbf{u}_i\right] = +n_i e(\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) - \nabla p_i + m_i n_i \mathbf{g} - m_i \nu_{ie}(\mathbf{u}_i - \mathbf{u}_e) - m_i \mathbf{u}_i S_i$$

ZZH: $m_e \nu_{ei}(\mathbf{u}_e - \mathbf{u}_i) + m_i \nu_{ie}(\mathbf{u}_i - \mathbf{u}_e) = 0$

Stavové rovnice
adiabatická || izotermická

$$p_\alpha \rho_{m\alpha}^{-\gamma} = \text{const.}$$

$$p_\alpha = n_\alpha k T_\alpha, \quad T_\alpha = const.$$

Maxwellovy rovnice

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

Magnetohydrostatika

$$\mathbf{u}_e \cdot \nabla \mathbf{u}_e \approx 0$$

$$n_e = n_i = n$$

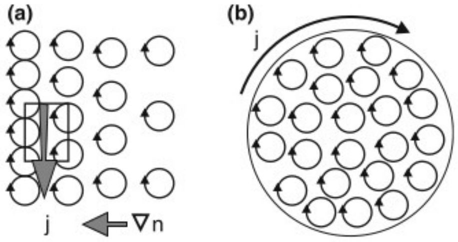
$$\mathbf{v}_m = \frac{m_e \mathbf{u}_e + m_i \mathbf{u}_i}{m_e + m_i}$$

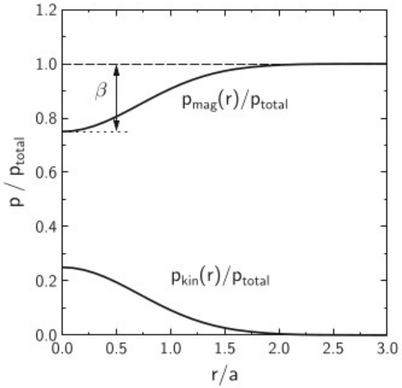
$$\rho_m = n(m_e + m_i)$$

$$p = p_e + p_i$$

$$\rho_m \frac{\partial \mathbf{v}_m}{\partial t} = \mathbf{j} \times \mathbf{B} - \nabla p + \rho_m \mathbf{g}$$

Diamagnetický drift





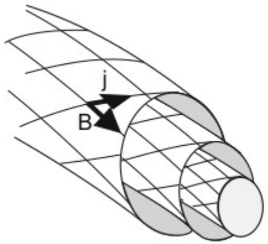
$$p_{\text{kin}} + p_{\text{mag}} = p_{\text{total}} = \text{const}$$

$$\mathbf{u}_\perp = \frac{\mathbf{E} \times \mathbf{B}}{B^2} - \frac{\nabla p \times \mathbf{B}}{qnB^2} = \mathbf{v}_E + \mathbf{v}_D$$

Statická rovnováha magnetizovaného plazmatu:

Pro $\frac{\partial \mathbf{v}_m}{\partial t} = 0$ → $$0 = \mathbf{j} \times \mathbf{B} - \nabla p + \rho_m \mathbf{g}$$

$$p_{\text{mag}} = \frac{B^2}{2\mu_0}$$



$$\nabla(p + p_{\text{mag}}) = \frac{(\mathbf{B} \cdot \nabla)\mathbf{B}}{\mu_0}$$

Zakřivení indukčních čar

Síla na jednotku objemu v EM poli:

$$\mathbf{f} = \varepsilon_0 \left[(\nabla \cdot \mathbf{E})\mathbf{E} + (\mathbf{E} \cdot \nabla)\mathbf{E}\right] + \frac{1}{\mu_0}\left[(\nabla \cdot \mathbf{B})\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{B}\right] - \frac{1}{2}\nabla\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \varepsilon_0 \frac{\partial}{\partial t}(\mathbf{E} \times \mathbf{B}) .$$

Magnetohydrodynamika

$$m_e\mathbf{u}_i + m_i\mathbf{u}_e = m_i\mathbf{u}_i + m_e\mathbf{u}_e + m_i(\mathbf{u}_e - \mathbf{u}_i) + m_e(\mathbf{u}_i - \mathbf{u}_e) = \frac{1}{n}\rho_m\mathbf{v}_m - (m_i - m_e)\frac{1}{ne}\mathbf{j}$$

$$\mathbf{j} = e(n_i\mathbf{u}_i - n_e\mathbf{u}_e)$$

$$nm_im_e\frac{\partial}{\partial t}(\mathbf{u}_i - \mathbf{u}_e) = ne(m_e + m_i)\mathbf{E} + ne(m_e\mathbf{u}_i + m_i\mathbf{u}_e)\times\mathbf{B} - m_e\nabla p_i + m_i\nabla p_e + n(m_e + m_i)\nu_{ei}m_e(\mathbf{u}_e - \mathbf{u}_i)$$

$$\frac{m_im_e}{e}\frac{\partial\mathbf{j}}{\partial t} = e\rho_m\left(\mathbf{E} + \mathbf{v}_m\times\mathbf{B} - \frac{\nu_{ei}m_e}{ne^2}\mathbf{j}\right) - m_i\mathbf{j}\times\mathbf{B} - m_e\nabla p_i + m_i\nabla p_e$$

Ohmův zákon $\partial\mathbf{j}/\partial t = 0$

$$\mathbf{E} + \mathbf{v}_m\times\mathbf{B} = \eta\mathbf{j} + \frac{1}{ne}(\mathbf{j}\times\mathbf{B} - \nabla p_e)$$

Specifický odpor plazmatu $\eta = \nu_{ei}m_e/ne^2$

Difuze magnetického pole

$$-\frac{\partial\mathbf{B}}{\partial t} + D_B\Delta\mathbf{B} = 0$$

$$D_B = \eta/\mu_0$$

$$\mathbf{B}(t) \propto \exp(-t/\tau_B)$$

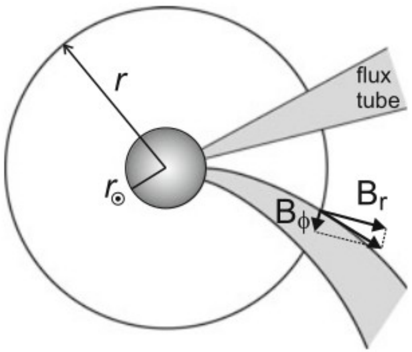
$$\tau_B = \frac{\mu_0\ell^2}{\eta}$$

Zamrznutí **B** pole

$\eta = 0$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}_m\times\mathbf{B})$$

$$\frac{d}{dt}\left(\frac{\mathbf{B}}{\rho_m}\right) = \left(\frac{\mathbf{B}}{\rho_m}\cdot\nabla\right)\mathbf{v}_m$$



Drift-difuzní aproximace bez B pole

Srážky s neutrálním plynem

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0$$

$$0 \approx m_\alpha n_\alpha \left[\frac{\partial}{\partial t}\mathbf{u}_\alpha + (\mathbf{u}_\alpha \cdot \nabla)\mathbf{u}_\alpha \right] = n_\alpha q_\alpha \mathbf{E} - \nabla p_\alpha - m_\alpha \nu_\alpha \mathbf{u}_\alpha$$

$$\boldsymbol{\Gamma}_\alpha = n_\alpha \mathbf{u}_\alpha = \mu_\alpha n_\alpha \mathbf{E} - D_\alpha \nabla n_\alpha$$

Difuzní koeficient $D_\alpha = \frac{kT_\alpha}{m_\alpha \nu_\alpha}$

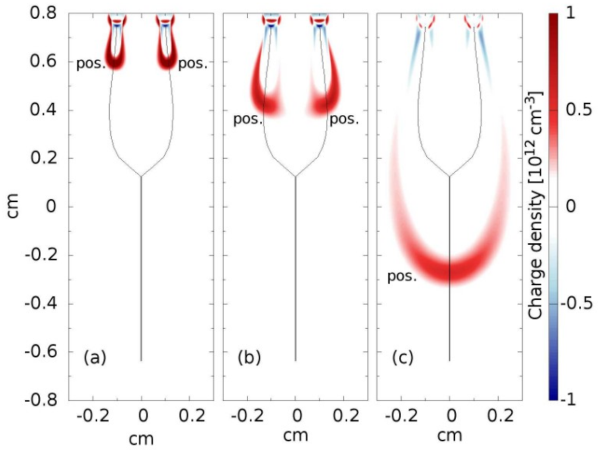
Pohyblivost $\mu_\alpha = \frac{q_\alpha}{m_\alpha \nu_\alpha}$

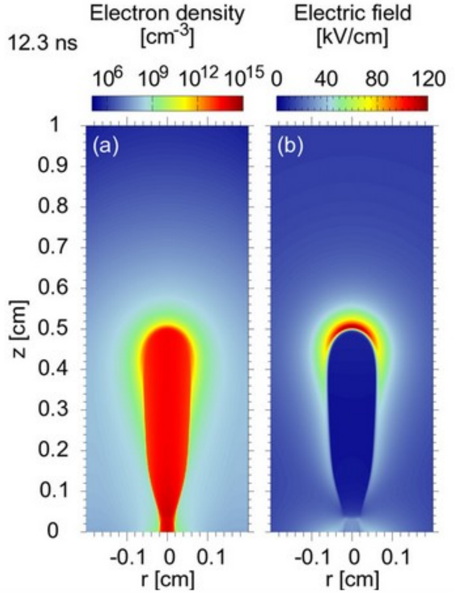
Einsteinova ralace: $\mu = |q| D / KT$

Šíření Ionizačních vln ve vzduchu:

$$\left.\begin{aligned}
\partial_t n_e + \nabla \cdot (n_e \mathbf{v}_e) - \nabla \cdot (D_e \nabla n_e) &= n_e \alpha |\mathbf{v}_e| - n_e \eta |\mathbf{v}_e| - n_e n_p \beta_{ep} + n_n \gamma + S_{ph}, \\
\partial_t n_p + \nabla \cdot (n_p \mathbf{v}_p) - \nabla \cdot (D_p \nabla n_p) &= n_e \alpha |\mathbf{v}_e| - n_e n_p \beta_{ep} - n_n n_p \beta_{np} + S_{ph}, \\
\partial_t n_n + \nabla \cdot (n_n \mathbf{v}_n) - \nabla \cdot (D_n \nabla n_n) &= n_e \eta |\mathbf{v}_e| - n_n n_p \beta_{np} - n_n \gamma,
\end{aligned}\right\}$$

$$\varepsilon_0 \nabla \cdot \mathbf{E} = -q_e (n_p - n_n - n_e), \quad \mathbf{E} = -\nabla V,$$





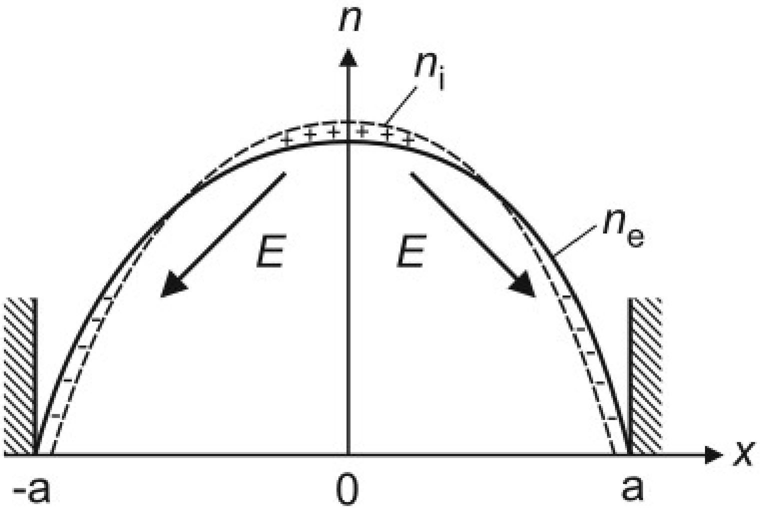
Ambipolární difúze

$$n_e = n_i = n.$$

$$\boldsymbol{\Gamma}_e = +n_e \mu_e \mathbf{E} - D_e \nabla n_e$$

$$\boldsymbol{\Gamma}_i = -n_i \mu_i \mathbf{E} - D_i \nabla n_i$$

$$\Gamma_e = \Gamma_i = \Gamma_a$$



$$D_i < D_a < D_e$$

$$\Gamma_a = -D_a \frac{dn}{dx}$$

$$D_a = \frac{D_i \mu_e + D_e \mu_i}{\mu_e + \mu_i} \approx D_i \left(1 + \frac{Te}{T_i}\right)$$

Ambipolární elektrické pole:

$$E(x) = -\frac{D_e - D_i}{\mu_e + \mu_i} \frac{1}{n} \frac{dn}{dx}$$

Řešení difuzní rovnice

$$\partial n/\partial t = D_a \nabla^2 n$$

$$n(\mathbf{r},t) = T(t)S(\mathbf{r})$$

Časová závislost

$$\frac{1}{T}\frac{dT}{dt} = \frac{D}{S}\nabla^2 S = -1/\tau$$

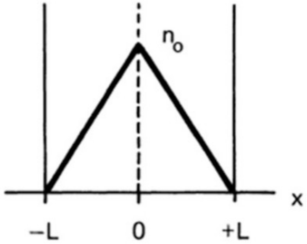
$$\frac{dT}{dt} = -\frac{T}{\tau}$$

$$T = T_0 e^{-t/\tau}$$

1D rovinná geometrie, rovnovážný stav se zdrojem v ose symetrie

$$\frac{d^2n}{dx^2} = -\frac{Q}{D}\delta(0)$$

$$n = n_0\left(1 - \frac{|x|}{L}\right)$$



1D rovinná geometrie, rovnovážný stav, zdroj v objemu.

$$\frac{\partial^2 n}{\partial x^2} + \frac{\nu_{ion}}{D_a} n = 0$$

$$\nu_{ion} = D_a \frac{\pi^2}{4a^2}$$

$$n(x) = n_0 \cos\left(\sqrt{\frac{\nu_{ion}}{D_a}}\, x\right)$$

$$\nabla^2 S = -\frac{1}{D\tau} S$$

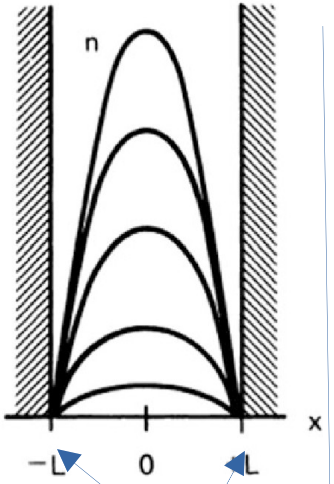
1D rovinná geometrie

$$\frac{d^2S}{dx^2} = -\frac{1}{D\tau} S$$

$$S = A\ \cos\frac{x}{(D\tau)^{1/2}} + B\ \sin\frac{x}{(D\tau)^{1/2}}$$

Základní difuzní mód

$$n = n_0 e^{-t/\tau} \cos\frac{\pi x}{2L}$$

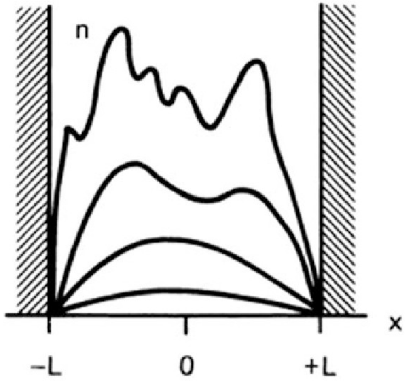


BC

$$\frac{L}{(D\tau)^{1/2}} = \frac{\pi}{2}$$

$$\tau = \left(\frac{2L}{\pi}\right)^2 \frac{1}{D}$$

1D rovinná geometrie



Počáteční rozdělení:

$$n = n_0\left(\sum_l a_l \cos\frac{(l+\frac{1}{2})\pi x}{L} + \sum_m b_m \sin\frac{m\pi x}{L}\right)$$

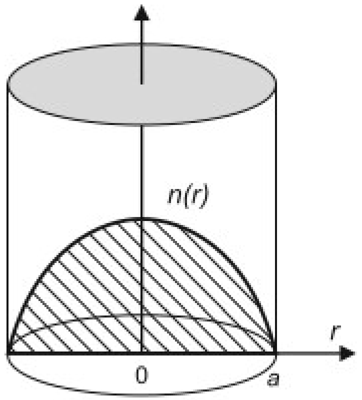
$$\tau_l = \left[\frac{L}{(l+\frac{1}{2})\pi}\right]^2 \frac{1}{D}$$

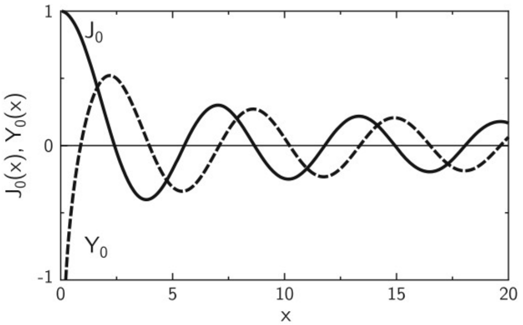
(Vyšší módy difundují rychleji)

$$n = n_0\left(\sum_l a_l e^{-t/\tau_l} \cos\frac{(l+\frac{1}{2})\pi x}{L} + \sum_m b_m e^{-t/\tau_m} \sin\frac{m\pi x}{L}\right)$$

Difuzní rovnice ve válcové geometrii

$$\frac{d^2S}{dr^2} + \frac{1}{r}\frac{dS}{dr} + \frac{1}{D\tau}S = 0$$

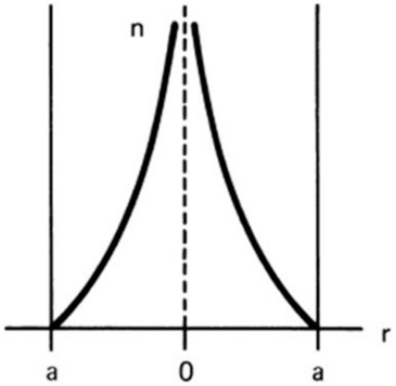




Stacionární řešení, zdroj na ose:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = 0$$

$$n = n_0 \ln(a/r)$$



Stacionární řešení pro konstantní ν_{ion}

Dostaneme rovnici

$$\frac{\partial^2 n}{\partial r^2} + \frac{1}{r}\frac{\partial n}{\partial r} + \frac{\nu_{ion}}{D_a}n = 0,$$

která je speciálním případem (pro $m = 0$) Besselovy rovnice (vyjde jako jedna z rovnic ze separace proměnných v Laplaceově rovnici ve válcových souřadnicích)

$$r^2 n'' + rn' + (k^2r^2 - m^2)n = 0.$$

Jejím řešením je kombinace Besselových funkcí $J_m(kr)$ a $Y_m(kr)$:

$$n(r) = AJ_m(kr) + BY_m(kr).$$

$Y_0(r)$ je záporná a singulární pro $r = 0$, řešením tedy bude $AJ_0(kr)$:

$$n(r) = n_0 J_0\left(\sqrt{\frac{\nu_{ion}}{D_a}}r\right).$$

Stabilní řešení bude existovat při rovnováze mezi difuzními ztrátami a ionizací: argument funkce J_0 musí být takový aby první kořen byl pro $r = a$:

$$J_0\left(\sqrt{\frac{\nu_{ion}}{D_a}}a\right) = J_0(2.405) = 0,$$

ν_{ion} a D_a jsou tedy vázány vztahem

$$\nu_{ion} = D_a\left(\frac{2.405}{a}\right)^2.$$